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Kinematics and Machine Design,

BY

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PREFACE

Recent developments in technical instruction have brought forward courses in machine design which are shorter and more condensed than those formerly offered. Students in these courses need to know the language of machine design, and the standard methods of designing the ordinary machine elements, as well as to be grounded in the more essential parts of kinematics. It is not essential that they know all the mathematical proofs lying behind and making valid the various equations they may have to use.

In order to furnish a text suited to short combined courses of this character the authors abridged and consolidated two texts which have proved successful over a period of years in giving fundamental training to college students in mechanical, electrical, and industrial engineering. They are "Kinematics of Machines" by G. L. Guillet and "Machine Design" by L. J. Bradford and P. B. Eaton. It is hoped that this volume will meet the requirements of a short combined course in these two subjects.

L. J. B.

G. L. G.

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August, 1943

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PART I

KINEMATICS OF MACHINES

CHAPTER I

GENERAL CONSIDERATIONS

1. Kinematics of Machines is that part of the subject of machine design which deals with motion of machine parts. The performance of useful work by a machine is dependent on the parts executing a definite cycle of motions. Hence kinematics is of prime importance in the design of machinery.

2. A Machine is a combination of parts of resistant materials having definite motions and capable of transmitting or transforming energy. A machine must always be supplied with energy from an external source. Its usefulness consists in its ability to alter the energy supplied so as to render it available for the accomplishment of a desired service.

A steam engine transforms the pressure energy of the steam into mechanical work which is delivered to the crankshaft. This machine, therefore, transforms one kind of energy into another.

A transmission gear may be used to connect two shafts which are required to rotate at different rates. Its function will then be to alter the mechanical work supplied to it, with regard to the speed of rotation and twisting moment, the result being to put the energy into a form more suitable for some specific purpose.

Resistant materials are those that do not easily become distorted or change their physical form when forces are applied.

3. A Mechanism is a combination of pieces of resistant materials whose parts have constrained relative motions. A machine is

composed of one or more mechanisms. When we speak of a mechanism, we think of a device that will produce certain mechanical movements and we set aside the question as to whether it is capable of doing useful work. A mechanism may or may not be able to transmit an appreciable amount of energy; a machine must do so. The latter is, therefore, a practical development of the former.

A working model of any machine, the works of a watch, and the moving parts of an engineering instrument are all termed mechanisms because in them the energy transmitted is very small, just enough to overcome friction, and the motions produced are the important consideration.

A structure is a combination of pieces of resistant material capable of carrying loads or transmitting forces, but having no relative motion of its parts.

4. A Link is a part of a machine or mechanism connecting other parts which have motion relative to it. A link may serve as a support, guide other links, transmit motion, or function in all three ways.

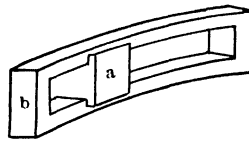
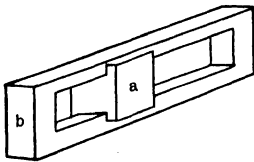


FIG. 1-1. Slider and straight guide. FIG. 1-2. Slider and curved guide.

Thus, in a steam engine, the connecting rod, crank, crosshead, and frame are links, since each functions in at least one of these ways. The common form of mechanism used in the steam engine is shown in Fig. 1-3. It has four links denoted by the letters *a*, *b*, *c*, *d*.

In connection with general mechanical work, the term "link" is applied to a slotted bar, as shown in Figs. 1-1 and 1-2, or to a section of an ordinary chain. In kinematics it is used in a more general way, as noted above.

Rigid links, such as the steam engine parts just mentioned, are capable of transmitting either a thrust or a pull. To this class belong most of the metal parts of machines. There are, however, many examples of flexible links, which in general are so constituted as to offer resistance in one manner only. Thus, tension links, such as ropes, belts, and chains, will transmit a pull but not a thrust, whereas compression or pressure links, such as the water in a hydraulic accumulator and pump system, or the oil in a hydraulic braking system on a car, are capable of carrying only a thrust.

When a number of links are connected to one another in such a way as to allow motion to take place in the combination, the result is called a kinematic chain. A kinematic chain is not necessarily a mechanism; it becomes one when so constructed as to allow constrained relative motion among its parts.

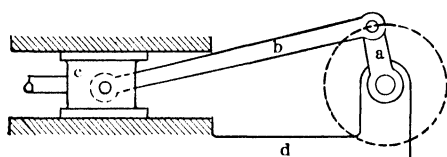
5. Motion can be defined as a change of position. Motion is always a relative term; that is, we cannot conceive of any motion of a body except by reference to another body. It is usual to regard the earth as a fixed body, and we therefore speak of the absolute motion of a body when we mean its motion relative to the earth. We apply the term relative motion to the movement of one body relative to another moving body, the earth again being considered as stationary. Strictly speaking, we have relative motion in both cases, and the above designations are only a conventional means of distinguishing them.

A body is said to have constrained motion when it is so guided by contact with other bodies, or by external forces, that any point on it is obliged to move in a definite path. Any link in a mechanism has constrained motion.

6. Inversion of a Mechanism. In any mechanism we have one link which is "fixed"; i.e., at rest relative to the earth, or to the body on which it is mounted. Exactly the same system of links may often be rendered suitable for a different purpose if the link originally fixed is allowed to move, while some other link is held stationary. Thus, in Fig. 1-3, the ordinary engine mechanism, *d*, is the fixed link. By fixing the crank *a* and allowing *d* to move, we obtain a device which is used as a quick-return motion

in certain machine tools. The latter mechanism is called an inversion of the former one.

It is important to note that the inversion of a mechanism **does**



not in any way alter the relative motion of the links which compose it.

In Fig. 1-3, for example, no matter which link is fixed, the motion of *a* relative to *d* is that

FIG. 1-3. Direct-acting engine mechanism.

of rotation, and *c* always slides in a straight line on *d*.

7. Classification of Motions. Most of the motions which occur in mechanisms fall into one of the following classes:

(a) Plane motion of a body is obtained when all points in it move in parallel or coincident planes. Examples are shown in Figs. 1-1, 1-2, 1-3.

Rectilinear motion is a form of plane motion in which all points in the body considered move in parallel straight lines. (See *c*, Fig. 1-3.)

(b) Helical motion is executed when a body rotates about an axis and at the same time moves parallel with the same axis, the two motions bearing a fixed ratio to each other. The motion of a nut on a threaded bolt is a very common example. Any point on a body with this form of motion describes a curve called a helix. Figure 1-4 shows the projection of a helical curve on a plane parallel with the axis of rotation.

The contact surfaces in helical motion are called a screw pair.

(c) Spherical motion is said to exist when a body is moving in such a way that any point in it remains at a constant distance from a fixed point, plane motion excluded. Any point on the body, therefore, moves on the surface of a sphere. Figure 1-5 shows a cross section of a tapered roller bearing. Generally the inner race *X* revolves with the shaft *Y*, and the outer race *Z* is stationary. Any point, such as *A*, on one of the rollers moves on the surface of a sphere *ALM*, whose center is at the point *O*; hence the rollers have spherical motion. Other examples are found in certain ball bearings, and in bevel gears, universal joints, etc.

8. The Displacement of a body is its change of position with reference to a fixed point. Both direction and distance are necessarily stated in order to define completely the displacement of a point or body.

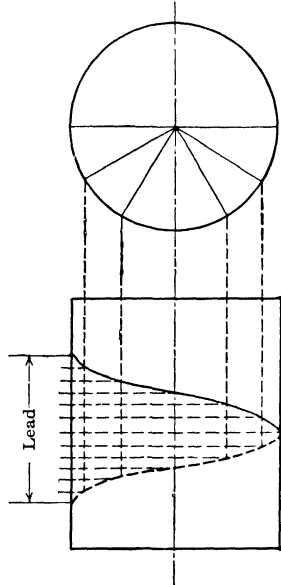


FIG. 1-4. Projection of a helix.

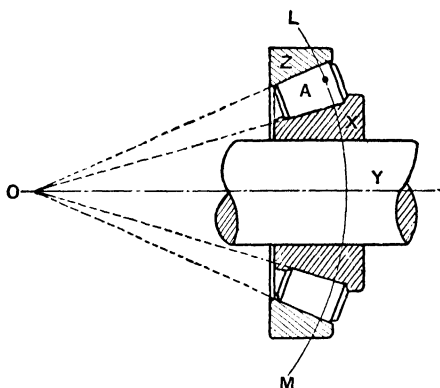


FIG. 1-5

9. Velocity is the rate of change of position or displacement of a body. A body may change its position by translation through space or by angular movement. Thus it may have **linear** or **angular** velocity.

Linear velocity is the rate of linear displacement of a point or body along its path of motion. It includes two factors, namely, speed and direction of motion. Linear velocity is often measured in feet per second, or in miles per hour, though other units are found more suitable in special cases. A linear velocity can always be represented graphically by a line, the direction showing the direction of movement and the length representing the magnitude of the velocity.

Angular velocity is the rate of change of angular position of a body or line. In order to state it completely, the sense of rotation should be given. Angular velocity is commonly measured by the angle turned through per unit of time. The angle may be expressed in radians, degrees, revolutions, etc. Thus, "radians per minute," "degrees per second," and "revolutions per minute" are often used as units.

10. Relation between Angular and Linear Velocity. Let B , Fig. 1-6, represent a point on a body which is rotating about O .

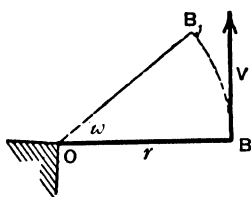


FIG. 1-6

O is therefore a fixed point, and B is moving, at the instant considered, in a direction perpendicular to the line OB , its linear velocity being V feet per second as indicated on the diagram.

Assuming that B continues to move at the same speed, after an interval of one second it will have reached the position B_1 , the arc BB_1 having a length equal to V , by definition. The angle ω turned through by line OB during the same interval is measured in radians by

$$\frac{\text{Arc } BB_1}{OB} = \frac{V}{r}$$

Therefore,

$$\omega = \frac{V}{r} \quad \text{or} \quad V = \omega r \quad (1-1)$$

where ω = angular velocity of the body in radians per unit of time. We may, therefore, state the following law.

The linear velocity of a point on a moving body is equal to the angular velocity of the body multiplied by the distance of the point from the center of rotation.

11. Acceleration is the rate of change of velocity with respect to time. Since velocity may be either angular or linear, we likewise have both angular and linear acceleration.

Linear acceleration is the rate of change of linear velocity. If a moving body or point has a linear velocity at a certain instant of

ac, and a second later the velocity has become 18 ft per sec. Thus, the acceleration for the interval is 10 ft per sec per sec. Note that, as calculated, this value is the average acceleration for the given one-second period. It does not tell us how the acceleration may have varied during the interval. More information would be necessary in order to determine the acceleration existing at a particular instant. Linear acceleration, like linear velocity, can always be represented graphically by a line which shows the magnitude and direction.

Angular acceleration is the rate of change of the angular velocity. When the angular velocity is measured in radians per second, our corresponding unit of angular acceleration will be "radians per second per second"; or, if in revolutions per minute, it may be either "revolutions per minute per minute" or "revolutions per minute per second."

12. Uniformly Accelerated Motion. When a particle or body starts from rest and is uniformly accelerated to velocity v in time t ,

$$\begin{aligned}s &= \text{average velocity} \times \text{time} \\ &= \frac{v + 0}{2} \times t = \frac{vt}{2}\end{aligned}\tag{1-2}$$

and

$$v = at\tag{1-3}$$

Therefore

$$s = \frac{1}{2}at^2\tag{1-4}$$

By elimination of t in equations (1-2) and (1-3) we have

$$v^2 = 2as\tag{1-5}$$

13. Normal and Tangential Acceleration. The velocity of a moving point may change in two ways: (a) its linear speed along its path may increase or decrease; or (b) the direction of its motion may change.

(a) The rate of change of speed in the direction of motion is the tangential acceleration, since this involves an acceleration acting along the path of motion.

(b) The change in the direction of the motion is normal or centripetal acceleration, which acts in a direction to the direction of the path of motion. We may then

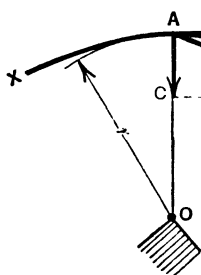


FIG. 1-7

along a line perpendicular to the path of motion and toward the center of curvature of this path. For example, suppose a point A (Fig. 1-7) is traveling along the curved path XY. O is the center of curvature of XY at the point A. Assume that the linear velocity of A is increasing. Then A at the instant considered has a tangential acceleration represented by the tangent AB to the path of motion.

Since point A is moving in a curve, it also has a normal acceleration toward the center of curvature, represented by the vector AC. For point A,

$$\text{Tangential acceleration} = \alpha \cdot r \quad (1-6)$$

$$\text{Normal acceleration} = \frac{V^2}{r} \quad \text{or} \quad \omega^2 \cdot r \quad (1-7)$$

where ω = angular velocity of the body on which point A is located and α = the angular acceleration of this body, both being measured in radian-second units.

Example. The flywheel for a metal shearing machine is 4 ft in diameter and rotates at a normal speed of 180 rpm. During the shearing period, which lasts 2 sec, the flywheel speed is reduced to a final value of 150 rpm. Assuming constant angular deceleration, determine the normal and tangential acceleration of a point on the rim at the instant when the speed is 160 rpm.

The angular velocity in radians per second corresponding to 160 rpm equals

$$\frac{160 \times 2\pi}{60} = 16.74$$

Tangential acceleration of a point on the rim equals

$$\omega^2 r = (16.74)^2 \times 2 = 562 \text{ ft per sec per sec}$$

If the wheel speed changes from 180 to 150 rpm in 2 sec, with constant acceleration, the value of this acceleration is

$$(150 - 180) \div 2 = -15 \text{ rpm per sec}$$

which equals

$$-\frac{15 \times 2\pi}{60} \quad \text{or} \quad -1.57 \text{ radians per sec per sec}$$

Therefore, the tangential acceleration of a point on the rim is

$$\alpha r = -1.57 \times 2 = -3.14 \text{ ft per sec per sec}$$

The negative sign indicates a deceleration.

14. Simple Harmonic Motion. When a point A (Fig. 1-8) moves in a circle with uniform velocity about a fixed point O , B ,

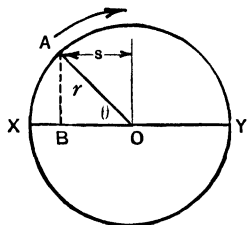


FIG. 1-8

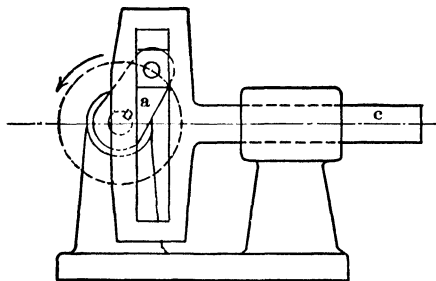


FIG. 1-9. Scotch yoke.

the projection of A on any diameter, such as XY , moves with simple harmonic motion.

The period is the time required for a complete revolution of the generating line OA (Fig. 1-8).

The amplitude is the distance between the points of motion, or twice the length of the generating radius

The phase is the angular position of the generating instant with respect to a reference line.

The mechanism shown in Fig. 1-9, commonly called yoke, gives simple harmonic motion to the link c when a rotates with uniform velocity.

15. Displacement, Velocity, and Acceleration for S.H.M.

pose OA (Fig. 1-8) starts from an initial position OX and revolves clockwise. Let ω equal its constant angular velocity. Then after a period of time t , the angle turned through will be $\omega \cdot t = \theta$, and the displacement of B from its mean position at O will be

$$s = OB = OA \cos \theta = r \cos \omega t \quad (1-8)$$

It can be shown that the velocity of B equals

$$v = -r\omega \sin \omega t \quad (1-9)$$

also, that the acceleration of B equals

$$a = -\omega^2 r \cos \omega t \quad (1-10)$$

Simple harmonic motion can therefore be defined as a motion in which the acceleration is directly proportional to the displacement, and acts in a direction toward the point of zero displacement.

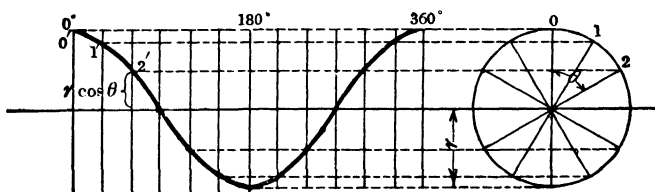


FIG. 1-10. Linear curve of displacement for body with S.H.M.

16. Linear Displacement Curve for S.H.M. In order to construct this curve, we take a base line representing 360° motion of the generating line (Fig. 1-10), and divide it into any number of equal spaces representing equal angles. Draw a circle whose radius is equal to that of the generating line, with center on the base line produced. Divide this circle into angles of the same value as the

base-line intervals. The zero position is at 90° to the base line, as indicated on the diagram. Project horizontally from points 0, 1, 2, etc., to points O' , $1'$, $2'$, etc. The latter are points on the

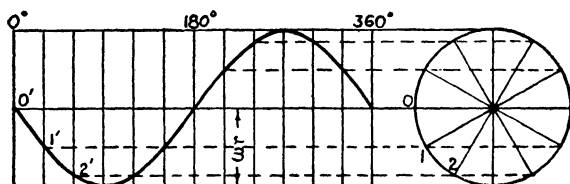


FIG. 1-11. Linear curve of velocity for body with S.H.M.

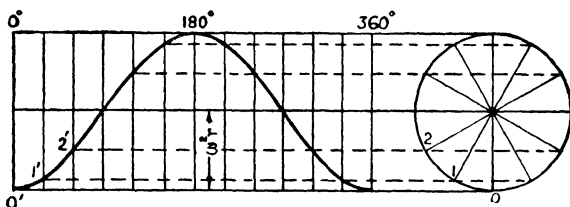


FIG. 1-12. Linear curve of acceleration for body with S.H.M.

linear curve of displacement on a base representing the crank angle, for it is evident that the ordinate for any angle θ is equal to $r \cos \theta$.

Figure 1-11 shows the construction for a linear velocity curve, the radius of the circle being equal to $\omega \cdot r$. Figure 1-12 illustrates the linear acceleration curve. Here the radius of the circle represents $\omega^2 \cdot r$.

17. Polar Curves for S.H.M. The construction for plotting a polar curve for the displacement of a point having S.H.M. is shown in Fig. 1-13. Let XY be the path of motion, its length representing the amplitude. Draw a circle with center O and XY as a diameter. Draw circles with OX and OY as diameters. Then these circles are polar displacement curves on an angle base as required. This may be proved as follows: Take a radius OR in any position, making an angle θ with XY . This intercepts one of the small circles at S . Join XS . Then the angle $XSO = 90^\circ$, since it is an angle subtended by a semicircle. Therefore,

$$OS = OX \cdot \cos \theta = r \cos \theta$$

Consequently, the intercept OS is the displacement of the point, by equation (1-8).

Polar curves for velocity and acceleration of a point with S.H.M. are shown in Figs. 1-14 and 1-15.

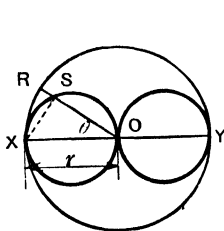


FIG. 1-13. S.H.M. — polar curve of displacement.

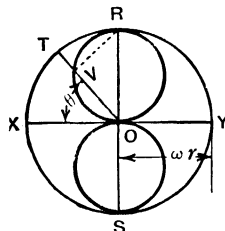


FIG. 1-14. S.H.M. — polar curve of velocity.

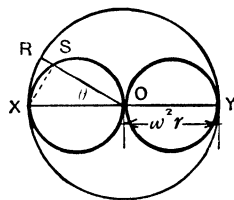


FIG. 1-15. S.H.M. — polar curve of acceleration.

18. Linear Velocity-time and Acceleration-time Curves. Frequently it is desirable to analyze the motion of a body, when data are available as to its position at certain instants, but no data are directly obtainable as to its velocity and acceleration. In-

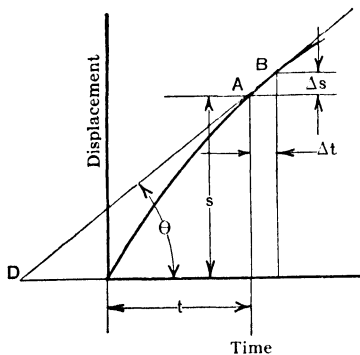


FIG. 1-16

formation regarding the latter quantities can be obtained if a linear displacement-time curve is first plotted and a graphical method is then used to derive velocity-time and acceleration-time curves from the displacement-time curve.

In Fig. 1-16 is shown a curve obtained by plotting the displacement of a moving body from a fixed reference point, on a base representing time. Thus at any point on the curve, such as A , the displacement and time are, respectively, s and t . At point B , after a short interval of time Δt has elapsed, the displacement is $s + \Delta s$ at time $t + \Delta t$. Now the average velocity during the short interval is v where $v = \Delta s / \Delta t = \tan \theta$. θ is the angle of slope of

the line BAD . In the limit where Δt is an infinitesimal, the distance from A to B also becomes infinitesimal, and the line BAD is then a tangent to the curve. Velocity v becomes the instantaneous velocity, that is, the velocity existing during the infinitesimal period dt . It follows, therefore, that the instantaneous velocity of a body is measured by the tangent of the angle of slope of the displacement-time curve at the point considered.

Since acceleration bears the same relation to velocity as velocity bears to displacement, it can be shown in the same manner that the instantaneous acceleration of the body is equal to the tangent of the angle of slope of the velocity-time curve at the point under consideration.

19. Graphical Construction of velocity and acceleration curves from a known displacement curve may be carried out as follows: Select any convenient number of points on the displacement curve (Fig. 1-17), as A_1, A_2, A_3 . Construct the triangles $A_1B_1C_1, A_2B_2C_2, A_3B_3C_3$ in which the AB lines are tangents to the curve and the horizontal BC lines are all of equal length.

The velocities at A_1, A_2, A_3 are proportional to the quantities $\frac{A_1C_1}{B_1C_1}, \frac{A_2C_2}{B_2C_2}, \frac{A_3C_3}{B_3C_3}$, since these are the tangents of the slope angles. By our construction, $B_1C_1 = B_2C_2 = B_3C_3$, and therefore it is evident that the velocities are represented graphically by

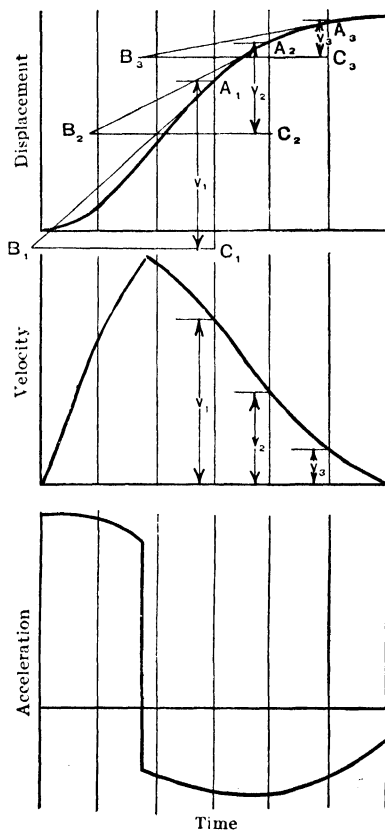


FIG. 1-17

the lengths A_1C_1 , A_2C_2 , A_3C_3 . These lengths are plotted as ordinates on a time base, giving the velocity-time diagram of Fig. 1-17.

Since acceleration bears the same relationship to velocity as velocity bears to displacement, a repetition of the construction just outlined, if applied to the velocity curve, will enable us to draw an acceleration-time diagram, as shown in Fig. 1-17.

A scale for a diagram is a statement of the numerical value of the quantity represented by unit distance on the diagram. Thus the velocity scale in Fig. 1-17 is a statement of the number of feet per second represented by an ordinate 1 in. long on the velocity-time diagram. To find the numerical value of this scale, let us assume that the displacement-time diagram is plotted to a displacement scale of 1 in. = 15 ft, and for time, 1 in. = 4 sec. Also assume that in the triangles $A_1B_1C_1$, $A_2B_2C_2$, etc., the BC side is always made 2 in. long, and therefore represents 8 sec.

Assume A_1C_1 to have any value, say $1\frac{1}{2}$ in. Then A_1C_1 will represent $15 \times 1.5 = 22.5$ ft. The corresponding velocity, at point A_1 on the curve, is therefore $22.5 \div 8 = 2.8125$ ft per sec. Now since A_1C_1 equals the ordinate v_1 on the velocity curve, $v_1 = 1.5$ in., representing 2.8125 ft per sec. Hence 1 in. represents $\frac{2.81}{1.5} = 1.875$ ft per sec. This is a statement of the velocity scale.

Starting with velocity and time scales, the acceleration scale can be similarly calculated provided the length of the base for the tangent triangles is known.

20. Relative Velocities of Three Bodies. The symbol V_{ab} means the relative linear velocity of point or body a with respect to point or body b . Considering three bodies, we have three relative velocities as follows: V_{ab} , V_{bc} , V_{ac} . There are also velocities V_{ba} , V_{cb} , and V_{ca} , but these are not independent quantities, since

$$V_{ab} = -V_{ba} \quad V_{bc} = -V_{cb} \quad V_{ac} = -V_{ca}$$

The truth of these statements is self-evident if we consider the following example. A passenger on an eastbound train sees any landmark receding to the west at the speed of the train. Thus

two vectors, representing velocity of passenger relative to earth and of earth relative to passenger, are of equal length but have arrows representing directions pointing, respectively, east and west, or one velocity is -1 times the other. Again, if the passenger walks forward through the train it is obvious that his velocity relative to the ground is the arithmetic sum of his velocity relative to the train (V_{ab}) and the velocity of the train relative to the ground (V_{bc}) or

$$V_{ab} \rightarrow V_{bc} = V_{ac}$$

The sign $\rightarrow$ is used to indicate the vectorial rather than the arithmetic sum. In Fig. 1-18 is shown the graphical method of summing the vectors V_{ab} and V_{bc} , the result being

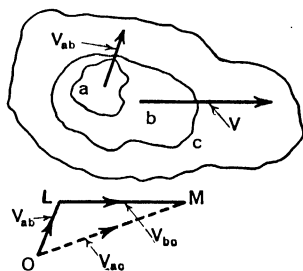


FIG. 1-18

equal to the closing line V_{ac} of the triangle. The direction of the arrows in this figure should be carefully observed since here errors often occur. Where we have a vectorial sum, as for V_{ab} and V_{bc} , the arrows point around the figure in the same sense. As a guide to correct vectorial summation let us write our vectorial equation as follows:

$$V_{a/b} \rightarrow V_{b/c} = V_{a/c}$$

If we treat the subscripts a/b , b/c as quantities to be multiplied, the result is $(a/b) \times (b/c) = a/c$. On the other hand, the sum $V_{a/b} \rightarrow V_{c/b}$ would equal V_{ac/b^2} by the same method, and this quantity has no significance as a velocity vector. This would show that the left-hand side of the equation is wrongly written.

When two bodies have known velocities with respect to a third, the velocity ratio is the quotient obtained by dividing one velocity by the other. Both velocities may be linear, or both may be

angular. If both are angular the velocity ratio should bear a negative sign when the two bodies turn in opposite senses.

Example. Find the velocity of the crankpin A (Fig. 1-19) on a locomotive driver when the locomotive is moving at the rate of 36.1 mph.

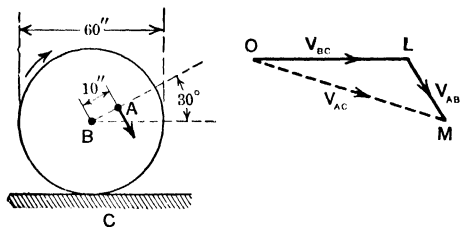


FIG. 1-19

Solution. The velocity of wheel center B relative to the track C is $\frac{36.1 \times 22}{15} = 53$ ft per sec.

The angular velocity of the wheel is $\frac{53 \times 12}{30} = 21.2$ radians per second.

The linear velocity of A relative to B ($v = \omega r$) is

$$\begin{aligned} 21.2 \times 10 &= 212 \text{ in. per sec} \\ &= 17.66 \text{ ft per sec} \end{aligned}$$

Starting from a point O , draw vectors OL and LM representing V_{BC} and V_{AB} respectively. The closing line OM of the triangle OLM is the required velocity V_{AC} . By this method OM is found to represent a velocity of 63.7 ft per sec at an angle of 14° with the horizontal.

PROBLEMS — CHAPTER I

1. A train moves at the rate of 45 mph. What is its velocity in feet per second? Ans. 66 ft per sec.

2. The wheel of an automobile is turning at the rate of 350 rpm. What is the angular velocity of the wheel in radians per second? What is the speed of the car in miles per hour? The tire diameter is 30 in.

3. A piece of cast iron 6 in. in diameter is to be turned in a lathe with a

cutting speed of 90 ft per min. At what speed must the work be rotated?

Ans. 57.3 rpm.

4. An automobile is accelerated uniformly from 10 to 30 mph in 8 sec. Find the value of the acceleration in foot-second units. If the wheels are 28 in. in diameter, find their angular acceleration in radian-second units.

5. A flywheel 4 ft in diameter is speeded up from 120 to 380 rpm with constant angular acceleration in 2 sec. Find the total acceleration of a point on the circumference of the wheel when the speed reaches 180 rpm.

Ans. 711 ft per sec per sec.

6. A point on a body moves along a curved path. At a certain instant the point has a velocity of 27 ft per sec, a tangential acceleration of 40 ft per sec per sec, and a normal acceleration of 54 ft per sec per sec. Find (a) the radius of curvature of the path, (b) the total acceleration of the point, (c) the angular velocity and the angular acceleration of the body.

7. A reciprocating steam engine has a stroke of 24 in. and rotates at 225 rpm. Find (a) the linear velocity of the crankpin, (b) the normal acceleration of the crankpin.

Ans. (a) 23.5 ft per sec. (b) 554 ft per sec per sec.

8. A revolving body has an angular acceleration of 30 radians per sec per sec. A point on it is distant 12 in. from the axis of revolution. When the speed of the body reaches 200 rpm find the normal and tangential acceleration of the point, and show how the total acceleration can be obtained graphically.

Ans. 30 ft per sec per sec; 438 ft per sec per sec.

9. What is meant by "simple harmonic motion"? Explain the meaning of the terms "period," "phase," and "amplitude" as applied to this form of motion.

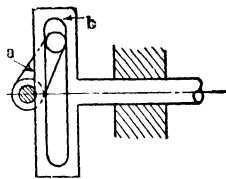
10. Show how to draw a linear curve of (a) velocity, (b) acceleration, for a point having S.H.M., given the period and amplitude.

11. Show how to draw a polar curve of (a) displacement, (b) velocity, for a point having S.H.M., given the period and amplitude. Prove your constructions.

12. A point has simple harmonic motion of period 2 sec, amplitude 6 in. Find (a) its displacement, (b) its velocity, when the revolving line is at 30° with the line of motion.

Ans. 2.6 in., 0.392 ft per sec.

13. In the mechanism shown in the accompanying figure, the crank *a*, 12 in. long, rotates at 100 rpm. Find (a) the velocity of *b*, and (b) the acceleration of *b*, when the crank *a* is at 60° with the line of stroke.



PROB. 13

14. The slotted link in a Scotch yoke is found to attain a maximum velocity of 6 ft per sec, the crank length being 4 in. At what average speed does the slotted link move if the crank rotates uniformly? What is the maximum acceleration?

Ans. 3.86 ft per sec; 108 ft per sec per sec.

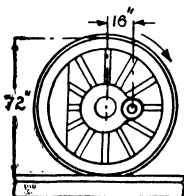
15. A polar curve for the velocity of a point with S.H.M. is drawn with a maximum ordinate of 2 in. (a) If the length of the generating radius is 6 in. and its angular velocity 150 rpm, what is the velocity scale for the polar curve? (b) If a polar acceleration curve for the same point has a maximum ordinate of 2 in. what is the acceleration scale?

16. How many different relative motions are there when three bodies are considered; when four bodies are considered? Show graphically how to find one of the relative velocities when the others are known.

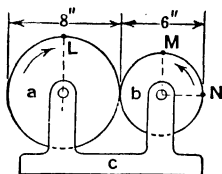
17. A locomotive has driving wheels 62 in. in diameter. The tender wheels are 36 in. in diameter. Find the angular velocity of each and the velocity ratio when the locomotive is traveling at the rate of 40 mph.

Ans. 22.7 rad per sec, 39.1 rad per sec; ratio 1 : 1.72.

18. A locomotive running at 60 mph has a stroke of 32 in. Find the velocity of the crankpin relative to the ground when the crank is in the position shown in the diagram.



PROB. 18



PROB. 20

19. A street car with wheels 30 in. in diameter is traveling at the rate of 35 mph. (a) What is the angular velocity of the wheels? (b) What is the linear velocity of the highest point on the tread of a wheel, relative to the track?

Ans. (a) 41 rad per sec. (b) 102.5 ft per sec.

20. The pair of friction wheels *a* and *b* shown in the diagram are respectively 8 and 6 in. in diameter. They roll together without slipping, *a* turning at 210 rpm. Show how to find graphically the relative velocities of point *L* to point *M*, of point *L* to point *N*, and of point *M* to point *N*.

21. An airplane has a landing speed of 50 mph. Find the velocity of the end of the propeller blade relative to the ground, at the instant when the blade is in a horizontal position when landing. The propeller is 7 ft in diameter and is turning at 550 rpm.

22. Two points lying on a revolving disk in the same radial straight line have a relative velocity of 60 ft per sec. If the disk turns at 420 rpm, what is the distance between the points?

23. The hour hand of a clock is 3 in. long and the minute hand 5 in. long. What is the relative velocity, in inches per minute, of points at the ends of the hands at nine o'clock?

24. A pilot flies a straight course from city A due north to city B, 400 miles distant. His plane has an air speed of 180 mph. A cross wind blows due east at 60 mph. In what direction must the plane be headed, and how long will the trip take? *Ans.* N $19^{\circ} 30'$ W, 2 hr 21 min.

25. A boatman heads his boat across a river 1 mile in width. He can propel the boat at the rate of 4 mph. The current in the river flows at 5 mph. How long does it take him to reach the opposite bank? What is his velocity relative to the shore in direction and magnitude? How far downstream does he land?

CHAPTER II

VELOCITIES IN MECHANISMS

1. General. Parts of machines with plane motion may be divided into three groups: (a) those with angular rotation about a fixed axis; (b) those with angular movement but not about a fixed axis; and (c) those with linear but not angular motion.

Whatever the plane motion of one body relative to another, the motion may be studied by use of the **instant center**. The instant center is defined in two ways:

(a) When two bodies have plane relative motion, the instant center is a point on one body about which the other rotates at the instant considered.

(b) When two bodies have plane relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.

2. Locating Instant Centers. Instant centers are extremely useful in finding the velocities of links in mechanisms. Their use sometimes enables us to substitute for a given mechanism another which produces equal motions and which is mechanically more serviceable. The methods of locating the instant centers are therefore of great importance. When a body has constrained motion, any point on it traces out a point path in space. When the motion is plane, the point path is a plane figure of some sort. Very frequently we can easily find the point paths corresponding to the motions of two points on a body. When this is the case the instant center is found as follows: In Fig. 2-1 body c has plane motion relative to d . Curves RS and TV are respectively the paths traced out on d by two points A and B on c . The instantaneous motions of the two points must be along tangents v and v_1 to the paths of motion, and the instant center must be so placed as to give motions in these directions. To cause A to move in

direction v , the body must be swung about a center somewhere on the line KL perpendicular to v . Likewise, to cause B to move along v_1 , the center must be somewhere along MN . The intersection of these lines at O_{cd} is the only point which will satisfy both requirements, and this point is therefore the instant center.

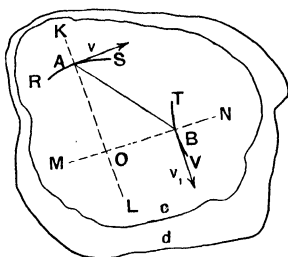


FIG. 2-1

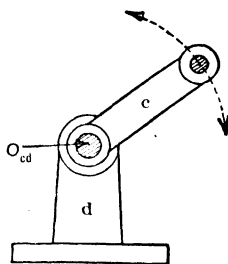


FIG. 2-2

3. Special Cases. (a) When two links in a mechanism are connected by a pin joint, as for example c and d , Fig. 2-2, it is evident that the pivot point is the instant center for all possible positions of the two bodies, and is therefore a permanent center as well as an instant center.

(b) When a body has rectilinear motion with respect to a second body, as in Fig. 2-3 where block c slides between the flat guides d , the instant center is at infinity. This must be the case, since if we take any two points, such as A and B on c , and draw KL and MN perpendicular to the directions of motion, these lines are parallel and meet at infinity.

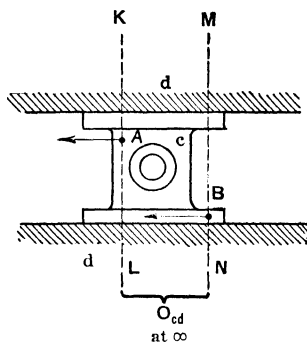


FIG. 2-3

(c) Where two bodies **slide** on one another, maintaining contact at all times, as c and d , Fig. 2-4, the instant center must lie along the perpendicular to the common tangent. This follows, since the relative motion of contact point O_1 on c to contact point O_2 on d is along the common tangent XY ; otherwise the two surfaces

would either separate or cut into one another. Relative motion along the common tangent can be effected only by swinging about a center somewhere along the perpendicular KL ; hence the instant center is on this line.

(d) Where one body rolls on the surface of a second, as in Fig. 2-5, where c rolls on d , the instant center is then the point of contact, since at this point the bodies have no relative motion.

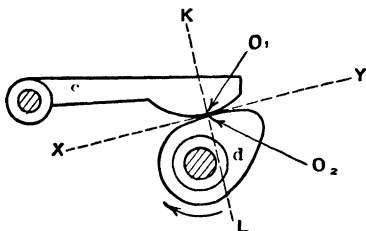


FIG. 2-4

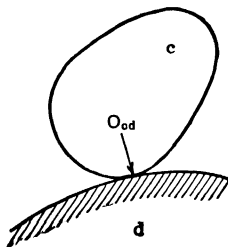


FIG. 2-5

4. Kennedy's Theorem. This theorem states that **the instant centers for any three bodies having plane motion lie along the same straight line.** It is proved as follows:

Let a, b, c (Fig. 2-6) be any three bodies having plane motion with respect to one another. Let O_{ab}, O_{bc}, O_{ac} be the three instant centers.

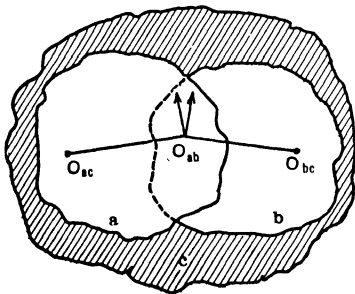


FIG. 2-6

O_{ab} is a point on either a or b , because it is an instantaneous pivot about which one body swings with reference to the other. First consider O_{ab} as a point on a . Then it is moving relative to c about the instant center O_{ac} , and its direction of motion is perpendicular to the line $O_{ac}-O_{ab}$. Next consider O_{ab} as a point on b .

It is now moving relative to c about the instant center O_{bc} , and its direction of motion is perpendicular to the line $O_{bc}-O_{ab}$. But the point O_{ab} cannot have two different motions relative to c at the

same instant. Therefore, the perpendiculars to the lines $O_{ac}-O_{ab}$ and $O_{bc}-O_{ab}$ must coincide. Consequently, $O_{ac}-O_{ab}-O_{bc}$ is a straight line.

Kennedy's theorem is very useful in locating instant centers in mechanisms in cases where two instant centers for three links are known and the third has to be found. Examples given later in the chapter illustrate its application for this purpose.

5. Number of Instant Centers. In any mechanism having plane motion, there is one instant center for each pair of links. The number of instant centers is therefore equal to the number of pairs of links. With n links, the number of instant centers is equal to the number of combinations of n objects taken two at a time, namely, $\frac{n(n-1)}{2}$.

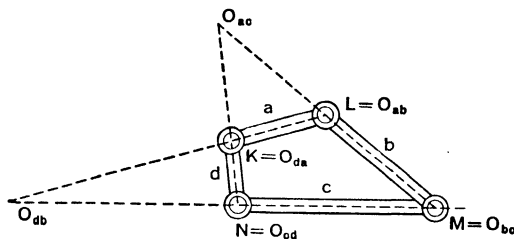


FIG. 2-7

6. Quadric-crank Mechanism. (Fig. 2-7.) This consists of four links connected by turning pairs at K, L, M, N . The number of instant centers is, by Art. 5, $\frac{4 \times 3}{2} = 6$. Four of these centers are found at the pivot points, by Art. 3(a). These are O_{ab}, O_{bc}, O_{cd} , and O_{da} . The remaining two, namely, O_{ac} and O_{db} , may be located (a) by the point-path method of Art. 2, or (b) by the use of Kennedy's theorem. Both methods will now be described.

(a) In applying the point-path method to find the instant center of two bodies, it is convenient to consider one of the bodies as being fixed and then to note the direction of motion of two points on the other. Thus, to find O_{ac} , we regard c as a fixed link and

observe the direction of movement of points K and L . Point K is moving about a center at N in a direction normal to KN . Likewise, L is moving about M in a direction normal to LM . The intersection of KN and LM is, therefore, O_{ac} , by Art. 2.

In a similar way, by regarding b as fixed, we can find O_{db} at the intersection of KL and MN .

(b) In using Kennedy's theorem to find O_{ac} , we select two groups of bodies, each group consisting of the two bodies a, c , plus a third. The instant centers for each group must lie in one straight line, by the theorem. Taking a, c, b as one group, O_{ac} must lie on a straight line with $O_{ab}-O_{bc}$. Taking a, c, d as the other group, O_{ac} must lie along $O_{da}-O_{cd}$. Therefore, O_{ac} is at the intersection of the lines $O_{ab}-O_{bc}$ and $O_{da}-O_{cd}$. The instant center O_{db} is found in the same manner.

Figure 2-8 shows the location of the instant centers in the slider-crank mechanism which has four links connected by three turning pairs and one sliding pair. Three centers, O_{ad}, O_{ab}, O_{bc} , are at the pin joints. O_{cd} is at infinity and O_{db}, O_{ac} are found by Kennedy's theorem.

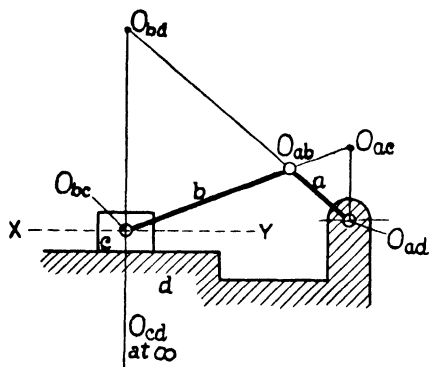


FIG. 2-8

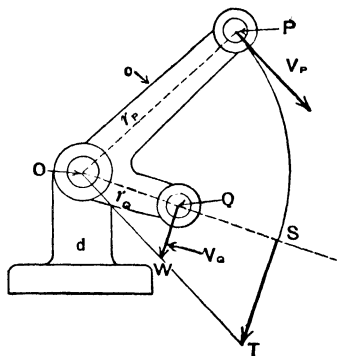


FIG. 2-9

7. Velocities from Instant Centers. Points on One Link.

When a body revolves about a fixed center, as c (Fig. 2-9), which is pin-connected to the fixed frame d , any point on the moving body has a velocity which varies directly as its distance from the

center of rotation. The velocity of a point P on c is $V_P = \omega_{cd} \cdot r_P$. Similarly, any other point Q on the same body c has a velocity $V_Q = \omega_{cd} \cdot r_Q$.

Dividing the two equations,

$$\frac{V_Q}{V_P} = \frac{r_Q}{r_P} \quad \text{or} \quad V_Q = V_P \times \frac{r_Q}{r_P} \quad (2-1)$$

As P and Q are any two points on the body c , it follows that if the velocity of one point on a moving body and the location of the pivot point are known, the velocity of any other point can be found. A simple graphical construction is often found convenient. A vector V_P (Fig. 2-9) is drawn perpendicular to r_P , representing the known velocity of P . With O as center and radius OP , a circle is drawn intersecting OQ , produced if necessary, at S . Since S and P are the same distance from the center of rotation, their velocities are equal in magnitude but differ in direction. The vector ST is drawn perpendicular to OS to represent V_P . Line QW , drawn parallel to ST , forms a triangle OQW , similar to OST . Therefore,

$$\frac{QW}{ST} = \frac{OQ}{OS} = \frac{r_Q}{r_P} = \frac{V_Q}{V_P} \quad \text{from equation (2-1)}$$

Hence, if ST represents V_P , then QW represents V_Q to the same scale. As instantaneous conditions only have been considered, this graphical construction applies whether the point about which the body rotates is an instant center or a permanent center.

8. Velocities from Instant Centers. Points on Different Links.

Very frequently it is necessary to find the velocity of a point on a certain link of a mechanism from the known velocity of another point located on a different link. Several methods are usually available, each having advantages for particular cases. A discussion of these methods follows.

(a) **Direct method.** In applying this method, we first find the instant center for two links, one containing the point of known velocity and the other containing the point whose velocity is to be determined. Regarding this instant center as a point where the links in question have a common velocity, we can work through

it from one link to the other. For example, in Fig. 2-10 is shown a quadric-crank mechanism in which link d is fixed. We shall assume that the velocity of a point P on link a is known and that we require the velocity of a point Q on c . Our first step is to find the instant center O_{ac} .

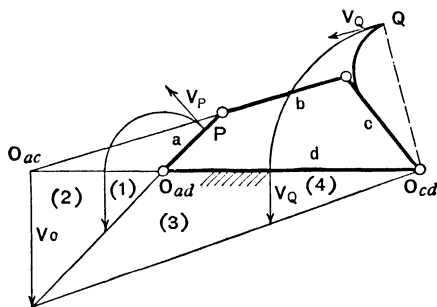


FIG. 2-10

Considering the two points P and O_{ac} : these are points on link a , and since the velocity of P is known, the velocity of O_{ac} (V_0) can be found graphically by the method of Art. 7. Thus link a pivots about O_{ad} and the triangle (1) is drawn, one side representing V_P . Triangle (2) similar to (1) will have a side representing the velocity of O_{ac} (V_0) as indicated. As a point in link c , O_{ac} has the same velocity, V_0 ; hence we now know the velocity of one point on c and can find the velocity of any other point, such as Q . Since link c pivots about O_{cd} , we construct the triangle (3) and then draw the similar triangle (4). The latter has a side representing V_Q . This length is set off perpendicular to $Q-O_{cd}$ and represents the velocity of Q in magnitude and direction.

The construction can be applied to any form of mechanism, provided that the instant center for the two links on which the points are located is accessible. When it is not accessible (as for example when it is located at infinity), some other method must be used.

(b) **Connecting-link method.** This is a step-by-step method whereby we start with the link on which is located the point of known velocity, and work through its instant center with respect

to a connecting link, then along the connecting link to its instant center with respect to the next link. Continuing in this manner, we finally reach the link containing the point whose velocity is required. In general, it is necessary to begin by locating all instant centers with respect to the fixed link (or pivot points) and the instant centers of each link with respect to its adjacent link.

For comparison with the direct method, we shall use the same mechanism (Fig. 2-11) as was shown in Fig. 2-10, namely, a quadric-crank mechanism, in which the velocity of P on link a

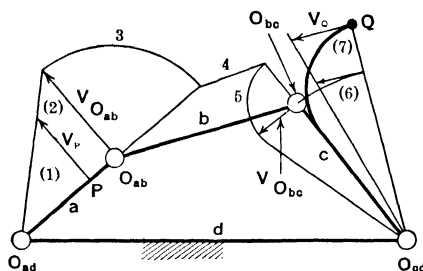


FIG. 2-11

is known and the velocity of Q on c is required. In this example links a and c are connected by link b , and we work through the latter from a to c .

We first locate O_{ad} , O_{ab} , O_{bc} , O_{cd} , as shown in the figure. The vector V_P , at 90° to $P-O_{ad}$, represents the known velocity of P . Considering the two points P and O_{ab} as points on a , we make the construction (by Art. 7) shown by the similar triangles (1) and (2) to find the velocity vector $V_{O_{ab}}$ for the point O_{ab} .

Next, taking the points O_{ab} and O_{bc} (two points on b), we construct arc 3, draw line 4 parallel to b , and swing arc 5 through 90° , obtaining vector $V_{O_{bc}}$ as the velocity of O_{bc} . O_{bc} and Q are both points on c which pivots about O_{cd} . Drawing the similar triangles (6) and (7), we find vector V_Q representing the required velocity.

In this example, pin joints connect the three links, a , b , and c . The connection between these links may, however, take any form and the method can be applied to any mechanism, provided that the needed instant centers are accessible.

The direct method may require the use of instant centers, the location of which requires much labor. For this reason the connecting-link method is sometimes preferable.

9. Linear Velocities by Resolution. Knowing the magnitude and direction of motion of one point on a moving body, and the direction of motion of a second point on the same body, the magnitude of the velocity of the latter point can be found by resolution. This method depends on the fact that the distance between the two points is constant, if the body is rigid.

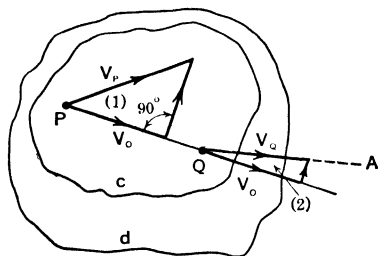


FIG. 2-12

Let P and Q (Fig. 2-12) be two points on a body c in motion with respect to body d . The velocity of P is indicated in magnitude and direction by the vector V_P at the instant considered. The point Q has motion in the direction QA at the same instant. The

distance PQ is constant, and so the components of the velocities in a direction parallel to PQ must be the same; otherwise the distance between them would increase or decrease. By drawing triangle (1) we find the vector V_O , the component parallel to PQ . Triangle (2) can now be drawn, since the vector V'_O , representing the component of Q 's velocity parallel to PQ , is equal to V_O ; also one side is perpendicular to PQ and the third lies along QA . The side last mentioned is V_Q and represents the velocity of Q .

By working from point to point through connecting links, the method of resolution can be used in many cases to find the velocity of any point on a mechanism when the velocity of one point, not necessarily on the same link, is known.

Example. In Fig. 2-13 is illustrated a compound mechanism often used in shapers as a means of driving the ram which carries the cutting tool. The slope of link d has been exaggerated in order to illustrate the construction more clearly.

We shall suppose that the velocity of the point P on the driving crank a is known and that the velocity of the point Q on the ram e is required.

Point P on link a and a coincident point P' on link c must have the same velocity normal to the line of slide of b on c . If this were not the case, P would move off the line $R-O_{cf}$; this is an impossibility, owing to the constraining effect of the sliding pair.

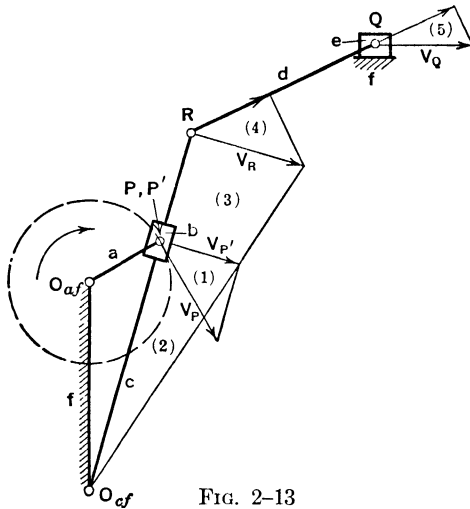


FIG. 2-13

If V_P is resolved into two components parallel and normal to $R-O_{cf}$ by drawing triangle (1), then the normal component represents $V_{P'}$, the velocity of point P' on c .

P' and R are two points on the link c pivoting about O_{cf} . By use of the graphical construction given in Art. 7 and shown by triangles (2) and (3), we find vector V_R representing the velocity of R . The resolution method cannot be used here, because $V_{P'}$ has a zero component along $R-O_{cf}$.

Finally, R and Q are points on the link d and therefore have equal velocity components along d . The resolution method, requiring the construction of triangles (4) and (5), fixes the length of the vector V_Q which represents the velocity of Q .

10. Angular Velocities. When two bodies are in motion it can be shown that their instantaneous angular velocities with respect to a third body are inversely as the distances from their instant center to the instant centers about which they are pivoting on the

third body. Thus, in Fig. 2-14, a and b are two bodies in motion with respect to c . The three instant centers, O_{ac} , O_{ab} , O_{bc} , are assumed to be located as shown in the figure, lying in one straight line in accordance with Kennedy's theorem.

O_{ab} is a point common to a and b . As a point in a , its instantaneous linear velocity is $\omega_{ac}(O_{ab}-O_{ac})$. As a point in b , O_{ab} is moving with a linear velocity $\omega_{bc}(O_{ab}-O_{bc})$. Therefore,

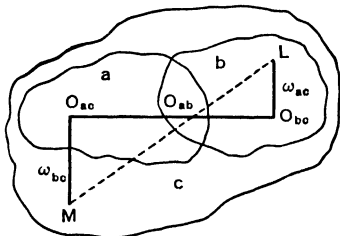


FIG. 2-14

$$\omega_{ac}(O_{ab}-O_{ac}) = \omega_{bc}(O_{ab}-O_{bc})$$

or

$$\frac{\omega_{ac}}{\omega_{bc}} = \frac{O_{ab}-O_{bc}}{O_{ab}-O_{ac}}$$

When one of these angular velocities is known the other can be determined graphically. The construction is indicated in Fig. 2-14. Suppose ω_{ac} is known and ω_{bc} is to be determined. Draw $O_{bc}-L$ perpendicular to $O_{bc}-O_{ac}$ of a length representing ω_{ac} . Join $L-O_{ab}$ and produce this line to meet $O_{ac}-M$, parallel to $O_{bc}-L$. From similar triangles,

$$\frac{O_{ac}-M}{O_{bc}-L} = \frac{O_{ab}-O_{ac}}{O_{ab}-O_{bc}} = \frac{\omega_{bc}}{\omega_{ac}}$$

Hence, $O_{ac}-M$ represents ω_{bc} to the same scale as $O_{bc}-L$ represents ω_{ac} .

When O_{ab} falls between O_{ac} and O_{bc} , the bodies a and b turn in opposite senses; but when O_{ab} is on $O_{ac}-O_{bc}$ produced, the bodies a and b turn in the same sense.

Example. Figures 2-15 and 2-16 show the same slider-crank mechanism in two positions. In each case, assuming that the angular velocity of the crank a (ω_{ab}) is known, show how to find graphically the angular velocity of link d (ω_{db}).

Construction. Find the three instant centers for links a , d , and b (the fixed link). These centers lie on one straight line, by Kennedy's theorem. Draw the triangle $L-O_{db}-O_{ad}$ in which $O_{db}-L$, perpendicular to $O_{db}-O_{ad}$, represents the known angular

velocity ω_{ab} . Draw $O_{ab}-M$, parallel to $O_{db}-L$, meeting $L-O_{ad}$, produced if necessary, at M . Then $O_{ab}-M$ represents the desired angular velocity ω_{db} .

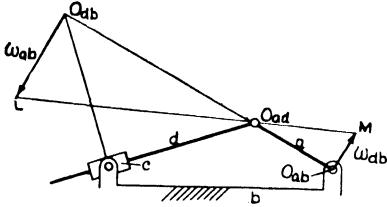


FIG. 2-15

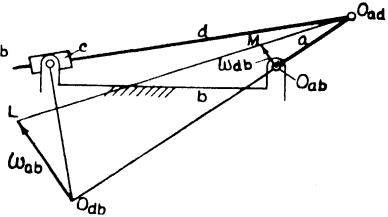
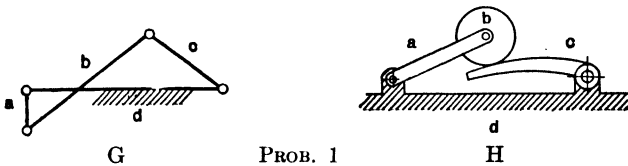
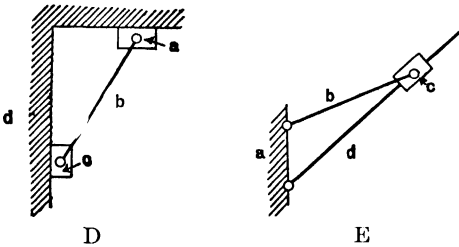
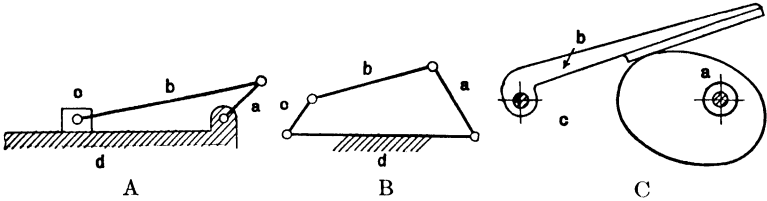


FIG. 2-16

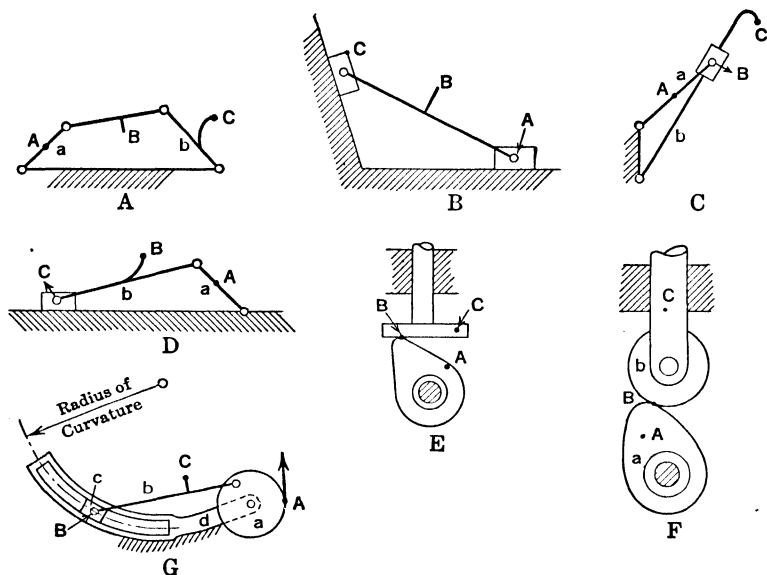
PROBLEMS — CHAPTER II

1. Locate all the instant centers in the mechanisms shown in Figs. A to H. In each case explain the methods employed.



PROB. 1

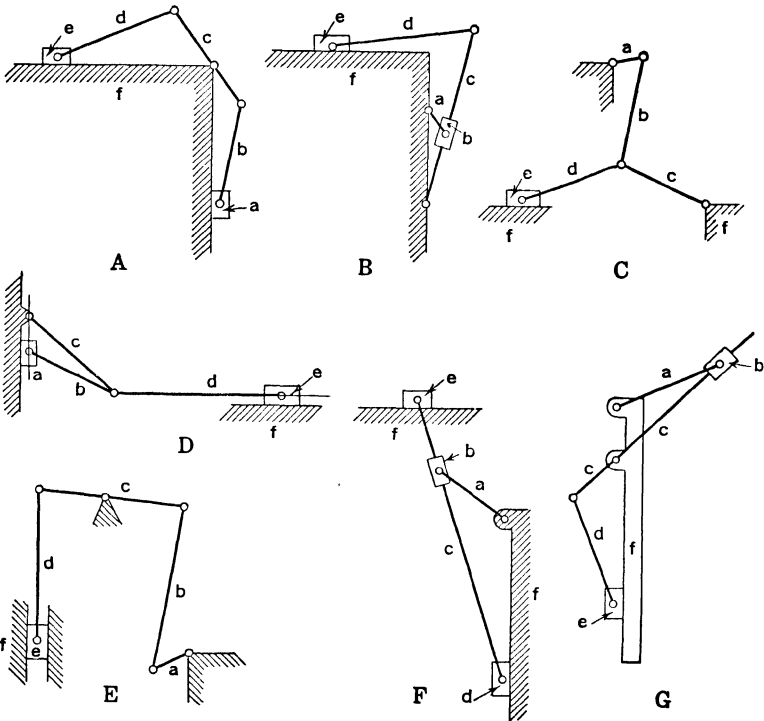
2. In Figs. A to G, assume a known velocity for the point *A* and show how to find graphically the velocities of points *B*, *C*. Mark the positions of all instant centers used in the constructions you employ.



PROB. 2

3. In Figs. A, C, D, F, and G of Prob. 2 assume a known angular velocity for link *a* and find the angular velocity of link *b*.

4. In the compound mechanisms shown in Figs. A to G (a) determine the number of instant centers, (b) locate all the instant centers.



PROB. 4

CHAPTER III

SLIDER-CRANK MECHANISMS

1. General. The uses of the slider-crank mechanism in its various forms are so many and important that it merits careful consideration. It can be described as a simple four-link mechanism with plane relative motion among its parts, three pairs of constraining

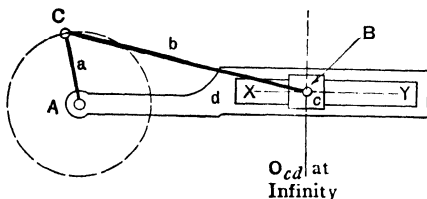


FIG. 3-1

surfaces being pin joints and the fourth a slider and guide allowing relative rectilinear movement of a pair of adjacent links.

2. First Inversion. Sliding-block Linkage. In this mechanism, shown in Fig. 3-1, link d becomes the stationary member. As ap-

plied to reciprocating engines, d is the frame, a the crank, and b the connecting rod. Link c is the piston in some engines having no crosshead; in others it consists of the crosshead, piston rod, and piston, since these parts move as a single rigid piece of material.

The mechanism is said to be "offset" when (as in Fig. 3-1) the straight line XY , which is the path of motion of the point B , does not pass through point A .

The crank, in practical engines employing this mechanism, generally rotates with an angular velocity which is approximately constant. For purposes of design it is necessary to analyze the velocity and acceleration of the piston. The analysis is commonly made under the assumption that the crank velocity is exactly constant, the error involved being of small proportions.

3. Piston Velocity. Graphical Method. The construction in Fig. 3-2 is as follows. The center line of the connecting rod b is produced to meet at D a line AD drawn in a direction perpendicular

to the line of stroke. It can be shown that the distance AD represents the piston velocity to the same scale as the crank line AC represents the crankpin velocity. This statement is proved as

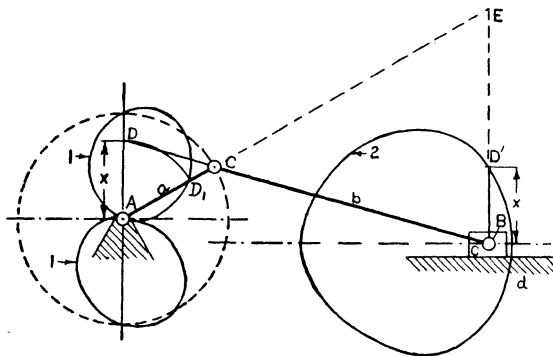


FIG. 3-2

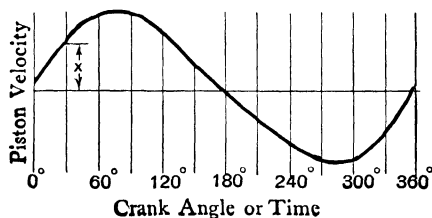


FIG. 3-3

follows. Produce AC to meet at E a line BE drawn perpendicular to the path of B . Then E is O_{bd} , and hence

$$\frac{\text{Linear velocity of } B}{\text{Linear velocity of } C} = \frac{EB}{EC}$$

Also

$$\frac{EB}{EC} = \frac{AD}{AC} \quad (\text{from similarity of triangles } BEC \text{ and } CDA)$$

Therefore,

$$\frac{V_B}{V_C} = \frac{AD}{AC} \quad \text{or} \quad V_B = \frac{V_C}{AC} \times AD \quad (3-1)$$

Now V_C is the crankpin velocity, which is constant when the crank rotates at a uniform speed. AC , also, is a fixed length.

Consequently, we may write

$$\text{Piston velocity} = V_B = \text{a constant} \times AD$$

When AD has a length of 1 in., $V_B = \frac{V_C}{AC} \times 1$; that is, 1 in.

represents $\frac{V_C}{AC}$ units of velocity. As an easy way of remembering the scale we may note that the piston velocity is represented by the length AD to the same scale as the crank length AC on our drawing represents the crankpin velocity.

A polar curve of piston velocity on a crank-angle base is shown at (1) in Fig. 3-2. Point D_1 on this curve is found by making the length AD_1 equal to AD .

A velocity-displacement curve is also drawn at (2) in Fig. 3-2. Point D' on this curve corresponds to the initial position of the mechanism and is found by erecting an ordinate BD' equal to AD .

A velocity-time curve (Fig. 3-3) is constructed by plotting the same velocity ordinates on a base on which equal crank angles are represented by equal spaces; time and angular crank displacement are proportional to each other and, the crank velocity being constant, the same base will serve for both. Thus, distance x in Fig. 3-3 is made equal to the length similarly marked in Fig. 3-2.

4. Piston Acceleration. Klein's Construction. This construction is used for finding a vector whose length represents the piston acceleration in the direct-acting engine mechanism. In Fig. 3-4, point D is found by producing BC to meet AD perpendicular to the line of stroke of the piston. Next, a semicircle CLB is then drawn, with BC as diameter. This is intersected at E by an arc drawn with C as center and CD as radius. From E a line EGH perpendicular to BC is drawn, meeting at H a line AH parallel to the line of stroke.

Then it can be shown that

$$\text{Piston acceleration} = \frac{AH}{AC} \times \text{crankpin acceleration} \quad (3-2)$$

The crankpin acceleration is entirely a normal acceleration (assuming that the crank rotates at constant velocity) and equals

$$\omega^2 \cdot r \quad \text{or} \quad V_C^2 \div r$$

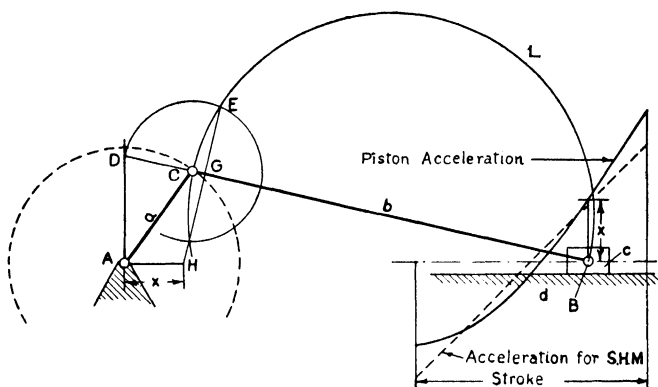


FIG. 3-4

For calculation of scale we may therefore say that AH represents the piston acceleration to the same scale as AC represents the normal acceleration of the crankpin center C . Hence

$$1 \text{ in.} = \frac{\omega^2 \cdot r}{AC} \text{ units of acceleration}$$

5. Piston Displacement, Velocity, and Acceleration. Analytical Method. Where it is necessary to calculate these quantities instead of using a graphical method, the following formulas may be used when the cylinder is not offset (see Fig. 3-5).

$$\begin{aligned} \text{Piston displacement} &= r (\cos \theta + \sqrt{n^2 - \sin^2 \theta}) \quad (3-3) \\ &\text{(from mid-stroke)} \end{aligned}$$

$$\text{Piston velocity} = -r\omega \left(\sin \theta + \frac{\sin 2\theta}{2n} \right) \quad (3-4)$$

$$\text{Piston acceleration} = -r\omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right) \quad (3-5)$$

where n = connecting rod length $\div$ crank length. The velocity

and acceleration formulas are approximate but the accuracy is sufficient for most cases.

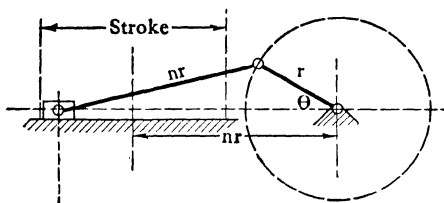


FIG. 3-5

Minus signs indicate velocity and acceleration in a direction opposite to that representing positive displacement from mid-stroke.

6. Quick-return Motion. The sliding-block mechanism can be used as a quick-return motion when offset as shown in Fig. 3-6.

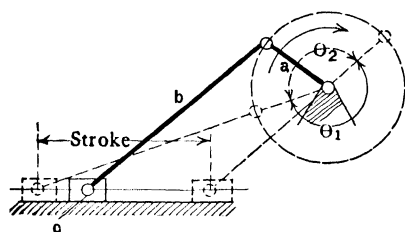


FIG. 3-6

That is, the piston link c executes its strokes to right and left in unequal periods of time. The mechanism is shown by broken lines in the two positions where the piston link has reached the end of its travel to right and left, respectively. At these positions

the crank and connecting-rod links lie along the same straight line. With the crank turning clockwise, the piston stroke to the left takes place while the crank rotates through the angle θ_1 and the return stroke requires a crank movement of θ_2 . Assuming a constant crank velocity, the time ratio of the two strokes is equal to θ_1/θ_2 . This ratio is unity when the offset is zero and increases with the offset.

7. Second Inversion. Swinging-block Linkage. In this mechanism, shown in Fig. 3-7, the link b , corresponding to the connecting rod in the direct-acting engine mechanism, is the fixed link. Figure 3-8 illustrates the application to an oscillating steam engine, link c taking the form of a cylinder pivoted so as to

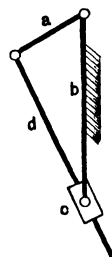


FIG. 3-7

oscillate about trunnions at *B*. Link *d* becomes the piston and piston rod.

Figure 3-9 shows the crank-shaper quick-return motion, another application of the second inversion. Link *a* is the driving crank, attached to which is the block *d*. The latter slides between

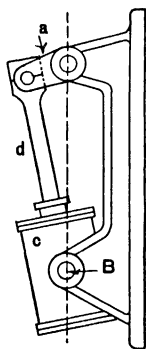


FIG. 3-8

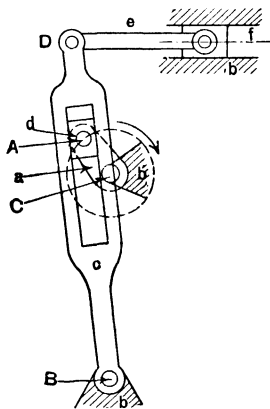


FIG. 3-9

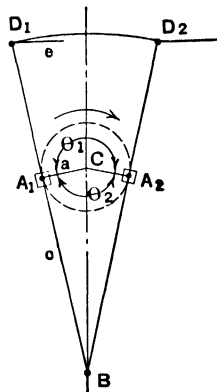


FIG. 3-10

guides formed on lever *c*, driving ram *f* through rod *e*. As applied to the shaper, ram *f* carries the cutting tool. This has a reciprocating motion, the return stroke being performed at a higher speed than the cutting stroke. Supposing, for example, that the crank turns clockwise; the lever *c* will reach its extreme position to the left when the crank is at A_1C (Fig. 3-10) perpendicular to BA_1D_1 . Likewise, *c* will reach the other extreme position when the crank is in position A_2C . The crank meanwhile turns through an angle θ_1 . The return stroke takes place during a crank movement θ_2 . Consequently, assuming constant crank velocity, the ratio of times of cutting to return strokes is equal to θ_1/θ_2 . This ratio can be given any required value by proper choice of ratio $\frac{\text{length } AC}{\text{length } BC}$.

8. Third Inversion. Turning-block Linkage. This is illustrated in Fig. 3-11, link *a*, corresponding to the crank in the direct-acting engine mechanism, being the fixed link. Figure 3-12 shows the Whitworth quick-return motion, a well-known application

often employed in machine tools and in other cases where it is desired to produce a reciprocating motion with a rapid return stroke.

In Fig. 3-12, *b* is the driving crank rotating with constant velocity and driving the slotted link *d* by means of block *c*. Link *d* revolves with variable velocity, and a connecting link *e* may be attached to drive a reciprocating member if required.

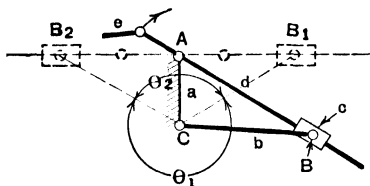


FIG. 3-11

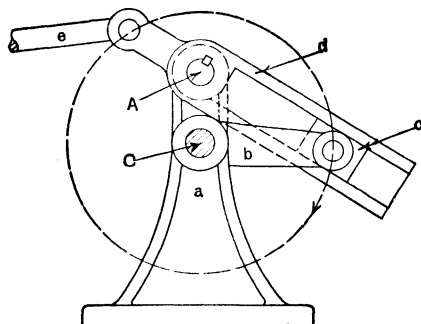


FIG. 3-12

Referring to Fig. 3-11, link *d* turns clockwise from a horizontal position AB_1 through 180° to position AB_2 , while the driving crank is turning through angle θ_1 . It executes the next half-revolution while the driving crank moves through an angle θ_2 . The time ratio is therefore θ_1/θ_2 .

Reduction of the length of link *a* without altering that of *b* will cause the ratio θ_1/θ_2 to decrease, its value becoming unity in the limit when *a* has zero length.

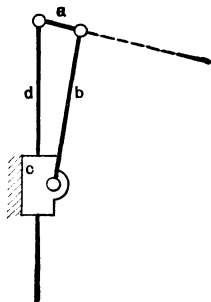


FIG. 3-13

9. Fourth Inversion. Fixed-block Linkage.

The remaining inversion of the slider-crank chain is obtained by fixing the block *c* (Fig. 3-13). The most common application of this linkage is found in hand-operated water pumps. It is also used in certain direct-

driven reciprocating steam pumps. In the hand-pump application, *c* becomes the pump barrel and *d* the pump rod, to the lower end of which the plunger is attached. The dotted extension of *a* forms the pump handle.

PROBLEMS — CHAPTER III

1. A steam engine has a stroke of 12 in. and the connecting rod is 24 in. long. Show how to find graphically the piston velocity and acceleration when the crank makes an angle of (a) 45° and (b) 90° and (c) 120° with the line of stroke. Also determine the velocity and acceleration scales in foot-second units if the the drawing of the mechanism is one-fourth full size and the engine rotates at 240 rpm.

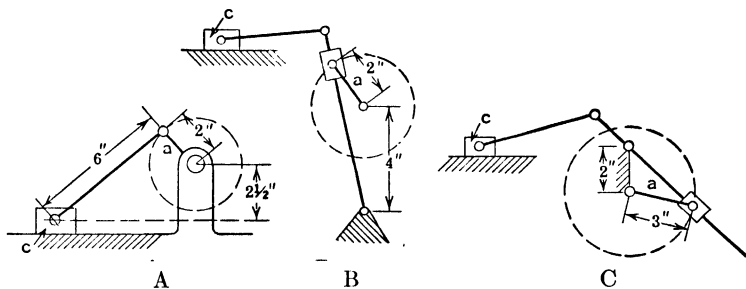
Ans. 1 in. = 8.38 ft per sec; 1 in. = 210.5 ft per sec per sec.

2. A steam engine, 8-in. bore and 10-in. stroke, runs at 300 rpm. Assuming a constant angular velocity of the crank, calculate the piston velocity and acceleration corresponding to the following crank angles: 0° , 45° , 90° , 135° , 180° . The connecting rod is 25 in. long.

Ans. At 45° , Vel. = 10.56 ft per sec; acc. = 290.5 ft per sec per sec.

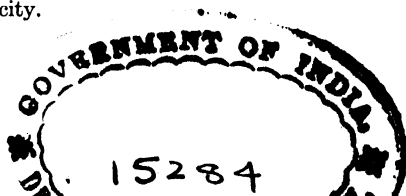
3. If the average piston speed of an internal combustion engine, $3\frac{1}{2}$ -in. bore and 5-in. stroke, with a connecting rod 11 in. long, is 3000 ft per min, at what speed does the engine run? Find, also, the maximum piston velocity and maximum piston acceleration. At what angles are these maximum values obtained?

4. In a four-cylinder gasoline engine with a stroke of $4\frac{1}{2}$ in. and a connecting rod 9 in. long, two cranks are on the top dead center when the other two are on the bottom dead center. Compare the piston accelerations for this position of the cranks if the rpm is 3600. What is the average piston velocity?



PROB. 5

5. Find graphically the time ratio of advance to return for the driven block *c* in the mechanisms shown in Figs. A to C, assuming that the driving crank *a* rotates with constant angular velocity.



CHAPTER IV

ROLLING CONTACT

1. Conditions for Rolling Contact. When two bodies in contact move with respect to each other in such a way that there is no relative motion at the point of contact, the bodies are said to have **pure rolling contact**. It follows that the points in contact have, for the instant, the same velocity relative to a third body. Moreover, by Art. 2, Chapter II, the instant center of the two bodies is located at the contact point.

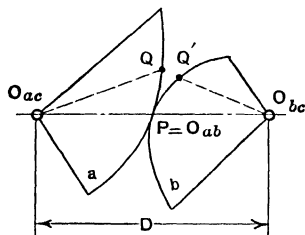


FIG. 4-1

When two bodies with pure rolling contact turn about instant or permanent centers on a third body, the point of contact must always lie on the straight line joining these centers. This may be proved by reference to Fig. 4-1, in which a and b have rolling contact and turn respectively about centers O_{ac} and O_{bc} . P is

the point of contact at the instant and, since at this point no relative motion exists, P is the instant center O_{ab} . By Kennedy's theorem (Art. 4, Chapter II), O_{ac} , O_{bc} , and O_{ab} lie in one straight line.

It has just been shown that the point of contact of a pair of rolling bodies is on the line joining their instant centers or pivots. We shall consider the case where two bodies with pure rolling contact turn about fixed pivots; in Fig. 4-1, O_{ac} and O_{bc} now become permanent centers as well as instant centers. If we select any point Q on the profile of body a , measure the length of the profile from P to Q , and lay off an equal length PQ' along the profile of b , then evidently, when the bodies rotate, at some time Q and Q' will coincide; otherwise slippage will have taken place.

Since Q and Q' meet on the line $O_{ac}-O_{bc}$,

$$(O_{ac}-Q) + (Q'-O_{bc}) = (O_{ac}-P) + (P-O_{bc}) = D \quad (4-1)$$

where D is the distance between the permanent centers. Thus' bodies of any form may have pure rolling contact; but they will have a fixed distance between their centers of rotation, and will therefore be capable of turning about permanent centers on a third body, only when the condition stated by equation (4-1) holds true. This condition is that the **sum of the radiants** to any pair of points which pure rolling will bring in contact **must be constant**.

2. Profile Construction. The following approximate construction may be used to find the profile of a body which is required to have pure rolling contact with a second body of known form, both bodies being assumed to rotate about permanent centers.

The construction is based on the properties of rolling bodies discussed in Art. 1. Let a (Fig. 4-2) be the body of known form rotating about O , and suppose it is required to find the profile

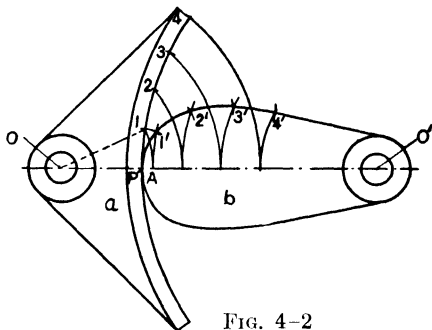


FIG. 4-2

of a second body rotating about O' which will roll with the first. By joining O and O' we find P , which must be the contact point in the position of the mechanism shown in the figure. A convenient number of points 1, 2, 3, etc., are selected on the profile of a . To find a point on b which will come in contact with 1 as the bodies roll together, we use O as center, radius equal to $O1$, and draw an arc intersecting OO' at A . We next use P as center and radius $P1$, constructing a second arc. Finally, with O' as center and radius $O'A$, we strike an arc intersecting the latter at $1'$. Now the sum of the distances $O1$ and $O'1'$ equals OO' , by construction. Since the distances $P1$ and $P1'$ are equal, if the profiles of the bodies between these points are sensibly similar, the lengths of these profiles will also be approximately equal, and hence 1 and $1'$ will

coincide during rotation. Point $1'$ may therefore be taken as a point on the profile of b . Point $2'$ is found in similar manner by making the distance $1'2'$ equal to 12 , and $O'2'$ equal to OO' minus $O2$. Finally a smooth curve is drawn through $P, 1', 2'$, etc. This construction becomes exact when the points $1, 2, 3$ are an infinitesimal distance apart.

3. Angular Velocity Ratio. In Fig. 4-3 two bodies in rolling contact make contact for the instant at a point P . If V_P is the linear velocity of the common point and we consider P as a point on a ,

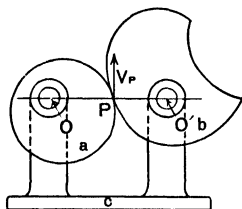


FIG. 4-3

$$V_P = \omega_{ac} \times OP$$

Regarding P as a point on b ,

$$V_P = \omega_{bc} \times O'P$$

Thus,

$$\omega_{ac} \times OP = \omega_{bc} \times O'P$$

or

$$\frac{\omega_{bc}}{\omega_{ac}} = \frac{OP}{O'P} \quad (4-2)$$

In the case considered it is obvious that the bodies a and b rotate in opposite senses. If the two rotations are considered as positive and negative, respectively, the velocity ratio should bear a negative sign.

Instantaneous conditions only were considered; consequently, points O and O' in Fig. 4-3 **need only be instant pivots** and not necessarily fixed pivots.

The above equation, stated in words, means that the velocity ratio of a pair of bodies in rolling contact is inversely proportional to the distance from their point of contact to their respective pivots. When the point of contact falls between their pivots, they must rotate in opposite senses. When it falls to one side of both pivots, the reverse is true.

For constant velocity ratio, OP and $O'P$ must have a constant ratio; this occurs only when P occupies a fixed position on the

line of centers. A pair of circles are the only curves that fulfill this condition; consequently, bodies that roll together with constant velocity ratio must have circular sections perpendicular to their axes of revolution, and such bodies will have velocities inversely proportional to their radii.

4. Velocity Ratio of Rolling Cones.

Figure 4-4 shows two cones used to connect shafts meeting at an angle θ . The distances BC and CD are the radii of the circular bases. At C the cones have a common velocity, V_C . Then,

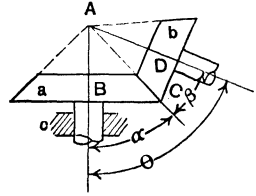


FIG. 4-4

$$V_C = \omega_{ac} \times BC = \omega_{bc} \times CD$$

or

$$\frac{\omega_{bc}}{\omega_{ac}} = \frac{BC}{CD}$$

The velocities are therefore inversely proportional to the radii or diameters of the bases.

Let α and β be the angles of the cones. From the figure,

$$\sin \alpha = \frac{BC}{AC} \quad (1)$$

$$\sin \beta = \frac{CD}{AC} \quad (2)$$

Dividing (1) by (2),

$$\frac{\sin \alpha}{\sin \beta} = \frac{BC}{CD} = \frac{\omega_{bc}}{\omega_{ac}}$$

But

$$\alpha = \theta - \beta$$

Therefore,

$$\frac{\sin (\theta - \beta)}{\sin \beta} = \frac{\omega_{bc}}{\omega_{ac}}$$

or

$$\frac{\sin \theta \cdot \cos \beta - \cos \theta \cdot \sin \beta}{\sin \beta} = \frac{\omega_{bc}}{\omega_{ac}}$$

Dividing numerator and denominator by $\cos \beta$,

$$\frac{\sin \theta}{\tan \beta} - \cos \theta = \frac{\omega_{bc}}{\omega_{ac}}$$

or

$$\tan \beta = \frac{\sin \theta}{\frac{\omega_{bc}}{\omega_{ac}} + \cos \theta} \quad (4-3)$$

In the same way it can be proved that

$$\tan \alpha = \frac{\sin \theta}{\frac{\omega_{ac}}{\omega_{bc}} + \cos \theta} \quad (4-4)$$

For rolling cones with internal contact (see Fig. 4-5) it may be proved in a similar manner that:

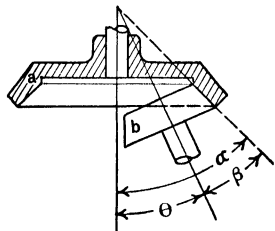


FIG. 4-5

$$\tan \beta = \frac{\sin \theta}{\frac{\omega_{bc}}{\omega_{ac}} - \cos \theta} \quad (4-5)$$

$$\tan \alpha = \frac{-\sin \theta}{\frac{\omega_{ac}}{\omega_{bc}} - \cos \theta} \quad (4-6)$$

These formulas enable us to calculate the cone angles when the angle between shafts and the velocity ratio are known.

5. Rolling Cones. As an alternative to the calculation of the angles of rolling cones, a graphical solution can easily be made.

OA and OB (Fig. 4-6) represent the axes of intersecting shafts to be connected by rolling cones with a speed ratio of 5 to 2. These may have either external or internal contact; the former will be considered first.

A distance of five units is laid off along OA , thus locating the point C . Similarly, point D on OB is found by making OD equal two units. From C , CE is drawn parallel to OB , and from D , DE parallel to AO . The intersection E is a point on the contact line of cones which give the desired speed ratio.

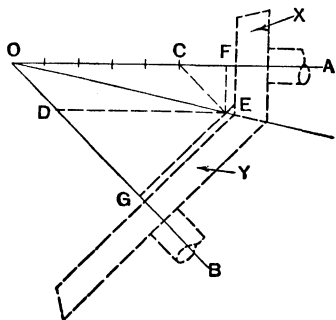


FIG. 4-6

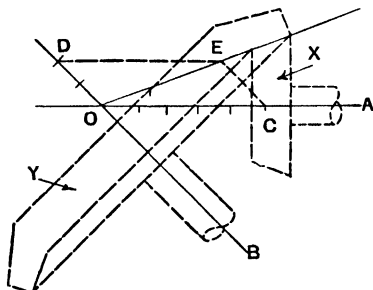


FIG. 4-7

Join OE and drop perpendiculars EF and EG to the axes AO and OB . From the geometry of the figure, it can be shown that

$$EF : EG = OD : OC = 2 : 5$$

Furthermore, a pair of perpendiculars drawn from any point on OE to the axes will have the same ratio of lengths. At x and y (Fig. 4-6) are shown two cone frustums drawn with a pair of these perpendiculars as base radii. The speed ratio is

$$\omega_y : \omega_x = EF : EG = 2 : 5$$

The cone angles to give the speed ratio specified are AOE and EOB .

Figure 4-7 illustrates the case where cones with internal contact are desired with the same speed ratio as before. The construction differs from that of Fig. 4-6 only to the extent that OD is laid off along BO produced. It should be noted that the use of internal instead of external contact reverses the sense of rotation of the driven member.

6. Rolling Ellipses. Two equal ellipses, initially placed as in Fig. 4-8, with all foci lying along the same straight line, and each turning about one of its foci; can be shown to have pure rolling

contact. If O and O' denote the foci that are the centers of rotation, the distance between these points is obviously equal to the major axis of either ellipse. It must be shown that the sum of the radii to any pair of points that the rolling of the curves will bring into contact is constant. (See Art. 1.)

Using the initial point of contact P as center and any radius, an arc is struck which cuts the curves at I and I' . Since chords PI and PI' are equal, from the symmetry of the figure it will be

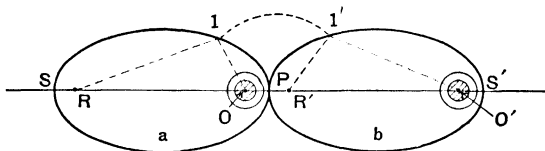


FIG. 4-8

evident that the elliptical arcs PI and PI' are of the same length. Therefore, pure rolling will bring I and I' together. We must show that $OI + O'I'$ equals OO' . If R, R' are the other two foci, from the properties of the ellipse we know that $OI + IR$ equals the major axis. But, since the ellipses are equal in all respects, IR equals $O'I'$. Therefore,

$$OI + O'I' = OI + IR = \text{major axis} = OO'$$

Thus the requirement of equation (4-1) is complied with.

The angular velocity ratio $\omega_b \div \omega_a$ of the ellipses in the relative position shown in Fig. 4-8 is equal to $OP \div O'P$, by Art. 3. When ellipse a has rotated 180° , S and S' will come in contact, and at that instant the velocity ratio will be $OS \div O'S' = O'P \div OP$. These are, respectively, minimum and maximum values of the velocity ratio, the latter being the reciprocal of the former. During each half-revolution of the ellipses, the ratio changes from one value to the other.

7. Ellipses for Desired Velocity Ratios. It is possible to construct ellipses which will give any desired variation in velocity ratio. Figure 4-9 illustrates the construction for a case where the driven ellipse is to have, as a maximum, three times the angular

velocity of the driver. It follows that the minimum velocity of the driven member is one-third that of the driver. The distance OO' , between the centers of rotation, is assumed to be known.

In Fig. 4-9, OO' is first divided into two parts, OP and PO' , such that $OP \div O'P = 3 \div 1$. If OS , equal to $O'P$, is laid off along OO' produced, then PS will be the major axis of one ellipse. PS'

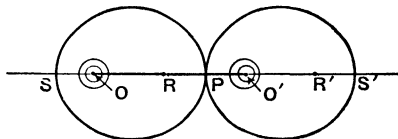


FIG. 4-9

equal to PS is the major axis of the second ellipse. The foci R and R' are located by making RP and $R'S'$ equal to PO' . When the foci and major axes are known, the ellipses may be drawn by any of the usual methods.

8. Friction Drives may be defined as those in which power is transmitted by the rolling contact of driving and driven members, friction at the contact surfaces being depended upon to avoid appreciable slippage. They are practical applications of mechanisms having rolling contact. Cylindrical wheels with internal or external contact are commonly employed to connect parallel shafts. Wheels taking the form of frustums of cones are used for connecting intersecting shafts. These may have external contact (as in Fig. 4-4) or internal contact (as in Fig. 4-5). The frustums must be those of cones having a common apex in order that pure rolling conditions may be approached.

Practically, a certain amount of slippage is bound to take place in a friction drive when power is being transmitted. This type of drive is most serviceable for light duty. A heavy contact pressure is necessary when transmitting a large amount of power; this tends to cause friction losses and wear on the bearings of the wheel axles as well as on the contact surfaces.

For the purpose of increasing the power which may be transmitted by friction wheels for a given contact pressure, the wheels are sometimes provided with V-shaped circumferential grooves. It can easily be shown, however, that such construction renders pure rolling contact impossible, and therefore tends toward increased wear and friction losses.

9. Brush Wheel and Plate. A friction drive of the form shown in Fig. 4-10 is sometimes used where it is desired to obtain a speed ratio which can be varied at will. The brush wheel *a* is usually the driver, making frictional contact with the driven plate *b*. Wheel *a* is mounted so that it may be shifted in an axial direction, thus moving in a line parallel to the surface of the plate. The speed ratio of driven and driving wheels depends on its position. Reversal of the sense of rotation is effected by moving *a* to the

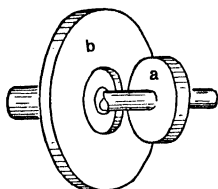


FIG. 4-10

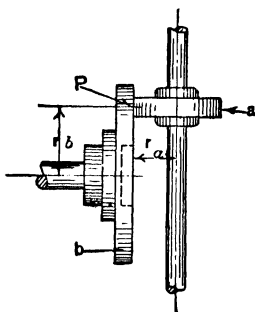


FIG. 4-11

opposite side of the disk axis. A depression at the center of the disk causes the two members to break contact when *a* is in mid-position, giving a "neutral" point in which no drive is obtained.

Pure rolling takes place only when *a* has no sensible thickness and consequently makes point contact. Such a condition in the practical machine could be approached if the power transmitted were very small. A wide brush wheel with line contact will have pure rolling contact at or near its center point, with increasing sliding velocity as the edges are approached. This tends to cause rapid wear at the sides.

If *P* (Fig. 4-11) is the point at which pure rolling takes place, then V_P is the same when calculated from the angular velocity of either body. Therefore,

$$V_P = \omega_a \times r_a = \omega_b \times r_b$$

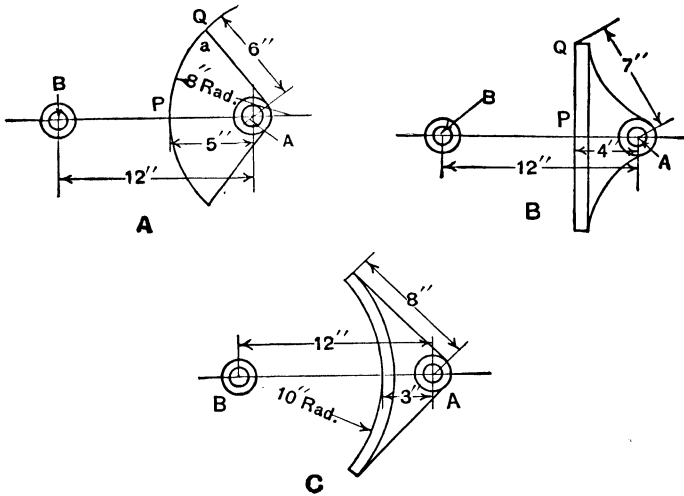
or

$$\frac{\omega_a}{\omega_b} = \frac{r_b}{r_a}$$

When ω_a is constant, ω_b will have its maximum value when r_b is least and its minimum value when r_b is largest. These radii may be selected to give the desired range of speeds. P , the point of pure rolling, is generally assumed to be located at the middle of the face of the wheel a .

PROBLEMS — CHAPTER IV

1. In Figs. A to C, show how to find the profile of a body rotating about fixed pivot B which will have pure rolling contact with the body turning about a fixed pivot at A . Calculate the angular velocity ratios when the contact point is at P and at Q .



PROB. 1

2. Two parallel shafts are at a distance of 15 in. center to center. They are to be connected by rolling ellipses so as to obtain a maximum velocity ratio of 4 : 1. (a) Find the lengths of their major axes and the distances between their foci. (b) What is the minimum velocity ratio of the shafts?

3. Rolling cones with external contact are required to connect two shafts at 60° , the velocity ratio to be 3 : 2. Show how to find graphically the cone angles.

4. Rolling cones with internal contact are required to connect two shafts at 30° , the velocity ratio to be 3 : 1. Show how to find graphically the cone angles.

5. A rolling cone with a center angle of 15° turning at 240 rpm makes external contact with a second cone which turns at 360 rpm. Find the angle between the shafts and the apex angle of the second cone.

6. Rolling cones with internal contact are to be used to secure a speed ratio of 4 : 1. The smaller cone has a center angle of 10° . What is the center angle of the larger cone, and what is the angle between the shafts?

7. A pair of equal rolling ellipses have a distance between their foci of 4 in. The major axes are 7 in. long. Find the maximum and minimum speed ratios.

Ans. 1 : 3.67; 3.67 : 1.

8. A brush-wheel-and-plate friction drive are required to connect two shafts, the driver turning at 300 rpm, and the driven plate to turn at a maximum speed of 100 rpm and a minimum of 25 rpm. The driving wheel is 5 in. in diameter and 1 in. wide. Find the maximum and minimum diameter required for the plate.

9. What maximum and minimum speeds are obtainable at the driven shaft in a brush-wheel-and-disk friction drive in which the wheel, 8 in. in diameter and $1\frac{1}{2}$ in. wide, rotates at 400 rpm, and in which the disk has a maximum diameter on the contact surface of 26 in., the minimum diameter being 14 in.? Allow 3 per cent for slippage.

CHAPTER V

CAM MECHANISMS

1. Cam Mechanisms are extensively used in machinery because of the ease with which the design can be carried out to produce any desired motion. The motions needed in machine parts are often of such a nature that it would be difficult to obtain them by any other mechanism of equal simplicity and practicability. Thus cam mechanisms are commonly used for operating valves in automobile, stationary, and marine internal combustion engines, in printing machinery, shoemaking machinery, automatic screw machines, stamp mills, clocks, locks, etc. It is difficult to find a machine of the type which we term "automatic" which does not employ one or more cam mechanisms.

All cam mechanisms are composed of at least three links: (a) the cam, which has a contact surface either curved or straight; (b) the follower, whose motion is produced by contact with the cam surface; and (c) the frame, which supports and guides the follower and cam.

2. Types: Disk or plate cam. Here the cam takes the form of a revolving or oscillating disk, as in Fig. 5-1, the circumference of the disk forming the profile with which the follower makes contact. The follower moves in a plane perpendicular to the axis of rotation of the cam.

Cylinder cam. (See Fig. 5-2.) In this mechanism the profile which acts on the follower is formed on the surface of the cylinder, in order to move the follower in a plane parallel to the axis about which the cam rotates or oscillates.

Translation cam. Here the cam profile is cut on one side of a block of metal or other material and the cam has reciprocating motion along a flat surface. (See Fig. 5-3.)

All cams may be regarded as wedges having surfaces of uniform

or variable slope, more frequently the latter. When the wedge is caused to slide back and forth on a flat surface it is called a **trans-**

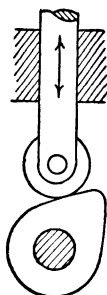


FIG. 5-1

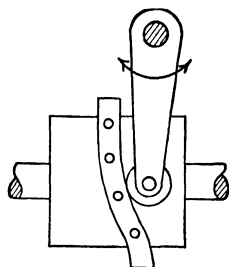


FIG. 5-2

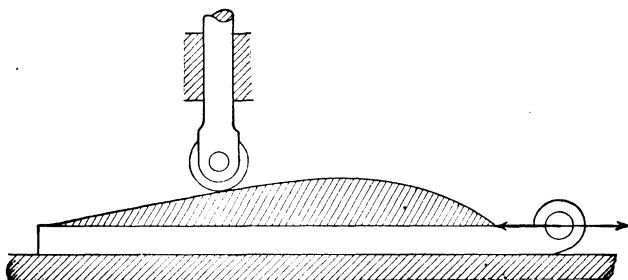


FIG. 5-3

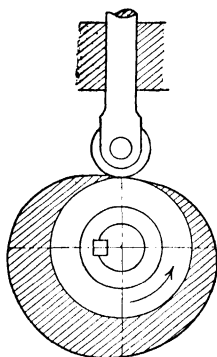


FIG. 5-4

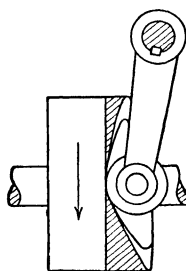


FIG. 5-5

lation cam (Fig. 5-3). When the wedge is wrapped around the circumference of a circular disk (Fig. 5-4) it becomes a disk cam.

When the wedge is bent to form a ring and applied to the flat end of a cylinder (Fig. 5-5) a cylinder cam results.

3. Constraint of Follower. In the mechanisms shown in Figs. 5-3, 5-4, and 5-5 it will be noted that the form of the cam is such that it does not completely constrain the motion of the follower, as no means of maintaining contact between cam and follower is indicated. Continuous contact is usually effected by utilizing either the forces of gravity or spring pressure.

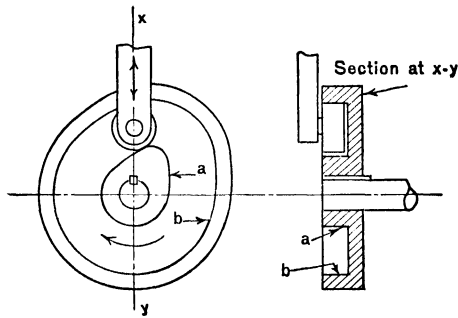


FIG. 5-6

4. A Positive-motion Mechanism (Fig. 5-6) is one in which the follower is compelled to move in a definite path by constraining surfaces and without the application of external forces. Failure to do so can be due only to breakage of some part. Methods of accomplishing this result will be taken up in detail in a later article.

DISPLACEMENT DIAGRAMS

5. Profile Design. The design of a cam profile is governed by the requirements in regard to the motion of the follower. These requirements depend on the function which the mechanism performs in the machine to which it is applied. The cycle of events for the follower, determined by such considerations, may call for certain "rest" periods during which no follower motion occurs, and certain periods of motion of a specified nature. It is generally found convenient to start on the cam-design problem by first making a graphical representation of the follower movement,

which is called a displacement diagram. This is a linear curve in which abscissas represent cam displacements and ordinates represent follower displacements. Since both members may have either linear or angular motion, these displacements may be either linear or angular, depending on the particular form of mechanism under consideration. Linear follower displacement is often referred to as the "lift," even though the movement may not be in a vertical direction.

Followers in practical applications frequently move exactly or approximately in accord with one of the following conditions:

- (a) Motion at constant velocity.
- (b) Motion with constant acceleration or deceleration.
- (c) Simple harmonic motion.

The Displacement Diagrams corresponding to these three cases, together with certain modifications, will be considered next.

The camshaft, where the cam has angular motion, is assumed to rotate at a constant speed. The discussion which follows is based on this assumption. Here the displacement curve is one in which the base represents time as well as cam displacement, the two quantities being proportional to each other.

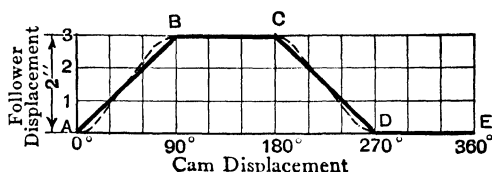


FIG. 5-7

6. Constant Velocity. In Fig. 5-7 is shown a displacement diagram for a cam mechanism in which the follower rises 2 in. during 90° motion of the cam at constant velocity, rests for 90° , falls 2 in. with constant velocity during 90° , and rests for the balance of the cycle.

When a body moves with constant velocity, its displacement is directly proportional to the elapsed time. Assuming a constant cam velocity, the follower displacement is consequently propor-

tional to the cam displacement. The curve AB for the first 90° must therefore be a straight line. During the second 90° period a horizontal straight line BC represents the rest period. The drop period during the next 90° of cam motion is indicated by another straight line CD , since here again we have constant velocity. DE is drawn horizontally for the final period.

For a practical application the diagram would probably be modified to the form shown by the broken lines, unless the cam turns very slowly. This is done to avoid sudden changes of motion when the lift begins and ends, and substitutes a gradual change of velocity which eliminates shock and noise.

7. Constant Acceleration. For any moving body with constant acceleration, $s = \frac{1}{2}at^2$, where s is the displacement, a the acceleration, and t the time interval. The distance moved is therefore proportional to the square of the time. Taking time intervals of 1, 2, 3, 4, etc., time units, the displacements of the body at the ends of these intervals will be relatively proportional to the quantities $1^2, 2^2, 3^2$, etc., or 1, 4, 9, etc. This principle is applied in the case of the displacement diagram shown in Fig. 5-8. Here the requirement is that the follower shall move a distance AC during a cam displacement AB . The construction is as follows.

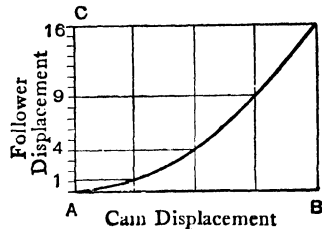


FIG. 5-8

AB is divided into any number of equal spaces; in the figure these are four in number. Each of these spaces represents an equal time interval under the assumption that the cam has uniform velocity. The follower displacements up to the ends of these intervals are proportional to the numbers 1, 4, 9, 16. But AC is the displacement at the end of the fourth interval. Therefore, we divide AC into sixteen equal parts and project from the first, fourth, ninth, and sixteenth divisions, as shown in the figure, thus locating points on the required curve.

8. Constantly Accelerated and Decelerated Motion. Acceleration lasting to the end of the follower travel would result in maxi-

imum velocity being attained just before the follower comes to rest, and this would cause a shock unless the cam speed were very slow. Consequently, the acceleration period should last only part of the lift interval and be succeeded by a "deceleration," which will bring the follower to rest gradually. Giving these quantities constant values will result in the smoothest cam action. The constant acceleration may or may not be equal to the constant deceleration; a cam profile can be designed to give any desired ratio of acceleration to deceleration. The displacement diagram for such a case will next be considered.

Let a_1 be the constant acceleration during the first part of the follower motion, s_1 and t_1 being the corresponding displacement and time. Let a_2 be the deceleration during the latter part of the motion, s_2 and t_2 being the displacement and time for the same interval. The ratio $\frac{a_1}{a_2}$ is the given acceleration-deceleration ratio.

Now $S = s_1 + s_2$, where S is the total follower movement.

If $v =$ the velocity at the end of the acceleration period,

$$v^2 = 2a_1s_1 = 2a_2s_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{s_2}{s_1}$$

Also,

$$v = a_1t_1 = a_2t_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{t_2}{t_1}$$

That is, the displacements and time intervals are to each other inversely as the acceleration-deceleration ratio.

Example. Draw the displacement diagram for a cam mechanism in which the follower moves 2 in. during 180° of cam displacement, acceleration and deceleration being constant and having a ratio of 3 to 1.

From the above discussion it will be evident that the displacements and times corresponding to the two intervals are in the ratio of 1 to 3. For the acceleration period the displacement is therefore one fourth of the total displacement and the period lasts for one fourth of the total time, ending at 45° cam displacement

(Fig. 5-9). This fixes the position of the point B on the 45° line, the ordinate being $\frac{1}{2}$ in. The construction for other points on the acceleration curve is the same as that used in Fig. 5-8. Points on the deceleration curve BC are found in the same way by working from C toward the left.

9. Practical Modification of the Constant-velocity Diagram. As noted in Art. 6, the displacement diagram for the constant-velocity cam

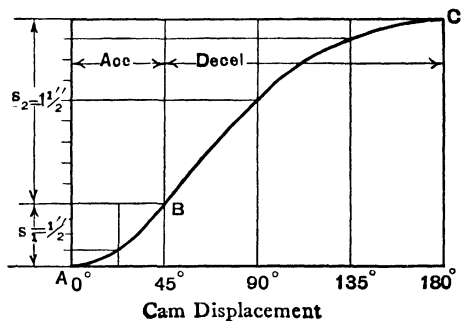


FIG. 5-9

is modified somewhat from the theoretical form in practical applications, for the purpose of avoiding sudden changes of velocity at the beginning and end of the lift period.

This modification can best be made by using a short period of constant acceleration at the beginning of lift, lasting until a suitable velocity has been attained. The follower then moves with constant velocity until near the end of the lift period, when a constant deceleration is applied, and the follower is brought to rest without shock.

The construction of the lift diagram for such a case will now be considered.

Suppose it is specified that the follower is to lift during 150° of cam motion, the displacements being those due to constant acceleration for 30° , constant velocity for 90° , and constant deceleration for the remaining 30° .

When a body is accelerated uniformly from rest to a velocity v in t units of time, it is evident that the average velocity for the period is $v/2$ and the distance moved is $vt/2$. If, instead, the body has a constant velocity v , it will move the same distance $vt/2$ in time $t/2$. Consequently, the follower in question will move the same distance during the first 30° , where it has constant acceleration, as it does in subsequent 15° intervals with constant velocity.

The total lift can therefore be regarded as composed of eight equal increments, the first being executed in the first 30° period,

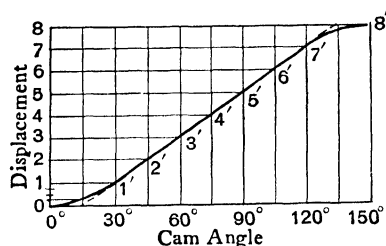


FIG. 5-10

the next six in the succeeding six 15° intervals, and the last in the final 30° period. We therefore divide the total lift (Fig. 5-10) into eight equal parts, obtaining the points 1, 2, 3, etc., and project from 1 to 1', 2 to 2', etc. Connecting 1', 2', 3' by a smooth curve completes the diagram. Intermediate points on the acceleration and deceleration curves may be found as in Fig. 5-8.

10. Simple Harmonic Motion. The construction of the displacement diagram for harmonic motion of the follower is the same as that described in Art. 16, Chapter I, for drawing a linear displacement curve of a point with harmonic motion. Figure 5-11

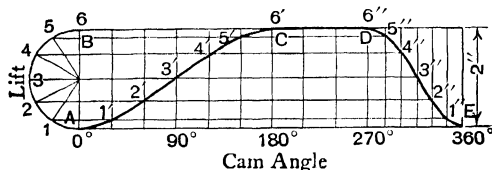


FIG. 5-11

illustrates a case in which the follower lifts 2 in. during 180° of cam motion, then rests for 90° , falls to initial position in 90° , and rests for the balance of the cycle.

A semicircle is drawn as shown, the lift being used as a diameter. The cam angle for the lift period, 180° , is divided into any convenient number of equal parts; in the figure each of these represents 30° . The semicircle is divided into the same number of equal arcs, and thus points 1, 2, 3, 4, etc., are found. Horizontal projection then locates points 1', 2', 3', etc., on the required curve. For the "drop" period, projection from the same points, 1, 2, 3, may be made if the cam angle corresponding to this period is divided into the same number of parts as the semicircle.

CAM PROFILE CONSTRUCTION

11. General Methods. So far we have only discussed the method of drawing displacement diagrams for required follower motions. The next step to be considered is that of finding the cam profiles necessary to produce these movements. The construction is altered in detail with different types of followers, but we may map out a general method which is applicable to all cases irrespective of the form of the displacement curve or the variety of follower in use. It can be applied to disk cams, cylinder cams, and translation cams, and comprises the following steps.

(a) The cam is considered as being the fixed link in the mechanism instead of the frame which carries the camshaft and follower guides. That is, we deal with an inversion of the actual mechanism. As noted in Art. 6, Chapter I, the relative motion of any pair of links remains unaltered when the mechanism is inverted. Therefore, the cam and follower will have the same relative motion whether the frame or the cam is considered as the fixed member.

(b) The portion of the follower that acts on the cam is then drawn in various positions which it will occupy at different instants during its cycle of motion relative to the stationary cam. The contact surface of the follower may consist of the surface of a roller, a knife edge, a flat, convex, or concave sliding face, etc.

(c) The cam profile is found by drawing a smooth curve tangent to the follower contact surface in all its various positions.

The contact surface of the follower is located as required in (b) by first finding the position of some selected point on the follower. The point chosen, which will be called the reference point, should be one that is easily located from data furnished by the displacement curve and also one from which the working face of the follower can be conveniently drawn. For example, where a roller is used, the roller center is the best point for the purpose; where the follower is flat-faced, the point where the follower axis intersects the contact face is satisfactory. Where the follower is convex or concave, the center of curvature is the best reference point.

DISK CAMS

12. Roller Follower. The follower is usually either guided so as to move with rectilinear motion, or it is pivoted so as to swing about a fixed point. The general method outlined in Art. 11

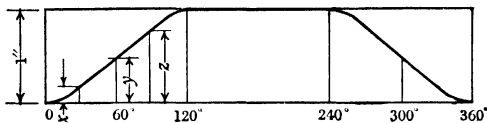


FIG. 5-12

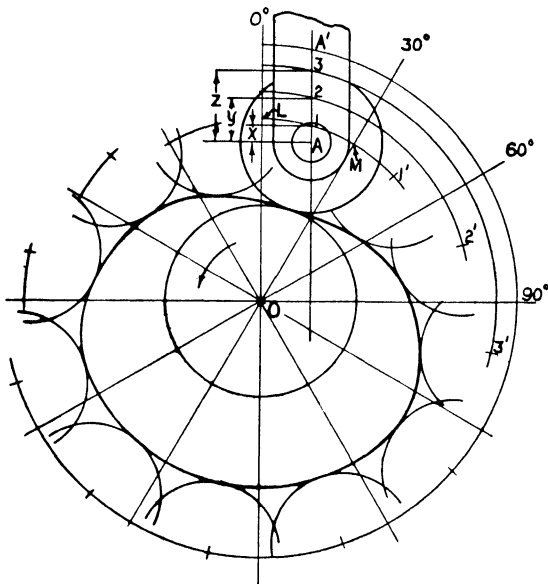


FIG. 5-13

applies in either case. The center of the roller is used as a reference point whose path is first determined, and from which the follower contact surface, namely, the circumference of the roller, is located in various positions.

Roller follower with rectilinear motion. The displacement diagram, Fig. 5-12, is assumed to specify the motion requirements.

The base circle (Fig. 5-13) is first drawn and the roller is located in its initial position touching this circle. The path of the roller center, $A-A'$, is drawn. A zero radius, for convenience, parallel to $A-A'$, is next located, and angular intervals of 30° are laid off from it about O . Keeping the cam stationary, the position of the roller center A is then found, after 30° displacement of the follower. The displacement diagram indicates a displacement x at 30° ; this distance is set off along $A-A'$, giving point 1 . With center O and radius $O-1$, an arc $1-1'$ is described in a sense opposite to that of the cam movement, and of such length as to subtend an angle of 30° at O . Point $1'$ can most easily be located by making chord $1-1'$ equal to chord $L-M$ or $1-L$ equal to $1'-M$.

Points $2'$, $3'$, $4'$, etc., are found in a similar way. Using these points as centers and the roller radius, corresponding positions of the follower contact surface are drawn. The required cam profile is evidently a curve drawn tangent to each of these circles. This curve is made as smooth as possible.

In Fig. 5-13, the line AA' does not pass through the cam axis, hence the follower is said to be offset. An offset is sometimes provided to reduce side thrust during the lift period.

13. Flat-faced Follower. Here two cases will be considered: (a) the case where the follower has rectilinear motion, and (b) the case where the follower has angular movement about a pivot.

(a) **Flat-faced follower with rectilinear motion.** Figure 5-15 illustrates this case. Assuming that the displacement diagram has been obtained and is of the form shown in Fig. 5-14, we proceed as follows.

Draw the base circle for the cam and divide it into convenient angular divisions. Draw the follower in its initial position BC , tangent to the base circle. Point A where the cam touches the base circle in this position is chosen as the reference point. Set off distances x , y , z , etc., obtained from the displacement diagram, along the path of motion of A , obtaining points 1 , 2 , 3 , etc. With O as center and $O-1$ as radius, swing an arc $1-1'$. Point $1'$ is the position of A after 30° displacement. Points $2'$, $3'$, etc., are found in the same way from displacements y , z , etc. Through $1'$ draw a line perpendicular to the radius $O-1'$; this represents the fol-

enlarged a certain amount, three of the lines will meet at a point; then the profile will have a sharp corner which is liable to wear away rapidly. Further increase in the base-circle diameter will cause this corner to disappear.

The necessary length of follower face BC in Fig. 5-15 can readily be determined by inspection of the figure. The face is usually a

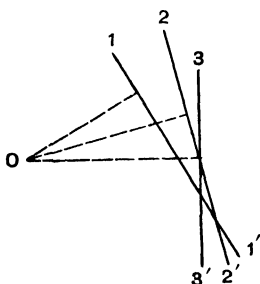


FIG. 5-16

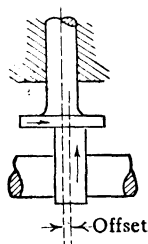


FIG. 5-17

circular disk, free to rotate about the follower axis. The point of contact is only on the axis in "rest" positions and moves out toward B or C as the follower velocity increases. The distances AB and AC must be great enough so that the contact point never passes B or C . By inspection of the diagram, the length of the longest tangent S can be found; AB and AC should be at least equal to S , and preferably slightly greater.

By offsetting the follower slightly, as shown in Fig. 5-17, a slow rotation of this member is induced. This tends to cause even wear on the contact face.

(b) **Pivoted flat-faced follower.** Figure 5-18 illustrates this mechanism, the follower turning about a fixed pivot at B . To construct the cam profile, any point, such as C , on the follower face is selected as a reference point. Arc $C-C'$, with center B , is the path of motion of C , assuming that the follower is to have a total displacement of θ° . The displacement diagram, Fig. 5-19, is drawn in the usual manner, the rectified length of $C-C'$ (a) being used to represent follower displacement. The form of the curve depends on the motion specifications. The construction for a

point on the cam profile at 30° cam displacement is indicated in the figure. Distance x represents the angular displacement of the follower at this instant; this distance is set off along the arc $C-C'$,

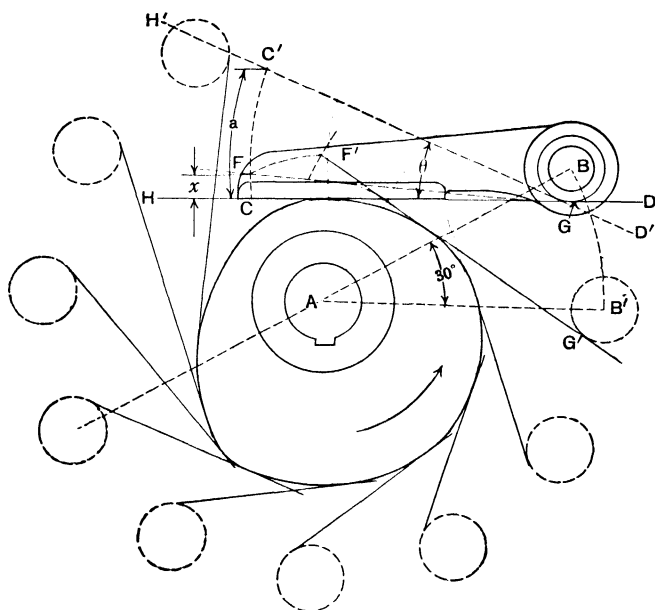


FIG. 5-18

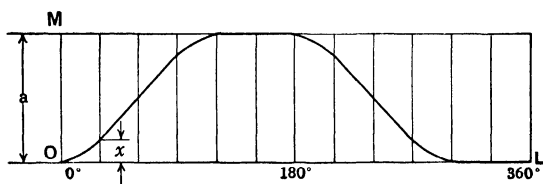


FIG. 5-19

thus giving point F . The follower is next rotated 30° in a sense opposite to that of the cam movement; this causes F to move to F' and B to B' . F' is easily located, since the angle $BAB' = 30^\circ$ and $BF = B'F'$. By drawing, with B' as center, a circle of radius BG , the tangent $F'G'$ will represent the new position of the follower

face. Repetition of this construction for other cam angles gives the series of lines shown in the figure to which the cam profile must be tangent.

14. Positive-motion Cam Mechanisms. This variety was mentioned in Art. 4 as being one in which provision is made for controlling the motion of the follower in both directions by the use of two contact surfaces. For disk cams this is accomplished in the following ways.

(a) By the use of a grooved disk and roller follower, as in Fig. 5-6.

(b) By providing two contact surfaces on the follower, located on opposite sides of the cam axis, both bearing on the same cam. (See Fig. 5-20.)

(c) By using two contact surfaces on the follower as in Type *b*, but causing each to bear on a separate cam. A brief discussion of each type follows.

Type (a). In Fig. 5-6, *a* and *b* are the two contact surfaces. The inner one, *a*, is constructed just as though the cam were of the ordinary non-positive kind. Then circles of diameter equal to that of the roller are drawn at convenient angular intervals, touching the surface *a*. A curve drawn to touch each of these circles on the outside will outline the surface *b*. A certain amount of clearance is necessary, the groove being made slightly wider than the roller.

It will be observed that when the roller rolls against *a* and slides on *b* it turns counterclockwise, whereas when rolling against *b* it turns clockwise. Each of these conditions exists at least once during a revolution, hence the roller must reverse its angular movement at least twice per revolution. This is bound to cause slippage, which may produce excessive wear at some points on the contact surfaces.

Type (b). When a flat-faced follower with reciprocating motion is used and the mechanism has the form shown in Fig. 5-20, the mechanism can be designed to give any required motion for 180°,

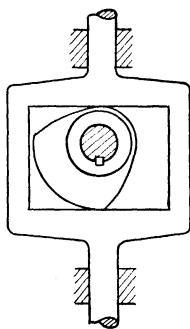


FIG. 5-20

subject only to limitations which apply to any cam with a flat-faced follower. The motion during the other half of the revolution, however, will be the same as that obtained during the first half, since it is caused by contact between the same cam profile and a follower face of the same shape, the only difference being that the direction is reversed. This type is not suitable, therefore, for an application in which the follower motion must be different on the lift and return strokes.

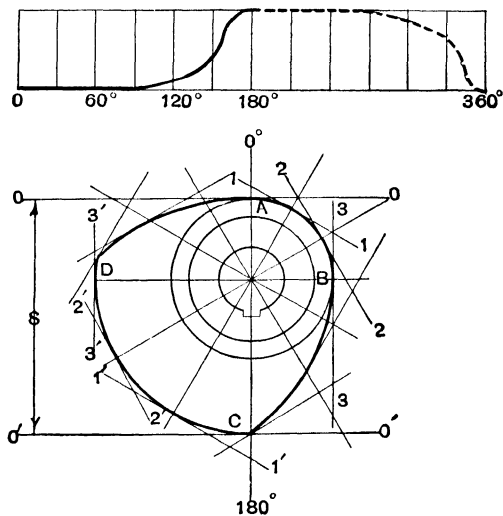


FIG. 5-21

Where the follower has angular motion about a pivot, the motion can be specified for 180° plus the angular displacement of the follower.

In Fig. 5-21 are shown an assumed lift diagram for 180° of motion and the details of construction for a cam mechanism as in Fig. 5-20. The half cam surface ABC is found by the construction of Art. 13. The lines $O-O$, $1-1$, $2-2$, etc., show the positions occupied by the upper face at the ends of equal angular intervals of 30° . The necessary distance S from the upper to the lower face of the follower is evidently equal to

$$\text{Base-circle diameter} + \text{lift}$$

To find the profile of the portion CDA of the cam, draw lines $0'-0'$, $1'-1'$, $2'-2'$, etc., respectively parallel to and at distance S from $0-0$, $1-1$, $2-2$, etc. Curve CDA is then drawn tangent to these lines. Clearance is provided by making the distance between the follower faces somewhat larger than S .

This construction produces a figure such that the distance between any pair of parallel tangents is constant.

Type (c). This consists of two disk cams, mounted on the same shaft, acting on a follower with two contact faces or rollers placed on opposite sides of the camshaft, each bearing on one of the cams. Although more complicated and expensive to make, this type imposes no limitations on the motion as does Type *b* with single disk. Neither does it have the undesirable reversal of roller rotation which occurs in Type *a*.

CYLINDER CAMS

15. Types. These may have a follower guided so as to move in a straight line along an element of the cylinder (Fig. 5-22) or a follower pivoted so as to move about an axis perpendicular to the cam axis (Fig. 5-24). The roller, if cylindrical, cannot have pure rolling contact because of the difference in the surface speed at top and bottom of the groove. Consequently it is sometimes made in the form of the frustum of a cone (Fig. 5-23) with the apex on the axis of revolution of the cam. Although this promotes pure rolling action, it also introduces an undesirable thrust, tending to move the roller away from the cam.

16. Cylinder Cam with Rectilinear Motion of Follower. Figure 5-25 shows the construction of a cylinder cam which meets the following specifications:

Cylinder diameter	6 in.
Roller diameter	$1\frac{1}{4}$ in.
Groove depth	$1\frac{3}{8}$ in.

The roller is cylindrical. The follower moves in a straight line along a cylinder element, rising from initial position with constant acceleration for 45° , with constant velocity for 60° , with constant deceleration for 45° to top position. It then rests for 30° and returns with the same motion as on the lift. The total lift is 4 in.

A cam angle-displacement diagram is first drawn (Fig. 5-25), the base being taken of such length that it represents the cylinder circumference (6π in.) to a convenient scale. Taking centers at a number of points along the curve, and with roller radius, draw

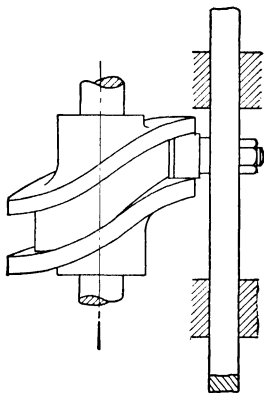


FIG. 5-22

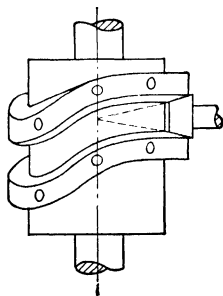


FIG. 5-23

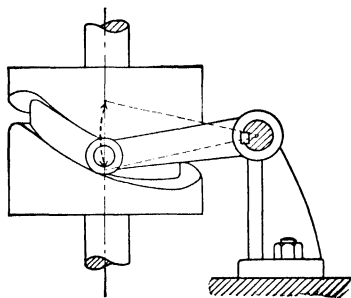


FIG. 5-24

circles as indicated in the figure. Draw curves tangent to these circles above and below; they will be the developed outline of the groove on the cylinder surface. If a templet of this form is made and wrapped around the cylinder, the required profile can be marked on the latter. The development of the root cylinder,

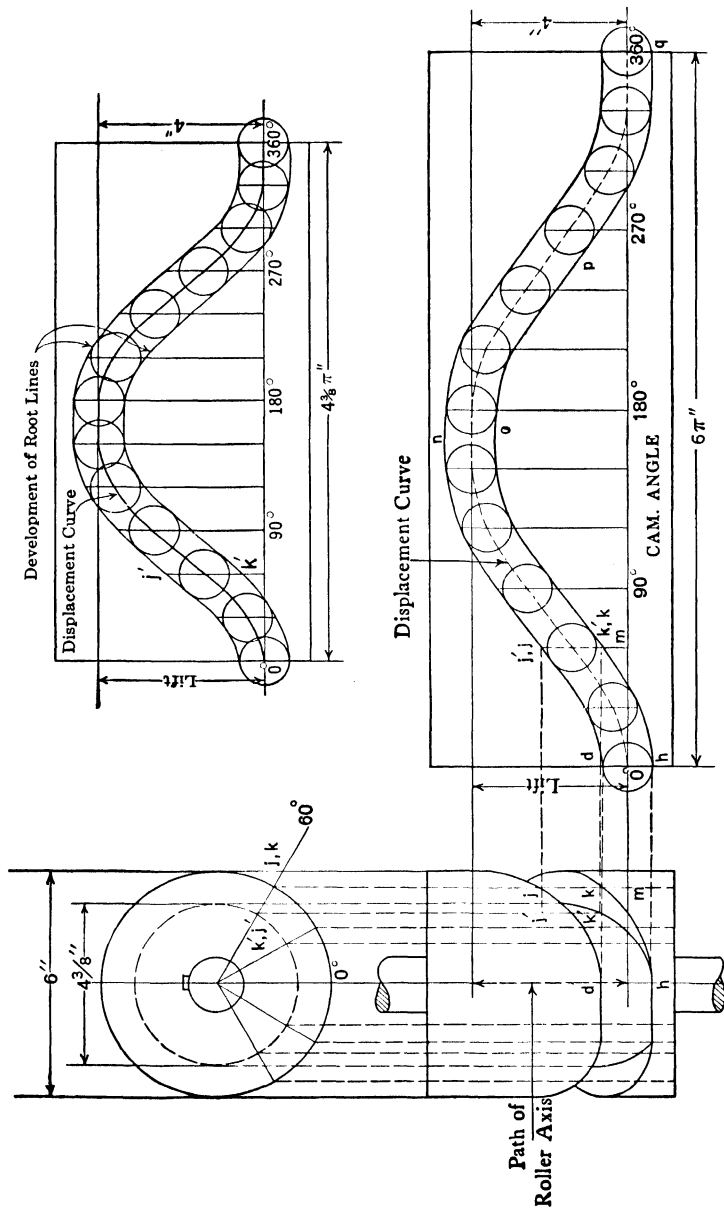


FIG. 5-25

of length $4\frac{3}{8}\pi$ in., is also shown in the figure. For strictly accurate results this is needed for drawing the elevation of the root lines. The shorter method, using only the outer cylinder development, is approximately correct for the root lines. (See Fig. 5-25.)

CIRCULAR-ARC CAMS

17. General. Many cams have profiles formed by circular arcs. There are three reasons for using such outlines in preference to other curves: (1) the drawing office specifications are more easily made for the use of the shop; (2) the process of manufacture is cheaper; (3) the completed cam can be checked more easily and with greater accuracy. Valve cams used in automobile and other internal combustion engines, as well as many others, are often of this class.

By proper choice of the radii and centers of the arcs, theoretical requirements as to follower movements can be approximated very closely. The process of design can be carried out by first drawing a displacement diagram of the desired motion to a large scale and from it laying out the cam. Then, by trial, arcs and radii are chosen which will approximate the true form. Finally, the resulting cam is checked by working back to a displacement diagram which is compared with the original one. If the revision of the cam is found to have altered the displacement curve to an unsatisfactory form, further revision may be necessary.

For high speed cams it is necessary to draw an acceleration curve for the follower, since the spring pressure needed in the non-positive type is dependent to a large extent on the weight of the follower and attached parts, and on the acceleration. Starting with the displacement diagram and treating it as a displacement-time curve, by the method of Art 18, Chapter I, we may construct a velocity-time and an acceleration-time curve, the latter giving the desired information for calculation of the spring.

18. Circular-arc Cams with Flat-faced Follower. In Fig. 5-26 is shown a simple form of cam of this type, consisting of a circular disk rotating about a point O other than its geometric center A . The distance r from the center A to the face of the follower is evidently constant for any position of the cam. Therefore, the

vertical motions of point A and the follower are the same. The vertical motion of A is simple harmonic when the cam rotates at constant angular velocity, since it is then moving in a circle about O at constant speed. It will be observed that B , the point of contact, is at the foot of a perpendicular from A on the cam face. If a perpendicular OC is drawn from O to AB , the following relationships hold, by Art. 15, Chapter I:

$$\text{Follower velocity} = \text{vertical velocity of } A = \omega \cdot OC$$

$$\text{Follower acceleration} = \text{vertical acceleration of } A = \omega^2 \cdot AC$$

where ω = angular velocity of the cam.

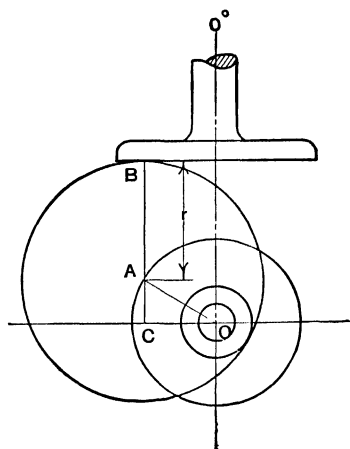


FIG. 5-26

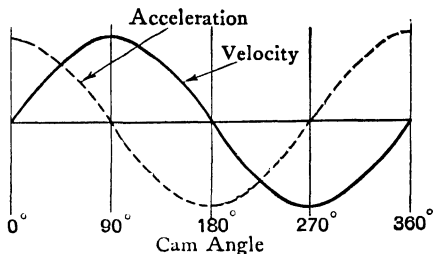


FIG. 5-27

Plotting the lengths OC and AC on a base representing cam angles will therefore determine points on the velocity and acceleration curves for the follower motion, as shown in Fig. 5-27. These are evidently sine and cosine curves.

The same construction may be used for a cam whose profile is made up of several circular curves; the follower motion will then be composed of corresponding sections of harmonics varying in amplitude and phase in accordance with the locations of the center of curvature.

Figure 5-28 illustrates the application to a portion of a cam outline composed of a circular arc KL with center at A' , and LM with center at A . Here the cam is kept stationary and the fol-

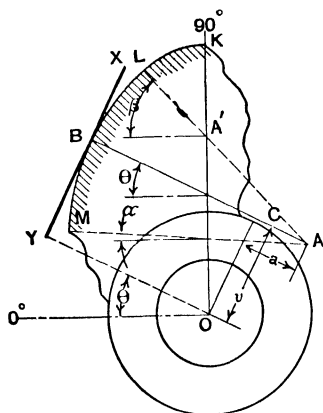


FIG. 5-28

lower is moved around it, as in previous cam constructions. When the angular displacement is θ from a reference position, the follower face lies along XY and touches the cam at the point B at the foot of the perpendicular AB . OC is drawn from O perpendicular to AB . From the above discussion it is evident that OC ($=v$) and AC ($=a$) represent graphically the velocity and acceleration of the follower at cam displacement θ . These distances are therefore transferred to the velocity and acceleration diagrams of

Figs. 5-29 and 5-30 and plotted as ordinates at angle θ . Other points on the curves are obtained in a similar manner. The follower begins contact with the arc LM

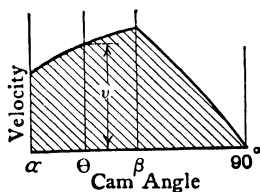


FIG. 5-29

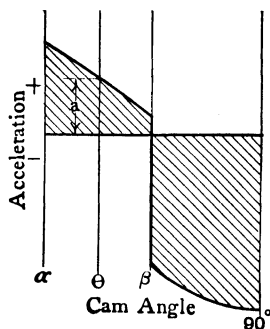


FIG. 5-30

at cam angle α and terminates contact at angle β when the cam is assumed to have counterclockwise rotation. In this event the acceleration represented by AC is positive and has consequent-

ly been plotted above the base line. If C should fall on AB produced, the sign would be the opposite.

Assuming that the cam turns at 900 rpm and that the cam is drawn twice full size, the velocity and acceleration scales can be found as follows.

The angular velocity of the cam (ω) = $2\pi \times \frac{900}{60} = 94.2$ radians per second.

The follower velocity

$$= \omega \times CO \text{ where } CO \text{ is actual size}$$

$$= \omega \times \frac{CO}{2} \text{ where } CO \text{ is twice full size}$$

$$= \frac{94.2}{2} \times CO \text{ in. per sec where } CO \text{ is in inches}$$

$$= 3.92 \times CO \text{ ft per sec}$$

For the velocity curve, therefore, 1 in. = 3.92 ft per sec. Similarly, for the acceleration curve, 1 in. = $\frac{\omega^2}{2 \times 12} = 369$ ft per sec per sec.

19. Circular-arc Cams with Roller Follower.

As pointed out by Goodman in his book "Mechanics Applied to Engineering," the motion of the follower in this mechanism is the same as the motion of the piston in a direct-acting engine of proper proportions. The cam mechanism is shown in solid lines in Fig. 5-31; the equivalent mechanism is indicated in broken lines in the same figure. The crank length R is equal to the distance from the center of curvature of the cam surface to the center of rotation of the cam. The connecting rod has a length equal to the sum of the radius of the cam surface plus the roller radius. Keeping this analogy in mind, it is evident that the follower velocity and acceleration may be determined by the same formulas or by the same

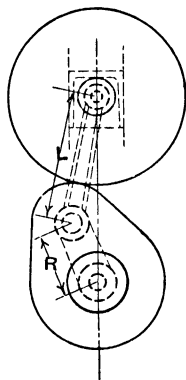


FIG. 5-31

graphical construction which we apply in finding the piston velocity and acceleration in the direct-acting engine.

PROBLEMS — CHAPTER V

1. Plot displacement diagrams for the follower motions as specified in (a) to (e). Show in each case sufficient construction lines, points, and notation to indicate the methods employed.

(a) A follower lifts with constant velocity to highest position during 120° of cam displacement, rests for 60° , and falls with simple harmonic motion during 45° to initial position, where it remains for the balance of the revolution.

(b) A follower lifts with constant and equal acceleration and deceleration to the top of its travel during 180° of cam movement, rests for 30° , returns to its initial position with constant velocity during 120° , and rests for the balance of the revolution.

(c) The follower motion is the same as for (b), except that during the lift period the acceleration is twice as great as the deceleration.

(d) A pivoted follower moves through a total angle of 20° . Its outward motion is accomplished with constant acceleration during a cam displacement of 45° , then at constant velocity for 90° , and finally with constant deceleration for 45° to the end of its travel.

(e) A pivoted follower, starting from the extreme outward position, moves in with constant and equal acceleration and deceleration to the other end of its travel, the total displacement being 25° during a cam movement of 90° . It then rests for 45° and returns to its initial position during 60° with motion of the same character as on the other stroke. A rest period makes up the balance of the revolution.

2. Why is it impracticable to use an unmodified constant-velocity cam at high speeds? How should it be modified in order to obtain best results?

3. A follower lifts $\frac{1}{2}$ in. during a cam displacement of 90° at constant velocity, the cam rotating at a constant speed of 120 rpm. (a) Find the velocity of the follower. (b) If the follower lifts with constant and equal acceleration and deceleration, find the value of the acceleration and the maximum velocity attained.

Ans. (a) 4 in. per sec. (b) 128 in. per sec per sec, 8 in. per sec.

4. A follower lifts $\frac{3}{4}$ in. during a half-revolution of the cam, the latter rotating at a constant speed of 480 rpm. The constant acceleration for the first part of the lift period is three times as great as the constant deceleration during the latter part of this period. Find the value of the acceleration and the cam displacement during which it takes place.

5. In each of the following cases, assume a displacement diagram to be given and show how to plot the cam profile which will give the required motion to the follower. Show sufficient construction lines and notation to indicate the method employed in each case.

(a) Disk cam rotating clockwise, with knife-edge follower moving in a straight line passing to the left of the cam axis.

(b) Disk cam rotating counterclockwise with roller follower having rectilinear motion, the roller center moving in a straight line which intersects the cam axis.

(c) Disk cam rotating counterclockwise with roller follower having rectilinear motion, the roller center moving along a straight line passing somewhat to the right of the cam axis.

(d) Disk cam, turning counterclockwise, having flat-faced follower with rectilinear motion. How is the necessary breadth of the follower face determined in this case?

(e) Disk cam, turning counterclockwise, with flat-faced follower pivoted to the right and somewhat above the cam axis.

6. Assuming that the form of the displacement curve is known, show how to find the cam profiles in each of the following positive-motion cam mechanisms.

(a) Single-disk cam turning clockwise with yoke-type follower having two parallel, flat contact faces. Follower has rectilinear motion.

(b) Single-disk cam turning clockwise with yoke-type follower carrying two rollers. Roller centers move in a straight line passing through the cam axis.

(c) Single-disk cam turning clockwise with yoke-type follower carrying two rollers. Follower pivots about a point on the right side of the cam axis.

(d) Slotted disk cam with two contact faces acting on a roller-type follower. The follower reciprocates along a straight line passing through the cam axis.

CHAPTER VI

MISCELLANEOUS MECHANISMS

1. Ratchets. A ratchet mechanism in its most common form consists of a device whereby two members capable of rotation are connected so that one will rotate the other in a certain sense, but not in the opposite sense.

Its use, however, is not limited to members having rotational motion. Often the driver has a reciprocating straight-line motion and compels linear motion of the driven member in one direction only.

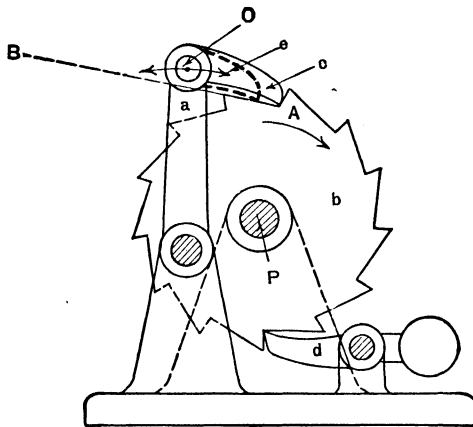


FIG. 6-1. Toothed ratchet.

The ratchet wheel *b* of Fig. 6-1 is turned in a clockwise sense by the action of pawl *c* when driver *a* turns clockwise. When *a* turns counterclockwise, *b* remains stationary. A second pawl, *d*, pivoted on the frame of the mechanism, is sometimes used to prevent the ratchet wheel from running backwards.

Two points should be noted in regard to the teeth on the ratchet wheel.

(a) The normal AB to the face of the tooth at the point of contact should pass between centers O and P . Otherwise the pawl tends to slip out of contact when the driving force is applied.

(b) A certain amount of "lost motion" may occur if the point of the pawl is not against a tooth face at the instant when the driver begins to move in the driving direction. This lost motion

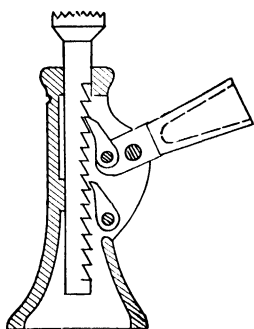


FIG. 6-2. Lifting jack.

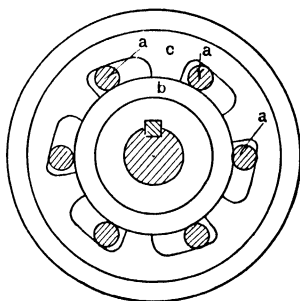


FIG. 6-3. Silent ratchet.

may vary from zero to the tooth pitch as a maximum, depending on conditions. A small pitch, therefore, insures a small amount of lost motion. By the addition of another pawl, e , of different length (see Fig. 6-1), the possible lost motion may be reduced to one-half its former value.

Figure 6-2 shows the mechanism of a lifting jack in which the driven link is given rectilinear motion when the handle is oscillated.

Figure 6-3 shows a form of silent ratchet. Rollers, a , are placed in slots which taper. Rotation of the driver b in a clockwise sense causes the rollers to move toward the narrow ends of the slots where they wedge tightly and thus lock together driver b and driven member c . This mechanism can be arranged to allow little lost motion, and the noisy action of the pawl-and-tooth type is avoided. Positive action is lacking, since friction alone is responsible for the transmission of motion to the driven member.

2. Elliptical Trammels. (Fig. 6-4.) This device is serviceable in drawing ellipses. In a form somewhat different from that

shown in the figure, it has been employed as an elliptical chuck for machining parts of elliptical section. It is a four-link mechanism containing two sliding and two turning pairs. The fixed link c has in this case elements of two sliding pairs. Any point A

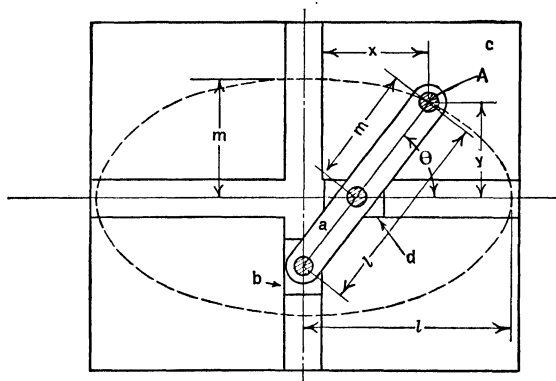


FIG. 6-4. Elliptical trammels.

on arm a can be shown to trace out an elliptical path on c . This is proved as follows.

Let x and y represent the coordinates of point A for any position of the mechanism. From the figure,

$$y = m \sin \theta$$

$$x = l \cos \theta$$

Hence,

$$\frac{x^2}{l^2} + \frac{y^2}{m^2} = \sin^2 \theta + \cos^2 \theta = 1$$

This is the equation for an ellipse with major axis equal to $2l$ and minor axis equal to $2m$.

3. Straight-line Motions. Where a link is required to have rectilinear motion, constraint is generally effected by the use of a "slide and guide" with plane surfaces in contact. Rectilinear motion can be obtained, however, by several mechanisms containing turning pairs only. These are generally termed straight-line motions. Such motions can be divided into two classes: (a) ap-

proximate straight-line motions, and (b) accurate straight-line motions.

The Watt straight-line motion is of historical interest only, since it is little used nowadays.

Watt employed it in his engines as a substitute for the present-day crosshead and guide, because in his time it was difficult to form accurate plane surfaces in metal. The device consists of a quadric-crank mechanism, a, b, c, d , as in Fig. 6-5. A point P on b traces out the dotted path shown in the figure. This will

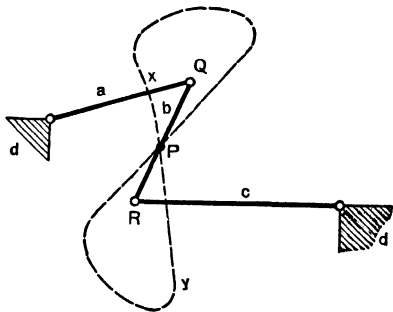


FIG. 6-5. Watt straight-line motion.

be observed to have a portion xy which is approximately straight, provided that point P is so selected that

$$PQ : PR = c : a$$

4. Crosby Indicator Motion. A mechanism shown in Fig. 6-6, composed of links connected by turning pairs, is used to connect the indicator piston rod with the recording pencil on the Crosby engine indicator. The pin A , connected to the end of the piston rod, is constrained to move in a vertical straight line. The pencil point at P traces out the indicator card on the drum b . In order that the instrument may present an accurate record of the pressure changes in the engine cylinder, it is necessary that the linkage connecting the piston rod and recording point fulfill the following requirements.

(a) It should cause the point P to move in a straight line parallel to the direction of motion of the indicator piston.

(b) It should magnify the piston motion in order to draw an indicator card of reasonable size, the ratio of magnification being constant for all positions of the mechanism.

Neither of these objects is accomplished with mathematical accuracy by the Crosby motion, but the errors involved are so small as to be negligible for practical uses.

The Peaucellier straight-line motion (Fig. 6-7) belongs to the class of accurate straight-line motions, since it can be shown that

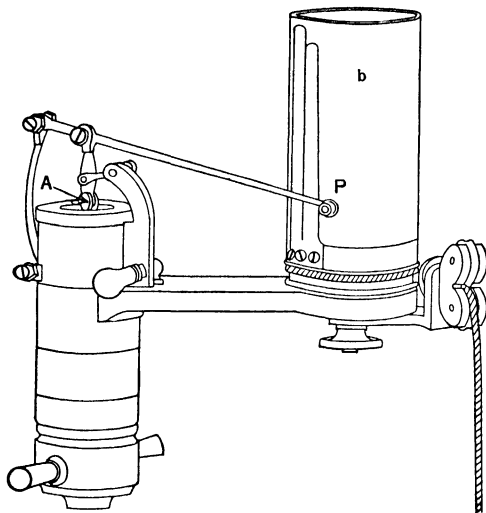


FIG. 6-6. Crosby indicator.

point *C* moves in a straight-line path *CD*, perpendicular to the center line of the fixed link *f*. The lengths of the links must have the following relative values:

$$a = b = c = d$$

$$e = f$$

$$g = h$$

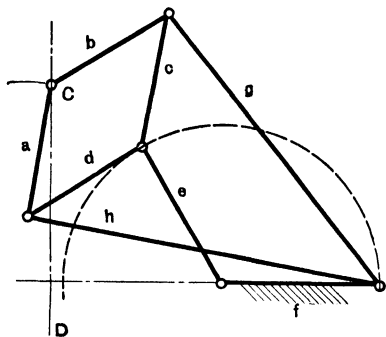


FIG. 6-7. Peaucellier straight-line motion.

This mechanism has too many joints to be of great practical importance.

5. The Pantagraph is generally used as a means of reproducing drawings or maps to a smaller or larger scale. It has

also been employed as a reducing motion in connection with engine indicators. Two forms are shown in Figs. 6-8 and 6-9. In both figures the length OA equals BC ; also OC equals AB . The

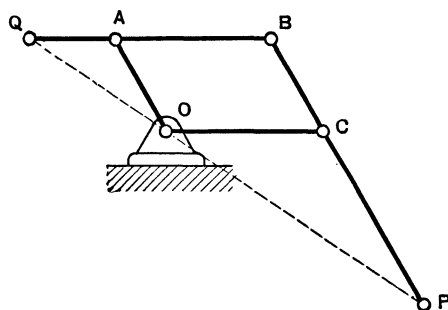


FIG. 6-8. Pantagraph.

figure $OABC$ is therefore a parallelogram, O being the fixed point or pole.

If the mechanism is placed in any position, any point P on BC is selected, and a point Q on AB is found such that O , Q , P fall in

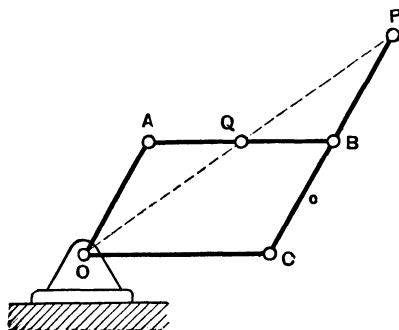


FIG. 6-9. Pantagraph.

the same straight line, it can be proved that points P and Q will trace out similar figures. In the practical form of the pantagraph, P forms the tracer point, which is run around a map or figure, and a copying point at Q will reproduce the diagram to a smaller scale.

6. Geneva Stop. In certain automatic machinery the Geneva stop is used when it is desired to obtain alternate periods of rest and angular motion for a driven member, when the driver rotates continuously in the same sense.

We find examples of it in watches, motion picture machines, can-making machinery, and indexing devices employed in the machine shop.

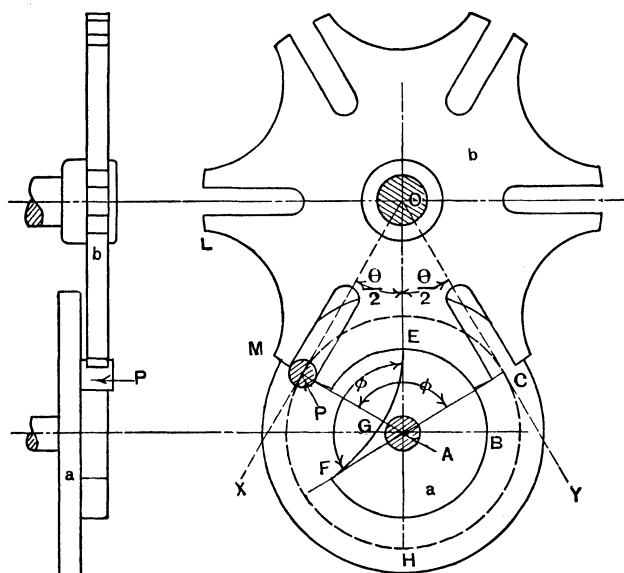


FIG. 6-10. Geneva stop.

It is suitable for movements of the driven link not exceeding 90° for each revolution of the driver. In Fig. 6-10 the driver *a* consists of a circular disk to which is attached a driving pin *P* and a second disk *EBFG*. The pin *P* engages for a certain portion of each revolution with radial slots in the driven member *b* which it carries forward through an angle θ . When pin *P* passes out of a slot the surface *EBF* engages one of the geometrically similar surfaces *LM* on the driven member and holds it stationary until *P* enters the next slot.

In order that the driven member may be started and stopped without shock, it is necessary that the mechanism be designed in such a way that in the position illustrated the lines OX , OY are tangent to the circle PCH through the axis of the driving pin.

Thus, in laying out the mechanism, an angle $\theta/2$ is measured on each side of the line OH , where θ is the desired angular movement of the driven shaft.

Any circle, as PCH , tangent to OX and OY , may be used as the path of the driving-pin axis. The radius of this circle is the crank length for the driving pin, and its center locates the axis of the driving disk. The arc EGF is chosen so that the surface will clear the points of the driven disk. The angle EAF (ϕ) must be made equal to the angle PAC in order that the driven member may be alternately locked in position and released at the proper instants.

PART II

MACHINE DESIGN

CHAPTER VII

FUNDAMENTAL DEFINITIONS AND CONSIDERATIONS

Before entering into a discussion of the process of the design of a machine or any of its parts, it is essential that certain terms which are frequently used be clearly understood. The more important of these are defined briefly in the following pages.

TERMS RELATING TO THE EFFECT OF LOADS ON MEMBERS

1. Load and Stress. A load is any external force applied to a machine member. Loads are classified according to the effect produced upon the internal structure of the member and also according to the manner of application. The former comprises tension, compression, and shear; the latter, live and dead loading.

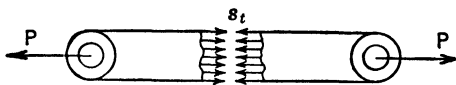


FIG. 7-1. Bar subjected to pure tension.

When a load is applied to a member the physical effect produced is a change of shape. The line of action of a direct tensile load, Fig. 7-1, passes through the center of gravity of every section, increasing the length while simultaneously decreasing the width and thickness. The internal structure of the bar resists the external load action, and internal forces (s) are set up which oppose the external forces. With direct tension loading, the internal forces distribute themselves uniformly over the cross section. To estab-

lish equilibrium the summation of the uniformly distributed internal forces must equal the external forces. The internal forces are known as stresses, and since they have a uniform distribution the following statement applies:

$$P = s \times A \quad (7-1)$$

where P = external force or load

s = stress intensity or unit stress

A = area of cross section at right angles to the line of action of the load.

In American practice P is measured in pounds and A in square inches; hence the expression for s results in pounds per square inch. Thus the stress intensity, or unit stress, is the force per unit area, pounds per square inch. Tension and compression loads, which act along the axis of the member, and pure shear loads, which act at right angles to it, may be treated by equation (7-1).

2. Deformation. Deformation is a change of shape. The total dimensional change is known as total deformation. Unit deformation is the change per unit of original length.

3. Unit Stress — Unit Deformation Diagram. Stress and deformation are conjugate in action since one cannot exist without the other. For this reason it is essential that the designer have an accurate conception of the exact relation between the stress in a machine member and its accompanying deformation. A member may be designed suitably as to strength, i.e., with a low unit stress, but fail to fulfill its function, especially if it is a constraining member, because of undue distortion. Another material may be substituted to remedy this defect. If the size and form of the original member are retained, the new material must have less deformation under the same stress which existed in the unsatisfactory member; i.e., it must be stiffer. The slope of the straight-line portion of the curve of Fig. 7-2 is proportional to the stiffness of the material and may be used to compare the relative stiffness of two or more materials. Referring to Fig. 7-2: A is the characteristic tension-test curve of a mild steel; B , that of brass. By inspection, steel is stiffer than brass.

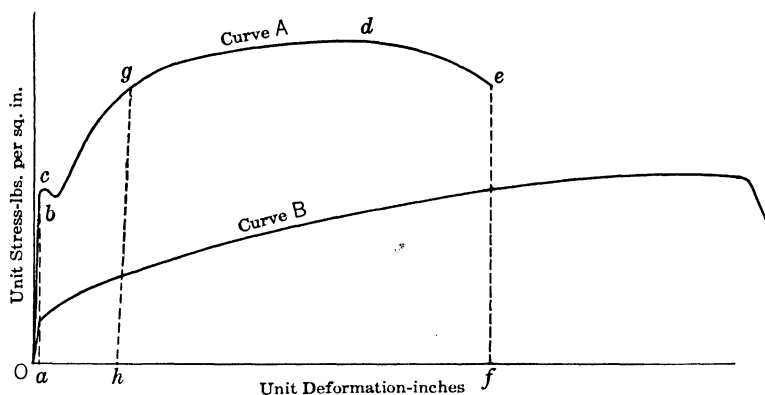


FIG. 7-2. Characteristic tension-test curves of mild steel and brass.

4. Bending Moments. Referring to Figs. 7-3 and 7-4: non-concurrent forces acting on a body produce moments which tend to make it rotate. If the body has no angular acceleration the sum of the moments of *all* the forces acting on the body, about

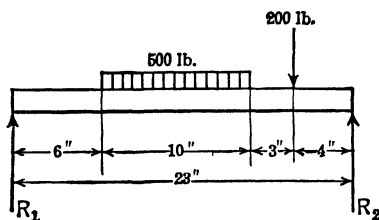


FIG. 7-3

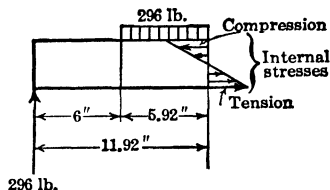


FIG. 7-4

any section, must be zero. That is to say, the sum of the moments of the external forces about that section must equal the moments of the internal stresses set up at the section. In short, the bending moment in any simple beam at any section is equal to the moment of either reaction minus the moments of all the loads between the reaction and the section considered. From the laws of mechanics we know that the maximum moment occurs where the vertical shear is zero, that is, where the sum of all the vertical forces either to the right or to the left of the section passes through zero. They

also tell us that where the neutral surface is straight, or nearly so, the internal stress couple is equal to the product of the stress induced at the outer surface and the section modulus of the member at the section considered. Equating this to the bending moment gives

$$M = s \frac{I}{c} \quad (7-2)$$

where M = the algebraic sum of the moments of all the external forces to the right or left of the section

I = rectangular moment of inertia of the section

c = distance from the neutral or gravity axis of the section to the outermost surface in tension or compression, as the case may be

s = maximum stress intensity.

Example. A simple beam having a rectangular cross section is $\frac{1}{2}$ in. wide and 3 in. deep. It is loaded and supported as shown in Fig. 7-3. Determine the maximum stress induced at the upper surface.

Solution.

$$R_1 = \frac{500 \times 12 + 200 \times 4}{23} = 296 \text{ lb}$$

$$R_2 = \frac{500 \times 11 + 200 \times 19}{23} = 404 \text{ lb}$$

Maximum moment occurs where the vertical shear is zero, that is, at

$$6 + \frac{296}{500} = 11.92 \text{ in. from } R_1$$

Cutting the beam at this point and considering the portion to the left of the section as a free body, we find it acted on by the forces shown in Fig. 7-4. The bending moment at the section of zero vertical shear is

$$M_{\max} = 296 \times 11.92 - 296 \times \frac{5.92}{2} = 2652 \text{ lb-in.}$$

Section modulus for a rectangle is $\frac{bh^2}{6}$. Therefore

$$\frac{I}{c} = \frac{\frac{1}{2} \times (3)^2}{6} = \frac{3}{4}$$

The maximum induced stress is

$$s = M \frac{c}{I} = \frac{2652 \times 4}{3} = 3536 \text{ lb per sq in.}$$

IMPORTANT PHYSICAL PROPERTIES OF MATERIALS

5. Elastic Limit. The point b , or elastic limit, is the stress corresponding to the highest point on the curve (Fig. 7-2, curve A) from which entire release of load does not leave a permanent stress or deformation in the material. The line Ob is sensibly straight. If the steel specimen is loaded until a unit stress b is reached, a unit deformation Oa results. Suppose the load to be gradually released. Upon successive decrements of load the corresponding deformation recedes from a toward O until, with zero load, no residual stress or deformation is apparent. The exact point is difficult to obtain in test, for it requires instruments of high accuracy and involves a considerable expenditure of time and effort.

6. Yield Point. Beyond b stress is not proportional to deformation, and the shape of the curve from b to e depends greatly upon the material under test. In such a ductile material as mild steel, a sharp bend occurs shortly after the elastic limit is reached. The point, c , of bend or yield is easy to determine with reasonable accuracy. The yield point may be defined as the unit stress corresponding to the point on the curve beyond which small increments of stress produce marked increments of deformation. In view of the ease with which the yield point of a material can be secured and the close proximity of this point to the elastic limit, the designer uses the yield point, instead of the elastic limit, as the limiting stress which cannot be exceeded.

7. Ultimate or Maximum Strength. The ultimate or maximum strength of a material is that stress intensity determined by considering the maximum test load to act over the original area of

the test specimen. The highest point on curve *A* of Fig. 7-2 is designated by *d* and denotes, in the engineering sense, the highest stress intensity within the material.

8. Breaking Strength. The breaking strength of a material is the stress intensity at the break per square inch of the original cross section of the test specimen. In ductile materials under tensile load, a distinct "necking" of the test specimen occurs shortly after the ultimate stress; i.e., ultimate strength is attained. A decrease in the load the specimen can carry accompanies the reduction in area, until fracture is effected. Thus in ductile materials the breaking strength is less than the ultimate strength. The designer is seldom concerned with the breaking strength where it differs from the ultimate.

9. Modulus of Elasticity. It has been noted that the line *Ob*, Fig. 7-2, is sensibly straight. Therefore unit stress is directly proportional to unit deformation between *O* and *b*. The quotient obtained by dividing the unit stress by the corresponding unit deformation, provided the elastic limit has not been passed, is designated the modulus of elasticity, *E*. Shear and compression may be similarly considered but, when not specially designated, *E* may be considered the modulus of elasticity due to the direct loading of tension. The following equation shows the relation between unit stress, unit deformation, and the modulus of elasticity for a material under a stress *within the elastic limit*:

$$\frac{s}{\delta} = E \quad (7-3)$$

where *s* = unit stress in tension, compression or shear

δ = unit deformation

E = modulus of elasticity.

It will be seen that the unit deformation of a material is inversely proportional to the value of *E*. A usual value of *E* for steel is 30,000,000 and for aluminum 10,000,000. Hence, for the same induced stress, within the elastic limit, the deformation of the latter is approximately three times that of the former. Steel is about three times as rigid as aluminum.

10. Work. When a force produces acceleration or maintains motion against a resistance it is said to do work. Work is a compound quantity involving both force and space or distance. Time does not enter. The measure of work is the product of the distance moved through by the body acted upon and the component of the acting force in the direction of motion of the body. Mathematically stated,

$$\text{Work} = F \times d \quad (7-4)$$

where F = component of the applied force in the direction of motion

d = distance moved through.

The definition given above holds whether the force produces or opposes motion.

Power is the rate of doing work or, in other words, the amount of work done in a unit of time. Mathematically stated,

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{F \times d}{T} \quad (7-5)$$

where T = time in which the work is accomplished. If the motion of the force is uniform through any time interval, the corresponding distance may be used. If, however, the motion of the force is variable, both the time and the distance must be infinitesimal. In either case the quotient of $\frac{d}{T}$ or $\frac{d}{dT}$ (d) is the velocity of the force, and power may be said to be the product.

Power = velocity $\times$ component of force in the direction of motion

$$= v \times F \quad (7-6)$$

The statements given above concerning work and power are of great importance and should be thoroughly mastered by the student.

11. The Effect of Sudden and Impulsive Application of the Load. The manner in which the load is applied to the members is of great importance. Loads applied suddenly produce stresses

greatly in excess of those which the same loads would produce if applied gradually. When a force is applied to a part of a machine, the machine part distorts somewhat and allows the force to move through a certain small distance; that is to say, the applied force does work on the part acted upon. If equilibrium is to be established, the work absorbed by the part must equal the work done by the applied force. When the applied force impinges with initial velocity the resisting member must absorb not only the work done after impingement, but also whatever kinetic energy the impinging body contains at the instant of contact.

Let

v = the velocity, in feet per second, at which the body is moving

h = height in inches through which the body would have to fall to attain the velocity v

W = weight of the body in pounds

g = acceleration due to gravity in free fall = 32.2 ft per sec per sec

A = area of resisting body

P = total resisting force

s = unit resisting stress

If the impinging body of weight W drops through a height of h inches before striking the resisting body and causes a deflection of Δ inches vertically after impingement, then the total vertical distance moved through by the impinging body will be $(h + \Delta)$ in. The total work done by this body will be $W(h + \Delta)$ in.-lb. The work absorbed by the resisting body is equal to the product of the maximum resisting force P and half the deflection, Δ . The work done and the work absorbed must be equal if the body is brought to rest, and we can write this equality

$$W(h + \Delta) = \frac{P\Delta}{2} = \frac{sA\Delta}{2} \quad (7-7)$$

Solving this expression for the stress, s ,

$$s = \frac{2W}{A} \left(\frac{h + \Delta}{\Delta} \right) \quad (7-8)$$

If the force is applied suddenly, but without initial velocity, h becomes zero and

$$s = \frac{2W}{A} \quad (7-9)$$

12. Effect of Repeated Application of the Load. Repeated application of a load will often cause a member to fail, even when the load is not great enough to stress the member to its elastic limit. This phenomenon is popularly referred to as being caused by the "crystallization" of the material. The popular notion is entirely erroneous, for the microscope has shown that all metallic materials of construction are crystalline in structure and that no perceptible change is caused by repeated application of the load.

A more authoritative designation is "the fatigue of metals," and this term is generally used in discussions of the subject, although it is no more accurately descriptive of what happens than is the term "crystallization."

Investigations show that the inability of metals to withstand repeated applications of fairly high stress is due to a lack of homogeneity in the material. This lack of homogeneity may be due to any one, or a combination, of the following causes:

(1) Internal:

- (a) Blow holes.
- (b) Pipes.
- (c) Slag inclusions.
- (d) Irregularity of crystalline structure.
- (e) Initial stresses due to mechanical work or heat treatment.

(2) External:

- (a) Shoulders and notches.
- (b) Scratches and tool marks.

Of these causes of nonhomogeneity, the first three can, in many cases, be greatly reduced in effect or entirely eliminated by the exercise of care in the production of the raw material. The external causes can also be guarded against by careful design and

workmanship in producing the finished article. The irregularity of crystalline structure is inherent in the material.

The actual process of failure of a member under repeated stress seems to be about as follows: Owing to the irregularity, a localized stress is set up which is in excess of the ability of the material at that particular point to withstand, and localized failure ensues. Professor H. F. Moore says, "Owing to the minute area over which it exists this localized failure produces no appreciable effect under a single load, but under a load repeated many times there is started from this area a minute crack at the root of which there exists high localized stress, and which under repetition of stress spreads until it finally causes failure."

The results obtained at the Illinois Experiment Station show that for a complete reversal of stress in bending the endurance limit may be safely taken as one-half the ultimate static strength for wrought ferrous materials. For stresses involving less than a complete reversal in bending, the following equation is proposed:

$$u_r = \frac{u}{2} \left(\frac{r + 3}{2} \right) \quad (7-10)$$

where u_r = the endurance limit for the stress ratio r

r = algebraic ratio of minimum to maximum stress occurring in the cycle (for complete reversal $r = -1$)

u = ultimate static strength of material.

Moore also tentatively recommends¹ that for completely reversed axial stresses the endurance limit be taken as 60 per cent of the endurance limit under reversed flexural stress. This value holds for complete reversal only. For all values of r from $+1$ to -1 the endurance limit may be stated as

$$u_r = 0.35u(1.86 + r) \quad (7-11)$$

The term "endurance limit" as used in the foregoing may be defined as the unit stress below which a metal is capable of withstanding an infinitely large number of applications of stress.

¹ Illinois Engineering Experiment Station Bulletin 142.

In other words, the endurance limit is the ultimate strength of the material under the particular conditions of loading considered.

13. Properties of Common Engineering Materials. The selection of the material to be used in making a machine part rests upon a sound compromise among many factors. Chief among these are strength, rigidity, weight, ease of manufacture, availability, resistance to heat and corrosion, cost, and appearance. In every case the material selected must function satisfactorily under the conditions imposed, and should do this with the minimum of first and maintenance costs.

Although the engineer uses in one way or another practically every known material, the vast majority of machine parts are made from a small number of materials. These, together with a brief description, are given below.

Cast iron. Cast iron is one of the cheapest and most abundant of engineering materials. It is readily cast into any desired shape, and resists well corrosion and a moderate amount of heat. If, however, it is subjected to prolonged heating at temperatures above 700°F it increases in size and decreases in strength. In the gray form it is easily machined. The white form is exceedingly hard and brittle and cannot be used alone. It is formed on the surface of gray iron by chilling and is valuable as a wear resistor. Cast iron, either gray or white, is unsuited to shock loading. Gray iron forms a satisfactory rubbing surface, if well supplied with lubricant, and not too heavily loaded.

Cast iron finds its principal uses in frames, cylinders, pistons, and other parts where great strength or lightness is not of the first importance.

The three general classes which differ from each other greatly both in chemical composition and physical properties are

- (a) Gray iron, in which a large part of the carbon is in the free or graphitic state.
- (b) White iron, in which most of the carbon is chemically combined with the iron.
- (c) Malleable iron, which is white iron that has been decarbonized by heating 60 to 72 hours at a temperature of about 1600°F in contact with an oxidizing material such as hammer scale.

Malleable cast iron has physical properties closely resembling those of mild steel. It is soft, quite ductile, can be bent or twisted to a considerable degree without failing, and can be machined without causing a serious reduction in strength. It finds its principal use as a substitute for forgings for such parts as connecting rods of pumps, gas engine valve rockers, and pipe fittings. Parts made from it have nearly the same properties as forgings of mild steel and are much more cheaply produced.

Steels. Steels may be divided into four general groups:

- (a) Carbon steels, forged.
- (b) Alloy steels, forged.
- (c) Stainless steel.
- (d) Cast steels, usually carbon steels.

Of these the forged **carbon steels** are by far the oldest and the most widely used. They present a wide range of physical properties, depending upon the percentage of carbon in them and the heat treatment to which they have been subjected. The low carbon steels are easily worked by the usual tools. They will stand a considerable amount of cold working without failing and are therefore adaptable to flanging and cold pressing. The higher carbon steels are readily worked in the annealed state but, if heat treated, are difficult to work except by grinding. Heat treatment causes volumetric changes that are likely to distort thin sections or cause severe internal stresses to be set up if abrupt changes of section are present in the piece. Steel does not resist corrosion well, and both wrought iron and cast iron are to be preferred where corrosion is a serious factor. Carbon steels are cheap and easily obtained and are generally used for all parts where a fair degree of strength combined with reliability and resistance to shock and repeated loading is desired.

Alloy steels are steels which contain a sufficient quantity of such elements as nickel, chromium, vanadium, tungsten, and molybdenum to affect, in a marked manner, the critical temperatures and physical properties. They should always be heat treated, as in the annealed condition the physical properties differ but little from those of carbon steels having the same carbon content. Heat-treated alloy steels show greatly increased strength

and ductility, and also an increased capacity to resist abrasion. They are expensive and not so plentiful as carbon steels. The endurance limit of these steels is high, but the ratio of endurance limit to elastic limit is not nearly so high as for the very low carbon steels. Alloy steels are used where great strength and resistance to shock and repeated loading are required and where expense is a minor item.

Stainless steels, containing large amounts of chromium and nickel, are highly resistant to corrosion by action of the weather, salt water, alkalies, or acids. Probably the most resistant is the one containing 18 per cent chromium and 8 per cent nickel, commonly known as 18-8 stainless. In general these steels are fairly strong, quite tough, and stay bright for long periods of time, especially if polished. They are, however, rather soft and have a low elastic limit. They cannot be hardened to the same extent as can the ordinary carbon and alloy steels. They are also much more expensive.

Cast steel has properties intermediate between those of forged steel and cast or malleable iron, and is used where greater strength than is obtainable from cast iron is required, and where the part has such a form as to make forging difficult or impossible. Unless great care is taken in the design and manufacture, steel castings may be unsound and unreliable. Cast steel has a much higher shrinkage than cast iron, and this necessitates the use of generous fillets and the avoidance of abrupt changes in section. Owing to the high degree of skill required for manipulation and the rapid deterioration of the furnaces and ladles used in producing them, steel castings cost more than cast or malleable iron, but are cheaper than forgings.

Brasses and bronzes are alloys of copper, the principal other ingredient being zinc in the former and tin in the latter. The general physical characteristics of the two classes of metals are the same. Both machine readily, are highly resistant to corrosion, and form excellent bearing surfaces when used with wrought iron or steel. Bronze resists corrosion better than brass. Both materials are extensively used for bearing linings to resist pressures greater than can be carried by babbitt, also for valves, condenser

tubes, pumps, and marine fittings where considerable strength combined with great resistance to corrosion is desired.

Aluminum is the lightest of all metallic engineering materials in common use. In the rolled state the tensile strength varies from 12,000 lb per sq in. for soft-annealed sheets to 22,000 lb per sq in. for hard-rolled, the ductility varying from 30 per cent in 2 in. for the former to 2 per cent for the latter. Special alloys which may be forged and heat treated may have an ultimate tensile strength of 55,000 lb per sq in. and a ductility of 16 per cent in 2 in. In the pure state aluminum is difficult to work, and can be soldered or welded only if special precautions are taken to remove the oxide and a special solder is used. It is highly resistant to corrosion by nitric acid or atmospheric agencies, but is readily attacked by alkalies. Its coefficients of expansion and thermal conductivity are high. For casting purposes aluminum is usually alloyed with copper, zinc, magnesium, or a combination of these elements. The resulting alloys have many of the characteristics of pure aluminum, but yield better castings. They are somewhat less resistant to corrosion than pure aluminum.

For ordinary design purposes the tensile strength of aluminum castings may be taken at 18,000 to 20,000 lb per sq in. Owing to the high coefficient of expansion the same precautions noted under steel castings must be observed in the design of parts to be made from cast aluminum. Aluminum melts at 1216°F and is therefore unsuited to parts subjected to high temperatures.

Babbitts are a class of alloys of tin, copper, antimony, and, sometimes, lead. Because the babbitts have low coefficients of friction when used in combination with steel, do not cut the journal nor adhere to it, and are easily and cheaply placed in the bearing, these alloys are widely used for bearing surfaces. They are weak and for that reason are used for surfacing only, the supporting part of the bearing being made of some stronger material, usually cast iron.

Plastics are made by combining fabric, paper, or wood flour with some synthetic resin. The mixture is then molded to the form desired and consolidated under heat and pressure. The resultant material is about half as strong as cast iron, is light,

TABLE 7-1
Properties of Materials

Material	Tensile Strength	Compressive Strength	Yield Point	Modulus of Elasticity	Elongation in 2 in. %	Remarks
<i>Cast iron, gray</i>	20,000	100,000	None	15,000,000	None	Easily cast and machined.
<i>Cast iron, nickel</i>	37,000			15,000,000		Used in place of forgings.
<i>Malleable iron</i>	56,000	56,000	37,000	25,000,000	22	
<i>Cast steel</i>						Shearing strength of steels is about 75% of the ultimate tensile strength.
Carbon (hard)	80,000	80,000	36,000	30,000,000	17	
Carbon (soft)	60,000	60,000	28,000	30,000,000	24	
Nickel (heat-treated)	125,000	125,000	98,000	30,000,000	16.5	
<i>Forged steel</i>						
(Carbon)						
S.A.E. 1010	58,000	58,000	43,000	30,000,000	40	
1020	65,000	65,000	45,000	30,000,000	35	
1035	80,000	80,000	50,000	30,000,000	32	
1045	108,000	108,000	70,000	30,000,000	20	
(Alloy) nickel						
S.A.E. 2330	82,000 to 180,000	82,000 to 180,000	60,000 to 150,000	30,000,000	26 to 15	
Ni-Cr S.A.E.						
3130	92,000 to 190,000	92,000 to 190,000	68,000 to 160,000	30,000,000	29 to 10	
Mo S.A.E.						
4140	110,000 to 180,000	110,000 to 180,000	85,000 to 155,000	30,000,000	21 to 10	
(Stainless) 18-8	85,000 to 95,000	85,000 to 95,000	30,000 to 40,000	30,000,000	55	
<i>Aluminum</i>						
Wrought alloy	13,000 to 61,000	13,000 to 61,000	4000 to 46,000	10,300,000	35 to 10	Endurance limit 5000 to 15,000
Sand cast	19,000 to 40,000	19,000 to 40,000	9000 to 27,000	10,000,000	4 to 2	Endurance limit 6500 to 7000
<i>Brass (cast)</i>	45,000	60,000		13,000,000	25	65% Cu, 35% Zn
<i>Bronze (cast manganese)</i>	65,000			15,000,000	25	
<i>Everdur (hard)</i>	113,000		75,000		5	
<i>Plastics</i>	6000 to 12,000			17,000,000		

resists water, acids, and alkalis, and has a good appearance. Moreover, it can be machined readily if desired. Because of its adaptability to mass production, it is finding a wide use for a variety of machine elements ranging from gears to lever handles.

14. Practical Considerations. In fixing upon the material to be used for any particular member the question of suitability cannot usually be separated from that of cost. A machine must be produced at as low a cost as is compatible with proper functioning of the parts in order that its selling price may be sufficiently low to give a reasonable profit and a wide market. In order that these ends may be attained the first cost of the material not only should be as low as possible, but also the cost of working it into the desired form must be low. Thus alloy steel could be used for the crankshaft of an ordinary steam engine and would be found to function with entire satisfaction. Its initial and machining costs would, however, be high compared with those of a crankshaft made from carbon steel which, though necessarily heavier for the same strength, would function well. If sold in competition with an engine equipped with a carbon-steel shaft the engine equipped with an alloy-steel shaft would be compelled to accept a smaller profit or lose the business. The designer should always remember that cost is one of the prime factors in manufacturing, and that, unless a machine can be made at a cost sufficiently low to enable its sale at a reasonable profit, its design and manufacture are a waste of time.

Having determined the magnitude and character of the forces acting on a machine member and selected the material with due regard for the nature of the work to be done by it, and for its cost, the last step in proportioning the part is the assignment of dimensions. In doing this the designer must make allowance for a number of factors in addition to those mentioned. The most important of these are:

- (a) Unknown and unforeseen loads.
- (b) Cooling stresses.
- (c) Defects in the material.
- (d) Inaccuracies in mathematical analysis.
- (e) Stresses due to handling.
- (f) Casting qualities.

Owing to the first four of these items the stress used in rational equations by the designer is greatly below that required to cause failure of the material used. The ratio of the ultimate static strength of the material, as determined by the ordinary laboratory methods, to the stress used in the design of a member is called the apparent factor of safety. It is obvious that the more uncertain any of the items mentioned above is, the lower must be the working stress and therefore the larger must be the factor of safety in order to insure the member against failure.

If the actual strength of the material under the conditions of loading is known and used in place of the ultimate static strength the ratio is called the real factor of safety. Thus in a member subjected to repeated, reversed bending the ratio of the ultimate strength in tension or compression to the working stress is the apparent factor of safety, whereas the ratio of the endurance limit to the same working stress is the real factor of safety.

The last two items are of a different nature. In being handled in the shop and in the process of erection a machine member may be subjected to loading that is entirely different in character and magnitude from that imposed by service. The designer must see to it that no dimension is so small as to make the piece fragile and

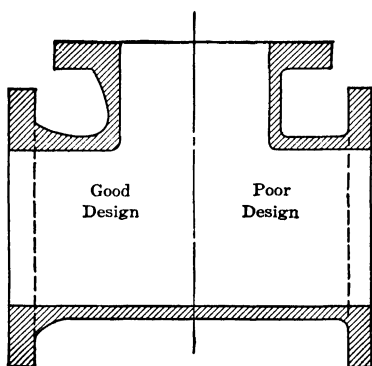


FIG. 7-5

likely to break in the handling incidental to manufacture, transportation, and erection. He must also see that the castings are sufficiently thick to prevent freezing of the metal before the mold is completely filled. He must also make allowance for cores shifting and producing walls that are too thin on one side. In making these allowances the designer should be guided by his experience and by what has been

found to be good or satisfactory practice.

Figure 7-5 shows a cast steel valve body. The right-hand half

of the section shows the original design. The left-hand half shows the design modified to minimize the danger of spongy metal in the heavy section. This is a factor of great importance in the design of castings.

The power of the driving device to operate a given machine may be estimated from two different bases:

- (a) The maximum energy requirement during the energy cycle.
- (b) The average energy requirement during the energy cycle.

The first of these is used where the fluctuation of energy during the energy cycle is small or where the maximum demand is small. It is the simpler, mechanically, of the two methods but is the more wasteful of power. The triplex pump with cranks set at 120° is a good example of a machine powered in this way. Here the variation of energy requirement during the cycle is small and the size of the driving device would not be greatly reduced if selected on the basis of the average energy supply.

The second method is used where the maximum energy requirement is greatly in excess of the average. In this case the driving device, if made large enough to provide for the maximum energy demand, would be much larger and more expensive than if made to provide for the average demand only. If the driving device is made just large enough to deliver the amount of energy required for one energy cycle in the time of that cycle, the machine can be kept running indefinitely if some storage reservoir of energy is provided to take up the excess of energy when the demand is light and give it up when the energy demand exceeds the supply.

This energy reservoir is an essential part of a machine when the driving device is based on the average energy requirement, since without it the machine would stall at the first peak load encountered. The ordinary belt-driven punching machine is an example of a machine powered on the basis of average energy requirement. In this case the energy reservoir consists of a fly-wheel in which the excess of energy delivered from the belt between strokes is stored as kinetic energy and surrendered at the time a hole is punched.

Electrolysis. When two unlike metals are joined and exposed to an electrolyte an electric current is established. This causes a rapid corrosion to take place in the metal standing higher in the electrical potential scale. Destruction of this metal may take place with astonishing rapidity. An instance is recorded in which a $\frac{3}{4}$ -in. steel bolt was used to join two pieces of copper immersed in sea water. The bolt was entirely destroyed in 24 hours.

Because of the destructive effects of electrolysis great care should be exercised in selecting materials which come in contact with an electrolyte. For this service it is better to use a single material. Thus a brass seat in a steel pipe union handling brine is certain to give trouble. An all steel union would be much better.

Table 7-2 lists the most generally used metallic engineering materials arranged in the order of their electrical potentials. The more widely separated the materials in this table, the more rapid will be the destruction of the higher one when the two are joined and placed in an electrolyte.

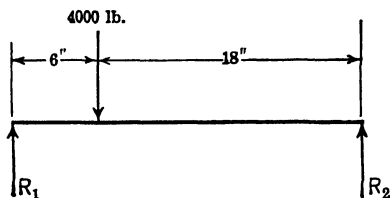
TABLE 7-2

Elements in Order of Electrical Potential

Magnesium	Manganese	Nickel	Copper
Aluminum	Zinc	Lead	Antimony
Chromium	Iron	Tin	Carbon

PROBLEMS — CHAPTER VII

1. The side links of a compressed air riveter have to sustain a load of 20,000 lb each. Assuming that the width of the cross section is to be four



PROB. 2

times the thickness and that the maximum allowable stress is not to exceed 10,000 lb per sq in., determine the dimensions for the links.

Ans. $w = 3$ in., $t = \frac{3}{4}$ in.

2. For the simple beam shown in the accompanying figure: (a) Draw the shear and moment diagrams. (b) Determine the maximum bending moment. (c) Deter-

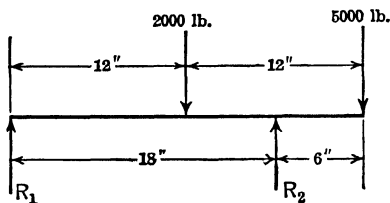
mine the vertical shear and the bending moment at the middle.

Ans. 3000 lb; 18,000 lb-in.; 1000 lb; 12,000 lb-in.

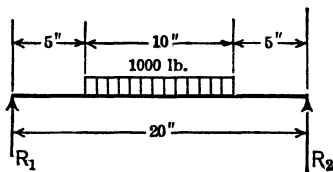
3. Assuming the beam of the figure in Prob. 2 to have a diameter of 3 in., determine the maximum tensile stress induced. *Ans.* 6794 lb per sq in.

4. A beam is loaded and supported as shown in the figure. (a) Draw the shear and moment diagrams. (b) Determine the maximum bending moment.

Ans. $R_1 = -1000$ lb; $R_2 = 8000$ lb; Max. B.M. = 30,000 lb-in.



PROB. 4



PROB. 5

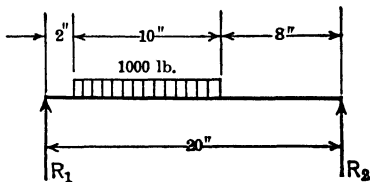
5. A simple beam is loaded and supported as shown in the figure. (a) Draw the shear diagram. (b) Determine the maximum bending moment.

Ans. 3750 lb-in.

6. A simple beam is uniformly loaded and supported as shown in the figure. (a) Draw the shear diagram. (b) Determine the maximum bending moment. (c) Determine the moment 8 in. to the left of the right-hand reaction.

Ans. 3412 lb-in. and 2800 lb-in.

7. A tensile specimen of steel has an original length of 8 in. and an area of cross section of 0.25 sq in. After a load of 6000 lb has been applied the length is found to be 8.0065 in. Find the unit stress and unit deformation induced and the modulus of elasticity.



PROB. 6

Ans. 24,000; 0.00081 in.; 29,600,000.

8. A cast iron member, circular in cross section and 8 in. long, is to withstand a dead load of 32,000 lb in compression. The member must not shorten more than 0.0032 in. and must have a factor of safety of at least 8. Assume the ultimate strength of the material to be 96,000 lb per sq in. and the modulus of elasticity to be 15,000,000 lb per sq in. Determine the diameter for the member.

Ans. 2.62 in.; say $2\frac{5}{8}$ in.

9. (a) A steel positioning piece $1\frac{1}{8}$ in. in diameter is to have a length of 10 in. when under a load of 15,000 lb. Assuming a modulus of elasticity of 30,000,000, determine the original length of the member.

Ans. 10.0017 in.

(b) Assume the piece to be made of bronze having a modulus of elasticity of 14,000,000 and determine the length of the piece. *Ans.* 10.0036 in.

10. The cover plate of a punch is subjected to an upward force of 66,000 lb while a hole is being punched. Sliding of the cover plate relative to the punch frame is prevented by means of two dowel pins made of steel. Assuming an allowable shearing stress of 11,000 lb per sq in., determine the diameter for the pins. *Ans.* 1.96 in.; say 2 in.

11. How many foot-pounds of work are required to stress a tension member having a cross-sectional area of 4 sq in. and a length of 5 ft to the elastic limit if the material is steel? Modulus of elasticity is 30,000,000 and the elastic limit 34,000 lb per sq in. *Ans.* 385.

12. (a) Determine the foot-pounds of work done in raising a weight of 600 lb a distance of 10 ft. *Ans.* 6000 ft-lb.

(b) If it takes 2 sec to raise the weight, what is the power in foot-pounds per second required? *Ans.* 3000 ft-lb per sec.

13. (a) A tractor hauls a load at a velocity of 5 mph. The pull which must be exerted by the tractor is 500 lb. What power in foot-pounds per minute is developed by the tractor? *Ans.* 220,000 ft-lb per min.

(b) What amount of work is done in moving the load half a mile?

Ans. 1,320,000 ft-lb.

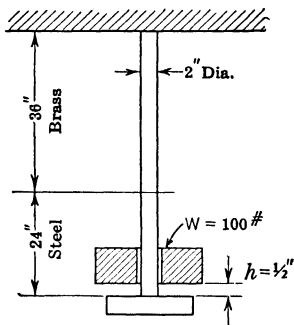
14. The piston rod of an engine is 2 in. in diameter and 3 ft long. The diameter of the piston is 15 in. and the maximum steam pressure is 100 lb per sq in. Assume that the steam acts as a load suddenly applied but without shock and, neglecting the column action, determine the maximum stress and deformation induced. *Ans.* 11,200 lb per sq in. and 0.0134 in.

15. A bar of steel 2 in. in diameter and 2 ft long is joined to one of brass 2 in. in diameter and 3 ft long. (See the accompanying figure.) The combination bar is subjected to impact from a weight W pounds falling through a height of $\frac{1}{2}$ in. Determine the weight required to induce a stress of 5000 lb per sq in. in the bar, and the maximum deflection. $E_{\text{steel}} = 30,000,000$. $E_{\text{brass}} = 13,000,000$.

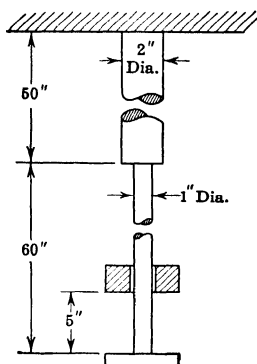
16. A weight drops through a height of 5 in. and impinges against the steel member shown in the figure in such a way as to place it in pure tension. The modulus of elasticity is 30,000,000. Determine the weight necessary to induce a maximum stress of 10,000 lb per sq in. in the bar.

17. Assume that the arrangement shown in the figure for Prob. 16 and described in that problem is horizontal and that the weight is moving horizontally with a velocity of 10 ft per sec. Determine the weight necessary to induce a maximum stress of 10,000 lb per sq in. in the member.

18. A compound bar 120 in. long is made of a bar of brass placed beside one of steel and fastened to it at each end. Each bar has a cross-sectional area of 2 sq in. Modulus of elasticity of steel is 30,000,000 and that of brass is 12,000,000. Determine the weight which, falling through a height of 6 in., will produce a stress of 10,000 lb per sq in. in the steel. *Ans.* 92.7 lb.



PROB. 15



PROB. 16

19. (a) The brasses of an automobile engine connecting rod have worn so as to allow play which gives shock loading equivalent to a force of 1500 lb falling through a height of 0.01 in. The connecting rod is 12.5 in. long and has a cross-sectional area of 0.5 sq in. It is made of steel having a modulus of elasticity of 30,000,000. Determine the stress induced by the inertia.

Ans. 15,390 lb per sq in.

(b) Assuming the ultimate static tensile strength of the rod material to be 125,000 lb per sq in., determine its endurance limit if the maximum compressive stress induced by gas pressure is 9600 lb per sq in.

Ans. 54,000 lb per sq in.

20. A reinforcing bar on a punch must sustain a shock equivalent to the impact of a weight of 500 lb falling through a height of $\frac{1}{2}$ in. The bar has an effective length of 30 in. and is to be made of 0.45 per cent carbon steel having an ultimate strength of 80,000 lb per sq in. The load produces pure tension. Consider the load to be repeated and, using a real factor of safety of 3, determine the diameter for the bar. $E = 30,000,000$.

Ans. 1.48 in.; say 1.5 in.

21. The piston rod of a four-cycle internal combustion engine is to be made of 3.5 per cent nickel steel having an ultimate tensile and compressive strength of 120,000 lb per sq in. The piston is 20 in. in diameter and sustains a maximum net gas pressure of 600 lb per sq in. The maximum inertia force is 150 lb per sq in. Take a real factor of safety of 5; considering the load to be applied suddenly but without initial velocity, determine the diameter required for the rod. Neglect column effect.

CHAPTER VIII

FRICITION, LUBRICATION, AND BEARINGS

1. Sliding Friction. The friction that is set up when one body moves with a relative motion of sliding with respect to another falls into two distinct classes:

- (a) Thin film or dry friction, which occurs when the surfaces are separated by layers of lubricant so thin as to be of molecular dimensions or are in true metallic contact.
- (b) Fluid film friction, which occurs when the surfaces are completely separated by a fluid film of measurable thickness.

Either of these types may exist in any bearing or, as frequently happens, both may be present in varying degrees in the same bearing. The laws governing the two types of friction are wholly different, and this should be borne clearly in mind whenever the subject of sliding friction is being considered.

The ratio of the frictional resistance to the normal pressure is called the coefficient of friction, and is expressed

$$f = \frac{F}{N} \quad (8-1)$$

where f = coefficient of friction

F = frictional resistance

N = normal pressure.

This expression applies particularly when flat surfaces are in contact.

If, for example, two surfaces are held together by a normal pressure of 120 lb and it is found that a force of 30 lb must be applied parallel to the surfaces to keep the bodies in uniform motion

relative to each other, the coefficient of friction is

$$f = \frac{30}{120} = 0.25$$

If the contact surfaces are cylindrical, one being convex and the other concave, as in the case of a journal and its bearing, the normal pressure is different at each element of the cylinders, varying from zero at about right angles to the load to a maximum approximately in line with the load. This makes the determination of the frictional resistance F , by means of equation (8-1), tedious. For this condition engineering practice has developed a simpler method of solution based upon the load on the bearing, the expression of the relationship being

$$\mu = \frac{F}{W} \quad (8-2)$$

where μ = coefficient of friction for cylindrical surfaces

F = frictional resistance

W = load on the bearing.

If, for example, a certain bearing sustained a load of 350 lb and it was found that the resistance to rotation was equivalent to a force of 70 lb applied at the contact surface, then

$$\mu = \frac{70}{350} = 0.20$$

In order that the results of experiment and experience may be made readily applicable to a wide range of sizes of bearings, it is customary to express F and W in pounds per square inch of projected area of the bearing, the projected area being equal to the product of the length and diameter of the bearing.

2. Work of Friction. In the design of machines it is frequently essential to know the amount of work lost in friction in order that artificial cooling may be provided where necessary, or efficiencies be calculated. In Chapter 7, Art. 10, work is defined as the product of a force and the distance moved through by it. If the

work of friction is desired, we have only to multiply the force of friction by the distance it travels. The product is the work of friction.

If, for example, the journal just used in the determination of the coefficient of friction of cylindrical surfaces is 2 in. in diameter, and the work of friction done in 1 revolution is desired, we have

$$\text{Force of friction} = 70 \text{ lb} = F$$

$$\text{Distance moved} = \pi \times 2 = 6.28 \text{ in.} = d$$

$$\text{Work of friction} = F \times d = 70 \times 6.28 = 439.6 \text{ in.-lb}$$

A point to be especially noted is that, if the force of friction is variable, the average force acting must be found before using equation (7-4).

3. Lubrication. Rubbing together two clean surfaces is attended by

(a) Abrasion.

(b) Loss of energy.

Where the surfaces in contact belong to members whose function is to transmit energy, it becomes desirable to reduce these results of rubbing to values as small as consistent with considerations of economy and reliability. To this end a third substance, known as a lubricant, is introduced between the rubbing surfaces. This substance may be a solid, such as graphite, talc, or white lead; a semi-solid, such as greases and soaps; or a fluid, such as the various oils, water, and, sometimes, air. In any case their primary function is to reduce abrasion, and their secondary, though in most cases equally important, function is to reduce the frictional resistance to relative motion of the surfaces. A lubricant does this by adhering to each of the surfaces formerly in contact and forming a more or less complete film between them. Relative motion becomes possible through the shearing of this film along a surface or surfaces within it. In this way the resistance of the film of lubricant to shear is substituted for the frictional resistance of the dry surfaces and, since this resistance is usually less than the frictional resistance of clean surfaces, loss of energy due to friction is reduced. Wear is reduced for two reasons:

- (a) The moving surfaces are in less intimate contact when a lubricant is used than when one is not used.
- (b) The particles of lubricant in actual contact with the moving surfaces have little if any motion relative to the surface with which they are in contact and, therefore, themselves cause little or no wear.

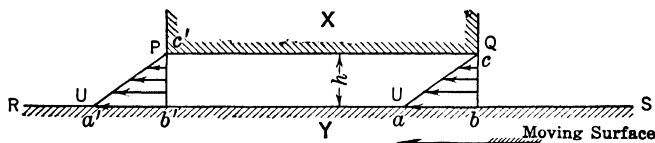


FIG. 8-1

4. Mechanism of Film Lubrication. In Fig. 8-1 let X be a fixed body having a plane surface PQ of area A , and let Y be a body moving with a velocity U with respect to X . The surface RS of Y is plane, is parallel to PQ , and is distant from it h . Both X and Y may be considered as submerged in a fluid lubricant. This lubricant adheres to the moving surface and is carried along until it comes under the fixed body X . The layer coming in contact with X will be stopped by it whereas that in contact with Y will continue to move with the velocity, U , of Y . If h is small, the velocity of any layer in the lubricant will be directly proportional to its distance from X . The arrows in the triangles abc and $a'b'c'$ show this variation graphically at entrance and at exit to the space between X and Y . The areas of these triangles represent the volumes of lubricant drawn into and discharged from the space by reason of viscous drag. Since the bases and altitudes of the triangles are equal, it follows that the volumes of lubricant drawn in and discharged are also equal and, therefore, no changes in pressure result from the motion of the moving surface.

If the fixed body X is inclined to the moving body Y as shown in Fig. 8-2, the equilibrium between the inflow and outflow, due to viscous drag, is destroyed. Triangle abc has the same base, U , as triangle $a'b'c'$, but its altitude h_1 is greater than the altitude h_2 of $a'b'c'$. Its area is therefore greater; this shows that viscous drag brings more lubricant into the space than it can take out.

This attempt to pack into the space between X and Y more lubricant than is taken out creates a pressure which is sufficient to expel the excess and which modifies the velocity distribution

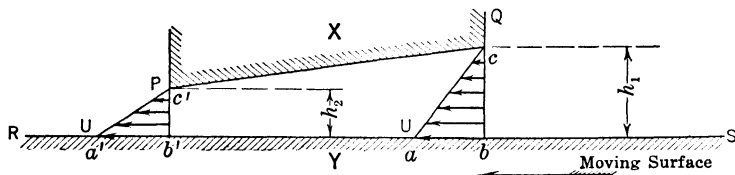


FIG. 8-2

in the lubricant. Figure 8-3 shows the resultant velocity distribution and the pressure built up. This pressure acts normal to the bounding surfaces, and therefore has a vertical component acting on X which is able to sustain a load.

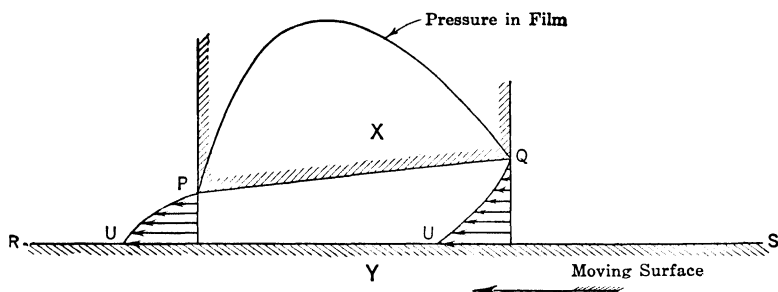


FIG. 8-3

Should the direction of motion of Y be reversed, more lubricant would be discharged than drawn in by viscous drag, and negative pressure would be developed to draw in enough lubricant to make good the deficiency.

From the foregoing it will be seen that a space between the rubbing surfaces which is convergent in the direction of motion is a necessity, if a load-carrying film is to be built up and maintained by the motion of the rubbing surfaces.

5. Rolling Friction. If a spherical, conical, or cylindrical body is rolled along a plane supporting surface, it meets with a certain

resistance to its progress. This resistance, called rolling friction, arises from the distortion of both the rolling body and the supporting surface as well as from the roughness of each. The exact nature of the phenomena causing rolling friction is not fully understood, hence a satisfactory mathematical solution of the problem is not available. Experiment, however, has shown that, if the surfaces in contact are reasonably smooth and do not distort abnormally the relationship between the load, dimensions of the rolling member and the resistance to rolling may be expressed by the following equation:

$$F \times r = W \times k \quad (8-3)$$

where F = force to be applied at the center of the roller to produce motion

r = radius of roller in inches

W = load on the roller

k = an experimental coefficient which varies with the nature of both the roller and the support. A good value for hardened steel rollers running on hardened steel races is 0.002 to 0.003.

6. Types of Bearings and Their Requirements. The portion of the rotating member which rests in the support is known as a journal, whereas the support itself is known as a bearing.

In order to function properly a bearing must meet the following requirements more or less fully:

- (a) It must have sufficient strength to carry any load which may be imposed upon it.
- (b) It must not distort unduly under the applied loads. This consideration of rigidity is usually the governing factor in the design of bearings for machine tools where accurate work is to be done.
- (c) The contact surfaces must be large enough to insure bearing pressures low enough to permit satisfactory lubrication and to keep wear within reasonable limits.
- (d) It must be constructed so that lubrication is easy and effective if the life of the bearing is to be long and the work lost in friction low.

- (e) It must dissipate the heat generated by friction as rapidly as it is formed, and do this without its temperature being greatly above room temperature.
- (f) It must be constructed so that wear can be compensated for by adjusting the parts of the bearing.
- (g) It must be capable of adjustment to bring it into alignment with other bearings supporting the rotating member.
- (h) It should keep the oil in and the dust out.

Bearings may be classified broadly in two groups upon the basis of the kind of relative motion involved.

- (a) Bearings in which the relative motion is sliding.
- (b) Roller and ball bearings in which the principal relative motion is rolling.

Of these two types the first is by far the more widely used, and is the simpler and the older. On the other hand, the second type gives lower friction losses, requires less attention, and shows less wear if properly designed and installed. The starting friction is also considerably lower.

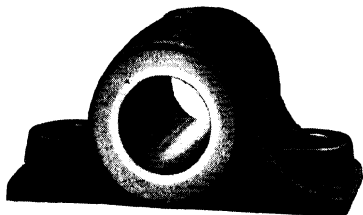
7. Forms of Radial Sliding Bearings. Using the method of adjustment for wear as a basis, there are three general types of sliding bearings carrying loads acting at right angles to the axis of the shaft. They are:

- (a) Solid.
- (b) Split.
- (c) Quarter box bearings.

These are shown in Figs. 8-4, 8-5, and 8-6.

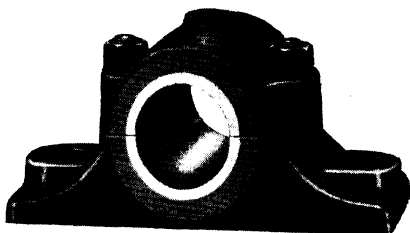
A solid bearing is a cylindrical shell of some material strong enough to withstand the loads put upon it. Usually some material such as cast iron or steel which combines low cost with strength is used. In order to secure a satisfactory surface for the steel or wrought iron journal to rub against, such bearings usually have a lining of babbitt or bronze. In a few instances the entire bearing is made of bronze; in others, especially in cheap and relatively unimportant work, the cast iron shell itself forms the rubbing surface. In places where the rubbing velocity is low or the

pressure light, either no provision is made to compensate for wear or else a renewable bushing is provided as shown in Fig. 8-4.



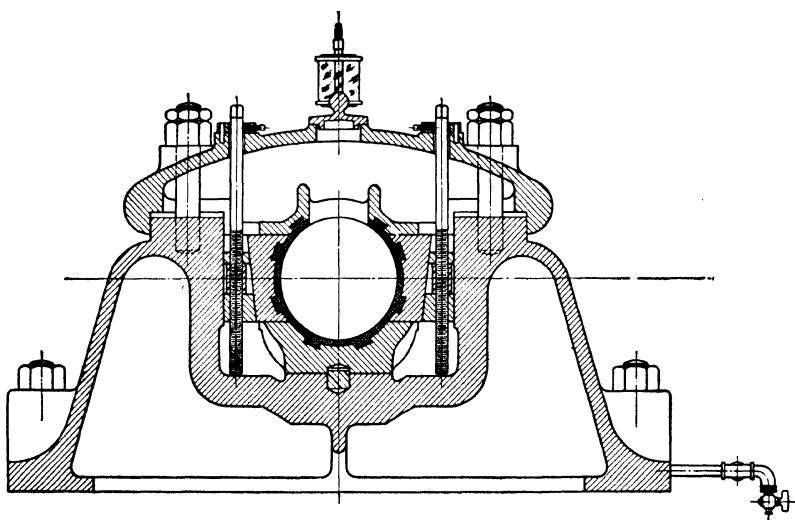
Dodge Manufacturing Corporation

FIG. 8-4. Solid bearing.



Dodge Manufacturing Corporation

FIG. 8-5. Split Bearing.



Arctic Ice Machine Co.

FIG. 8-6. Quarter box bearing.

If appreciable wear is anticipated the bearing is split as shown in Fig. 8-5 and wear is compensated for by removing some liners from the sides and drawing the halves of the bearing more closely together by means of the bolts securing the cap to the base.

These bearings are cheap, serviceable, simple machine members and are widely used. Most line shaft bearings and crankpin bearings of automobile engines as well as the bearings of many machine tools belong to this type.

If more efficient lubrication than is given by the simple oil hole or oil cup is desired, oil rings are provided as shown in Fig. 8-7. With this arrangement a continuous supply of oil is insured and almost perfect lubrication is secured. Lubrication starts automatically as soon as the journal begins to rotate and stops when it comes

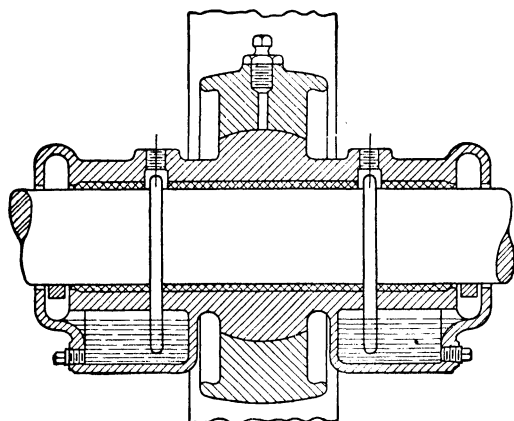


FIG. 8-7. Ball and socket bearing with ring oiling.

to rest. These bearings require very little attendance. It is usually necessary to fill the oil reservoir not more often than once in three months. In addition to being almost the only bearings used on motors and generators, they are used in the better installations of line shafting and on many engines.

Where it is especially desirable to preserve the original position of the journal and where the wear takes place in more than one direction, as in the main bearings of large engines, an arrangement shown in Fig. 8-6, known as a quarter box bearing, is used. Wear in a vertical direction downward is compensated for by placing thin shims under the bottom brass, thus returning the center of the journal to its original position. Wear in a horizontal

direction, either right or left, caused by steam pressure, is compensated for by raising the adjusting wedges behind the cheek brasses, thus maintaining the horizontal position of the journal. This preservation of the original position of the journal is especially important where the shaft is directly connected to the armature of an electric motor or generator, as very heavy loads due to magnetic attraction may be caused by the armature being off center with respect to the pole pieces. Quarter box bearings are usually found in large, important machines such as Corliss engines, large electric motors, and electric generators.

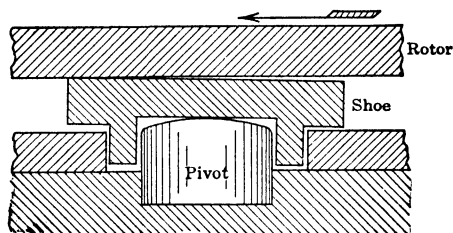


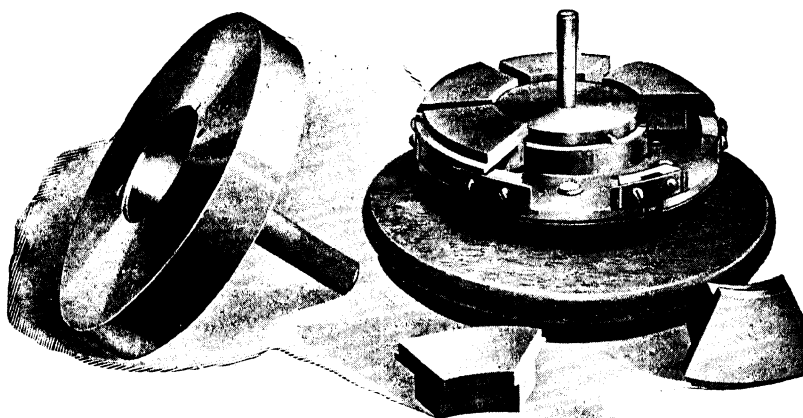
FIG. 8-8. Diagrammatic arrangement of Kingsbury thrust bearing.

8. Sliding Thrust Bearings. A thrust bearing is one whose function is to prevent axial motion of the rotating members. The loads which are applied to such bearings are parallel to the axis of the shaft.

As previously pointed out, it is highly desirable to secure a complete separation of the rubbing surfaces by an oil film. With the attainment of this in view Professor Kingsbury has perfected the thrust bearing whose essential features are shown in Fig. 8-8. It consists, in its essentials, of a collar attached to the shaft bearing against several segmental shoes. These shoes are pivoted so as to permit them to tilt freely in either a radial or a tangential direction, as operating conditions may require. The oil film forms as soon as the bearing starts to rotate and is maintained automatically by the adhesion of the oil to the collar, the tilting of the shoes preventing them from scraping it off.

The Kingsbury Machine Company states that the coefficient

of friction of these bearings is lower than that of any other known type, 0.0009 being a good average value.



Kingsbury Machine Works

FIG. 8-9. Model air-lubricated Kingsbury thrust bearing.

The Gibbs thrust bearing, illustrated in Fig. 8-10, is a modification of the Kingsbury bearing. In this case the segmental shoes

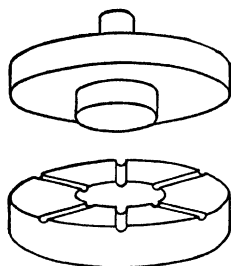


FIG. 8-10. Gibbs thrust bearing.

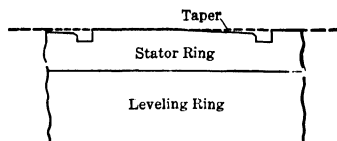


FIG. 8-11. Taper of shoes.

are rigid, easy ingress of the lubricant being secured by scraping the shoes to a taper as shown in Fig. 8-11. This bearing has proved very satisfactory for hydraulic turbine work. It is simpler and cheaper than the Kingsbury bearing, but lower bearing pressures

must be allowed for it. The normal range of pressures for the Kingsbury bearing is 250 to 500 lb per sq in., whereas the range for the Gibbs bearing is 150 to 300 lb per sq in. Both these bearings have proved extremely satisfactory for horizontal as well as vertical shafts and are widely used on steam and hydraulic turbines of the largest size, plate glass grinders, and centrifugal pumps. They have given excellent results at very low speeds as well as at high speeds.

9. Design of Sliding Bearings. The minimum diameter of bearing is fixed by two separate and entirely independent considerations. In the first place the diameter must be at least great enough to insure a journal of sufficient strength and stiffness and, secondly, it must be large enough to insure a unit bearing pressure which experience has shown to be satisfactory. Whichever of these factors demands the larger bearing diameter becomes the governing factor.

The allowable bearing pressure in pounds per square inch of projected area will depend upon

- (a) The character of the load, whether intermittent or constant.
- (b) The rubbing velocity.
- (c) The ability of the bearing to radiate heat.

If the load is constant, the likelihood of the lubricant being squeezed out is greater than if the load is intermittent. Since the presence of a lubricant is essential in preventing abrasion, lower unit bearing pressures are allowable if the load is constant than if it is intermittent. The bearing pressure in the eccentric of a power punch is sometimes as high as 5000 lb per sq in. of projected area, whereas in the main bearings of low speed engines it is only about 200 lb per sq in. of projected area.

Minimum film thickness. In bearing operation the most essential factor is the maintenance of an oil film between the rubbing surfaces. This must be thick enough to make amply certain that roughnesses on journal and bearing will not break through and come into metallic contact, especially since the shaft deflection may be higher and the oil viscosity lower than anticipated. One large company has adopted a value of 0.0005 in. as the minimum

value for oil-film thickness in bearings for steam turbines and electrical machinery. Where the finish of both shaft and bearing is exceptionally smooth and the bearing is short, thus minimizing the deflection of the shaft within the bearing, this value may be as low as 0.0002 in.

Radial clearance. No hard and fast statement can be made as to the best value of radial clearance. It can be stated in general, however, that small clearances give thin films and high friction, medium clearances give thick films and low friction, and large clearances give thin film and high friction. Probably diametral clearances ranging from 0.001 in. to 0.003 in. per inch of journal diameter will be most satisfactory as regards lubrication. Diametral clearances of 0.001 in. to 0.0013 in. per inch are probably proper for electrical machinery.

Location of oil grooves. As pointed out earlier, the load imposed on a perfectly lubricated bearing is carried by the pressure built up within the oil film.

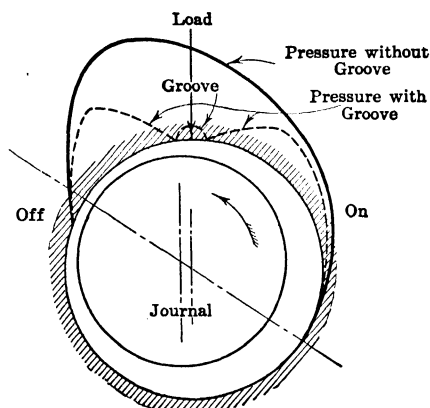


FIG. 8-12. Effect of oil groove on oil film pressure.

It is important, therefore, to place the oil grooves so that they do not interfere with the production of this oil-film pressure. Figure 8-12 shows the typical distribution of pressure within a bearing. It will be noticed that the maximum intensity of pressure occurs a little to the "off" side of the bearing from the line of action of the load. The maximum intensity of the pressure in the oil film of bearings with ordinary clearances varies from two and one-half to four times the bearing

pressure in pounds per square inch of projected area. Should a groove be cut in this region, the pressure at the groove will be the pressure at which oil is supplied to the bearing. Under these con-

ditions the pressure distribution will be about that shown by the dotted lines. The load-carrying capacity is proportional to the area under the pressure curve. From Fig. 8-12 it will be seen that the presence of a groove in the region of positive pressure causes a very large reduction in the carrying capacity of the bearing. Probably the best place to put grooves is at right angles to the line of action of the load. Under no circumstances should they run from regions of high pressure to regions of low pressure, for they would then act as drains running the oil from the bearing.

Heat-radiating capacity. If the conditions of bearing pressure and rubbing velocity are such as to render the work of friction high, the heat generated by this work must be carried away by some means other than mere radiation. The two most common ways of doing this are, first, by circulating an excess of lubricating oil to the bearing and, second, by water cooling the bearing shell. In the first case oil in large quantities is pumped into the bearing. There it absorbs the heat generated by friction and the overflow of the oil carries it away from the bearing. This overflow is passed through a filter and cooler and is then returned to the bearing to gather and bear away another load of thermal units. Water cooling is done in the same general way except that the water is circulated through passages in the bearing shell and does not come into contact with the bearing surfaces. The filter and cooler are also omitted, the overflow water being wasted.

Whether artificial cooling should be provided depends upon whether or not the bearing can radiate the heat equivalent of the work of friction without exceeding the maximum temperature compatible with the certain formation and maintenance of the film of lubricant. The amount of heat a bearing can radiate will depend on the character of the bearing and the allowable difference in temperature between the bearing and the surrounding medium. Since metals transmit heat far more readily than air, a thick bearing will radiate more heat for a given temperature difference than will a thin one. Also, if the bearing is connected to a large mass of metal around which air can circulate freely, its radiating capacity is still further increased. The maximum temperature allowable for a bearing will vary with the kind of lubricant

used, but bearing temperatures above 120°F must be termed high, and ordinarily they should never be allowed to exceed 140°F.

Mr. Axel K. Pederson gives the radiating capacity of bearings as

$$Q = \frac{(T_0 + 33^\circ)^2}{K} \quad (8-4)$$

where Q = the heat-radiating capacity of the bearing in foot-pounds per second per square inch of projected area

T_0 = the difference in temperature in Fahrenheit degrees between bearing and the cooling medium, usually the air of the room

K = an experimental constant, depending on the type of bearing and the air conditions surrounding it. For bearings of light construction in still air, $K = 3300$. For bearings of heavy construction, well ventilated, $K = 1860$.

It must be remembered that if heat is likely to be brought into the bearing by conduction along the shaft, as in steam turbine bearings, provision must be made to take care of this heat as well as that caused by friction.

If the bearing is made very long in comparison with its diameter, localized pressures of great intensity are likely to occur. The oil film is likely to be broken at these points, and the excessive heating which follows will cause destruction of the bearing. Experience has shown that certain ratios of length to diameter have given most satisfactory results. As might be expected, the greatest ratios of length to diameter occur in cases where alignment is easiest, as in bearings for line shafts and steam turbines. The smallest ratios are found where alignment is most difficult to maintain, as in engine crankpins. Table 8-2 gives the ratios of length to diameter of the most common types of bearings.

The method followed in the design of thrust bearings is, in general, similar to that for journal bearings. The force of friction on each elementary ring and the distance it travels are, however, different from those of any other ring in the bearing. The force of friction set up on the innermost ring is least and that on the outer-

TABLE 8-1

Allowable Bearing Pressures

Class of Bearing	Allowable Pressure in Pounds per Square Inch Projected Area
Slow speed with intermittent load, as the eccentrics of punching machinery.....	3000 to 5000
Wristpins	
H. S. steam engines (stationary)	1000 to 1800
L. S. steam engines.....	1000 to 1500
Gas engines.....	1500 to 2000
Automobile.....	1500 to 2500
Air compressor.....	400 to 1000
Locomotive.....	3000 to 4000
Main Bearings	
H. S. steam engines (stationary)	
Based on dead load.....	60 to 120
Based on steam load.....	150 to 250
Gas engine.....	500 to 700
Air compressor.....	115 to 140
Automobile crankshaft.....	1000 to 1500
Crankpins	
H. S. steam engines (stationary).....	900 to 1500
L. S. steam engines (stationary).....	800 to 1300
Gas engine.....	1500 to 1800
Air compressor.....	250 to 850
Locomotive.....	1500 to 2000
Miscellaneous	
Eccentric straps.....	70 to 140
Horizontal steam turbine.....	40 to 60
Heavy line shaft, babbitted.....	100 to 150
Light line shaft, cast iron.....	15 to 25
Slow speed step bearings.....	2000
Marine thrust bearings.....	45 to 55
Slides, cast iron on babbitt.....	200 to 300
Slides, cast iron on cast iron.....	40 to 100
Rolling mill main bearing.....	1800 to 2500

TABLE 8-2

Ratio of Length to Diameter of Bearings

Type of Bearing	$\frac{L}{D}$
Marine engine main.....	1 to 1.5
Stationary engine main.....	1.5-2.5
Stationary gas engine main.....	2-2.5
Ordinary shafting, fixed bearings.....	2-3
Ordinary shafting, spherical.....	3-4
Generator and motor.....	2-3
Machine tool.....	2-4
Crankpins, stationary engines.....	1-1.25
Wristpins, stationary engines.....	1-1.5
Steam turbines.....	2-3
Centrifugal pumps and fans.....	2-3

most is greatest. The distances traveled by these elementary frictional forces are likewise least at the inner edge and greatest at the outer edge of the collar or step. There is, however, a radius at which the total frictional force set up in the bearing could be concentrated without changing the work it does. This radius is called the frictional radius, and is

$$r_f = \frac{2}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) \quad (8-5)$$

where r_1 = inner radius of collar or step

r_2 = outer radius of collar or step.

10. Design of Bearing Cap. Wherever possible the load on a bearing should be directed against the body of the bearing and not against the cap. In some instances this is not possible, and the load must be carried by the cap. Even where the cap carries little or no load, it is customary to determine its thickness and the size of the bolts securing it to the base as if it were subjected to the full load. An automobile crankpin bearing is an example of this condition.

The bolts are designed as pure tension members, the allowable stress being given in Chapter XI, Table 11-3. Owing to the fact

that all the bolts holding a bearing cap to the base may not be set up to the same degree of tightness, thus throwing a greater portion of the load to some bolts than to others, each bolt is assumed to carry a load equal to four-thirds that which would have been its share had the total load on the bearing cap been equally distributed among the bolts.

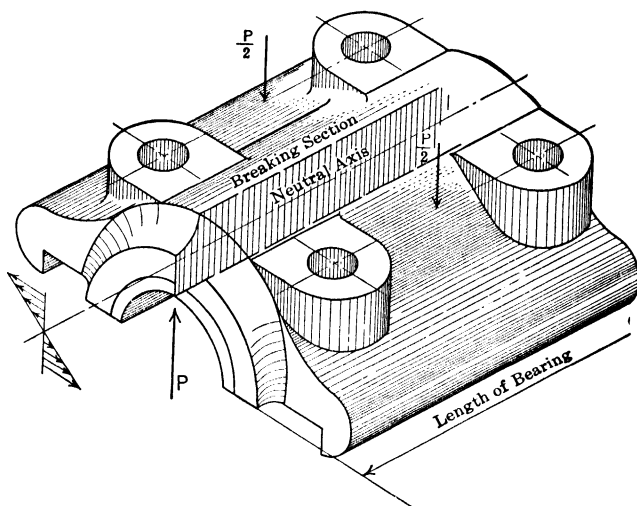


FIG. 8-13. Bearing cap considered as a beam.

The cap itself may be considered to be a beam. The conditions of both loading and supporting are indefinite, but a common assumption is that the cap is a simple beam supported at the centers of the bolts and loaded in the center. (See Fig. 8-13.) The width of the beam is taken to be equal to the length of the bearing. These assumptions, undoubtedly, are considerably in error on the side of safety, for we know that the load is somewhat distributed and that the ends of the bearing are more or less clamped by the nuts of the holding bolts. The last-named conditions give a beam which is partially fixed at the ends and which carries a load distributed (but not uniformly) over that portion which is in contact with the journal.

As the true conditions of loading and supporting are difficult, if

not impossible, to determine, the actual bending moment may be approximated by assuming it to be the mean between the maximum and minimum possible bending moments acting on the cap. The maximum possible bending moment would occur if the load were concentrated at the center of the cap and if the supports were knife edges placed at the axes of the holding bolts, as in Fig. 8-13. This would give

$$M_1 = \frac{Pl}{4} \quad (8-6)$$

where P = total load on the bearing

l = distance between center lines of holding bolts measured at right angles to the axis of the journal

M_1 = maximum moment induced. (This occurs in the center under the load.)

The minimum possible bending moment would occur if the cap were fixed rigidly except where it touches the journal, and if the load were uniformly distributed over the portion in contact with the journal. This would give

$$M_2 = \frac{Pd}{12} \quad (8-7)$$

where M_2 = the maximum moment induced by the most favorable conditions as stated above and occurs at the ends of the loaded portion of the cap

d = diameter of the journal.

The actual bending moment induced may be assumed to be the mean of these two extreme values

$$M = \frac{M_1 + M_2}{2} \quad (8-8)$$

The thickness of the cap can now be determined by assuming the cap to be a beam having a rectangular cross section with a width equal to the axial length of the bearing and subjected to the bending moment M given above.

11. Ball and Roller Bearings. Rolling friction is, in almost every case, less than sliding friction, and the ball and roller bearings have been developed to substitute rolling for sliding friction in journal and thrust bearings. They present the following advantages:

- (a) Low friction losses. This is especially true of ball bearings.
- (b) Durability. There is practically no wear.
- (c) Require no attendance.

To these may be added, in the case of ball bearings, the advantage of occupying very little axial space. Their principal disadvantages are:

- (a) Extreme accuracy required in both manufacture and mounting.
- (b) Large diameter.
- (c) Relatively high first cost.

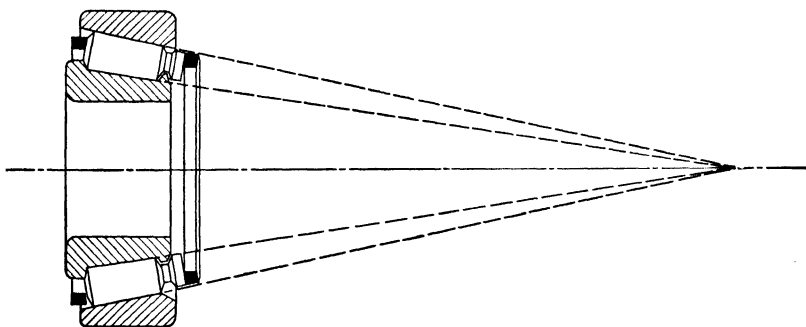
Ball bearings when properly made and mounted give as nearly pure rolling friction for journal and thrust bearings as it is possible to obtain. Pure rolling is not obtained simply because the materials of which the balls and races are made are not perfectly rigid. There is, therefore, as pointed out in Art. 5, a slight deformation of both ball and race caused by the imposed load. This changes the nature of the contact from point contact to contact over a small area, and introduces an element of sliding into the nature of the relative motion between the ball and race.

In order that each ball in a bearing may take its share of the load all must be true spheres, must be within 0.0001 in. of the same size, and must have the same elastic properties. It is impracticable to grind balls to such close limits, so reliance is placed on sorting them after manufacture. In order that the elastic properties may be as nearly the same as possible, both balls and races should be made from homogeneous materials, carefully hardened before the final grinding. It has been found that any scratches, pits, or roughness on the surfaces of the balls lead to rapid failure when in service. This is obviated by carefully polishing them.

12. Design of Ball and Roller Bearings. When the bearing is to be composed of cylindrical rollers, provision must be made for

keeping the axes of the rollers parallel to each other and to that of the shaft. The rollers must also be uniformly spaced.

Sometimes the rollers and races are made conical, as shown in Fig. 8-14. This scheme has the advantage of allowing adjustment for wear by setting the inner race farther into the cone of rollers. It also permits the carrying of large end thrusts, a thing which cannot be done with bearings having cylindrical rollers.



Timken Roller Bearing Co.

FIG. 8-14. Timken bearing.

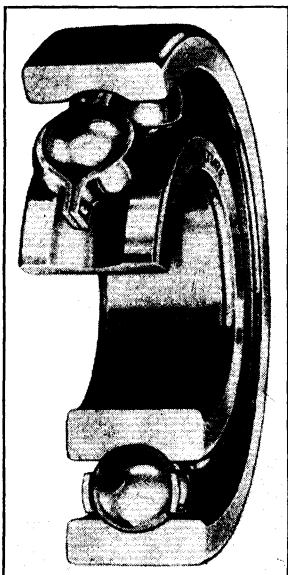
The apex angle of the roller must be small in order to prevent severe end thrust being set up by the radial load. Usually the apex angle is kept down to 6 or 7°. The elements of each roller and of the races should intersect at a common point lying in the axis of the shaft, as shown.

The most common type of ball bearing is that in which the ball has two points of contact with the races, as shown in Fig. 8-15. The contour of the raceways may be either flat or grooved as shown, but the grooved form has several advantages and is almost universally used. The advantages of the grooved race are:

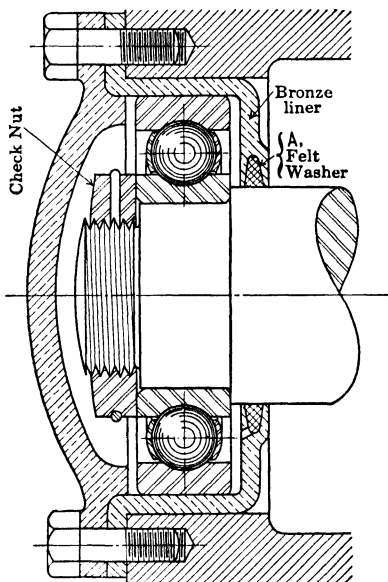
- (a) The balls are retained in the bearing.
- (b) Considerable end thrust can be carried.
- (c) Higher loads can be sustained.
- (d) A strong, rigid race is provided which is not likely to be distorted by the applied load.

Exhaustive experiments conducted by Professor Stribeck indicate that the carrying capacity of the ball bearings varies directly

as the square of the ball diameter and also directly as the number of balls. It is better, therefore, to select a bearing with a few large balls than one with many small balls. This choice gives, also, the added advantage of a reduced frictional resistance due



New Departure Manufacturing Co.



New Departure Manufacturing Co.

FIG. 8-15. Radial ball bearing.

FIG. 8-16. Mounting for radial ball bearing.

to the increased radius of curvature. For the actual carrying capacities for the various types of ball and roller bearings the student is referred to trade catalogs, in which such information is fully given.

The consensus among engineers who have given consideration to the subject of ball-bearing mounting is that the inner race should be a good fit on the shaft. A push fit is recommended where the bearing is to be disassembled occasionally, and a force fit in the other cases. When the push fit is employed the inner race should always be locked to the shaft by means of a suitable check

nut, as shown in Fig. 8-16. The outer race should be a push fit in the housing, so that it may creep and so distribute uniformly over its surface such wear as may occur. When the housing is made of some soft metal, as aluminum, it is well to use a bronze or malleable iron liner as in Fig. 8-16, which makes a force fit with the housing and a push fit with the race. If this is not done the race is likely topeen the soft housing and loosen, thus throwing the bearing out of alignment.

In every case great care should be taken to see that the seats of the races are as nearly round as possible. This precaution is necessary because each race takes the shape of its seat. Any out-of-roundness present in the seat appears in the race after assembly and may result in rapid destruction of the bearing.

Wherever possible it is desirable to employ bearings having only a single row of balls, as it is difficult to secure an even distribution of the load.

Ball bearings and straight roller bearings may be used singly if desired. Because of the end thrust developed by radial loads acting on them, tapered roller bearings must always be used in pairs and arranged so that the cones point in opposite directions. For this reason, as well as for securing greater capacity, tapered roller bearings having two or even four rows of rollers are sometimes found desirable.

Rust and dirt are two of the worst enemies of both ball and roller bearings, and every precaution should be taken to keep them out. Dust and dirt may be kept out by placing felt washers where the shaft passes through the housing, as shown at *A* in Fig. 8-16. Rust may be prevented by packing the bearing in grease or submerging it in oil. If this is done provision should also be made for replacing the lubricant. The presence of a lubricant in a ball or roller bearing increases the loss due to friction, but this is more than offset by the protection from rust.

Ball and roller bearings almost always fail by flaking or spalling of the surface of the race or roller. This is caused by the surface reaching the endurance limit of the material. Since this endurance limit is a function of the number of applications of the load, it follows that the load a high speed bearing can carry will be less

than that of a low speed bearing. All ball and roller bearing manufacturers recognize this fact by decreasing sharply the rated capacities of their bearings as the operating speeds increase.

The term "hours of life" when applied to ball and roller bearings means the number of hours that at least 90 per cent of the bearings will run without failure.

13. Selection of Ball and Roller Bearings. Ball and roller bearings are rated by the manufacturers on the basis of the radial load the bearings will carry for a certain number of hours, without more than 10 per cent of the bearings failing. The capacities of SKF bearings, for instance, are given for 500 hours; those of Timken bearings are for 3000 hours.

After the radial and axial loads are determined from a study of the machine in which the bearing is to operate, the problem becomes one of reducing the actual radial and thrust loads at the required speed and desired hours of life to an equivalent load at a speed of 500 rpm and 500 hours of life for ball bearings and 3000 hours of life for Timken roller bearings. A bearing which has at least this rated capacity is then selected from the tables in the manufacturer's handbook. Variations in load affect the capacity of the bearing and allowance for this must be made. Where the load acts in one direction and the variation is simple and continuous, the equivalent steady load may be expressed as

$$W = \frac{W_{\min} + 2W_{\max}}{3} \quad (8-9)$$

The SKF Company states that the equivalent radial load for their ball bearings is given by the equation

$$P = x(R + yA) \quad (8-10)$$

where P = equivalent radial load at the actual speed and hours of life

R = actual radial load

A = axial thrust load

y = an empirical factor depending on the size and type of bearing (Table 8-3 gives a few values of y .)

- $x = 1$, for self-aligning bearings or other bearings with inner ring rotating
 $= 1.33$ for non-self-aligning bearings with outer ring rotating.

TABLE 8-3

Approximate y Values for SKF Bearings

(For exact values see SKF data sheets or catalog.)

Type	Bearing Size	y	Type	Bearing Size	y
Ball Bearings			Ball Bearings		
Self-aligning bearings	13,300-13,304	2.5	Deep-groove bearings	34-39 6,200- 6,240X 6,300- 6,352X 6,403- 6,418	*
	1,200- 1,203	2.5			
	1,204- 1,205	3.0			
	1,206- 1,208	3.5			
	1,209- 1,211	4.0	Z plate bearings	36Z-39Z 6,200Z- 6,220Z 6,300Z- 6,318Z	1.5 to 1.0
	1,212- 1,222	4.5			
	1,300- 1,303	2.5	Snap-ring bearings	6,205N- 6,212N 6,304N- 6,310N	
	1,304- 1,306	3.0			
	1,307- 1,324	3.5	Angular contact bearings	7,205- 7,240 7,304- 7,340 7,405- 7,420X	0.33
	10,405-10,418	3.5			
	2,200- 2,206	2.0			
	2,207- 2,209	2.5			
	2,210- 2,222	3.0	Double-row deep-groove bearings	5,202- 5,222 5,301- 5,322 5,405- 5,417	1.1
	2,300- 2,304	1.5			
	2,305- 2,322	2.0	Roller Bearings		
Self-aligning adapter bearings	1,504A-1,505A	3.0	Spherical roller bearings	22,216-22,264 22,308-22,356X	3.0 2.0
	1,506A-1,507A	3.5			
	1,509A-1,511A	4.0	Adapter spherical roller bearings	22,516-22,540A 22,609-22,640A	3.0 2.0
	1,513A-1,522A	4.5			
	1,604A-1,606A	3.0			
	1,607A-1,622A	3.5			
	2,606A-2,622A	2.0			

* $y = 1.5$ for high speeds and 1.0 for low speeds.

If the life desired of the bearing is other than 500 hr, another correction must be made by applying what is termed the "life factor." The equivalent load Q for a life of 500 hr at the operating speed is shown by the equation

$$Q = PS \quad (8-11)$$

$$\text{where } S = \frac{(\text{desired hours of life})^{\frac{1}{3}}}{7.94}.$$

In order to select the required bearing from the catalog the value of Q must be further corrected by applying a speed factor, T :

$$T = \frac{7.94}{(\text{rpm})^{\frac{1}{3}}} \quad (8-12)$$

The calculated equivalent radial load for 500 hr of life and 500 rpm is

$$W = \frac{Q}{T} \quad (8-13)$$

The tables given in the catalog are entered with this value and a bearing having a rated capacity not less than W is selected.

SKF states that elevated temperatures, even of short duration, cause a reduction in carrying capacity as shown in Table 8-4.

TABLE 8-4

Temperature °F.	257	302	347	392	437	482
Reduction	5%	10%	15%	25%	35%	40%

Example. A ball bearing mounted on a spindle turning at 3370 rpm is to sustain a radial load of 800 lb and a thrust load of 500 lb. The desired life of the bearing is 6000 hr. Determine the rated load for which the bearing is to be selected, assuming it to be a deep-groove type.

Solution. Since this is a high speed bearing, y will be taken as 1.5 (Table 8-3).

Equivalent radial load at 3370 rpm and 6000 hr life

$$P = 800 + 1.5 \times 500 = 1550 \text{ lb}$$

Equivalent radial load at 3370 rpm and 500 hr life

$$Q = PS = 1550 \times \frac{(6000)^{\frac{1}{3}}}{7.94} = 3550 \text{ lb}$$

Equivalent radial load at 500 rpm and 500 hr life = rated load

$$= W = Q \div T = 3550 \div \frac{7.94}{3370^{\frac{1}{3}}} = 6700 \text{ lb}$$

Owing to the fact that a radial load on a tapered roller bearing produces some thrust, the procedure to be followed in selecting a roller bearing to meet a given set of conditions is somewhat different from that just outlined for ball bearings. The remainder of this article is abstracted from the Timken Reference Manual.

As in the selection of a ball bearing, the scheme is to find the load for a speed of 500 rpm and a life of 3000 hr which will be equivalent to the actual load at the given speed and desired hours of life. In addition to the correction factors to take care of the speed and life, a third factor, called the application factor, is introduced to provide for the special conditions met with in the service to which the bearing under consideration is to be applied. Values of this factor are given in Table 8-5. Since the bearings must be used in pairs, any difference in the load on the members of the pair causes an unbalanced thrust to act on the more lightly loaded bearing of the pair. This must be taken account of in calculating the radial load the bearing must sustain.

The foregoing may be stated as

Required rating at 500 rpm =

$$\frac{\text{calculated load} \times \text{life factor} \times \text{application factor}}{\text{speed factor}} \quad (8-14)$$

In the foregoing equation the life factor can be expressed as

$$K_L = \left(\frac{\text{hours of desired life}}{3000} \right)^{0.3} = \frac{(\text{hours of desired life})^{0.3}}{11.05} \quad (8-15)$$

TABLE 8-5

Timken Application Factors

Service	Factor	Service	Factor
Airplane landing gears . . .	1	Gear reduction units . . .	1
Auto wheels and differ- entials	1.75	Locomotives	1
Auto transmissions	1	Machine tools	1
Belt drives	1	Paper mill machinery . . .	1.33
Clay-working machinery..	2	Paper mill pumps	1.33
Electric motors and gen- erators	1	Dredge pumps	1.5
Engines	1	Refrigerating machinery	1
Fans and blowers	1	Road-making machinery	2
Farm machinery	2	Rubber machinery	1.5
Farm tractor wheels	1.33	Woodworking machinery	1
		Steel mill machinery . . .	1
		Stokers	1.33
		Textile machinery	1

the speed factor as

$$K_s = \left(\frac{500}{\text{rpm}} \right)^{0.3} = \frac{6.46}{(\text{rpm})^{0.3}} \quad (8-16)$$

If the bearings of a pair are of the same size and carry approximately equal radial loads and no thrust, the calculated load is that imposed by the external forces acting on the shaft. When thrust is present, the calculated load must be found by combining the thrust load with the radial load as shown.

Calculated load

$$= 0.66 \times \text{actual radial load} + K_T \times \text{thrust load} \quad (8-17)$$

where $K_T = \frac{\text{radial rating}}{\text{thrust rating}} =$ about 1.5 for standard Timken

bearings and 0.75 for steep-angle bearings, used where thrust predominates. Should the radial load calculated in this way be less than the radial load imposed, it should be disregarded and the selection made on the basis of the imposed radial load alone.

Example. A Timken bearing supporting one end of the worm shaft of a speed reducer turning at 1800 rpm is to carry a radial load of 242 lb and a thrust load of 550 lb. The desired life is 12,000 hr. Determine the rated capacity of the bearing to be selected.

Solution.

$$\text{Speed factor} = K_S = \left(\frac{500}{1800} \right)^{0.3} = 0.681$$

$$\text{Life factor} = K_L = \left(\frac{12,000}{3000} \right)^{0.3} = 1.51$$

$$\text{Application factor} = K_A = 1$$

Since the thrust load exceeds the radial load, a steep-angle bearing will be used. Then

$$\text{Calculated load} = 0.66 \times 242 + 0.75 \times 550 = 568 \text{ lb}$$

$$\text{Rated load} = \frac{568 \times 1.51 \times 1}{0.681} = 1260 \text{ lb}$$

Owing to the conical form of the rollers, the radial load on a Timken bearing produces a thrust which is taken by the other bearing of the pair. This thrust produces, in turn, a radial load on the second bearing which must be added to the radial load due to the external forces. If bearing *A* of a pair carries a radial load, the magnitude of this thrust on bearing *B*, of the pair, is given by the relationship

$$\text{Resultant thrust on } B = \frac{0.34 \times \text{radial load on } A}{K_{TA}} \quad (8-18)$$

$$\text{where } K_{TA} = \frac{\text{radial rating of } A}{\text{thrust rating of } A}.$$

In the absence of knowledge of the ratings of *A*, K_{TA} may be taken as 1.5 for standard and 0.75 for steep-angle bearings. The calculated radial load on *B* may now be expressed as

$$\begin{aligned} \text{Calculated radial load on } B \\ = 0.66 \times \text{radial load on } B + K_{TB} \times \\ (\text{resultant thrust on } B \pm \text{external thrust}) \end{aligned}$$

If this is less than the actual radial load it is disregarded and the design is based on the radial alone.

Example. Two bearings, *A* and *B*, constituting a pair, carry loads of 400 and 100 lb, respectively, owing to external forces. Determine the calculated radial load for bearing *B* at operating speed and desired life.

Solution. Assuming

$$\frac{\text{Radial rating } A}{\text{Thrust rating } A} = K_{TA} = 1.5$$

$$\text{Resultant thrust on } B = \frac{0.34 \times 400}{1.5} = 90.7 \text{ lb}$$

Assuming

$$\frac{\text{Radial rating } B}{\text{Thrust rating } B} = K_{TB} = 1.5$$

Calculated radial load on *B* at operating speed and desired life

$$\begin{aligned} &= 0.66 \times 100 + 1.5 \times 90.7 \\ &= 202.05 \text{ lb} \end{aligned}$$

14. Sliding Surfaces. Machine members such as engine cross-heads, planer tables, and shaper rams have a relative motion of translation with respect to their constraining surfaces. In addition, the motion is usually reciprocating. Because of this the lubrication of such surfaces is more difficult than that of journals, and lower bearing pressures must be allowed. Spooner gives 200 to 300 lb per sq in. for babbitt on cast iron and 40 to 100 lb per sq in. for cast iron on cast iron as the allowable bearing pressures for such service. For steam engine work bearing pressures of 30 lb per sq in. for high speed engines to 100 lb per sq in. for marine engines have been found to give satisfactory results in the design of crossheads and their guides.

For compensating for small amounts of wear either the slider or the guide is made adjustable. In engines the crosshead is provided with shoes as shown in Fig. 8-17. Two general methods for securing adjustment are in use. In the first, shown in Fig.

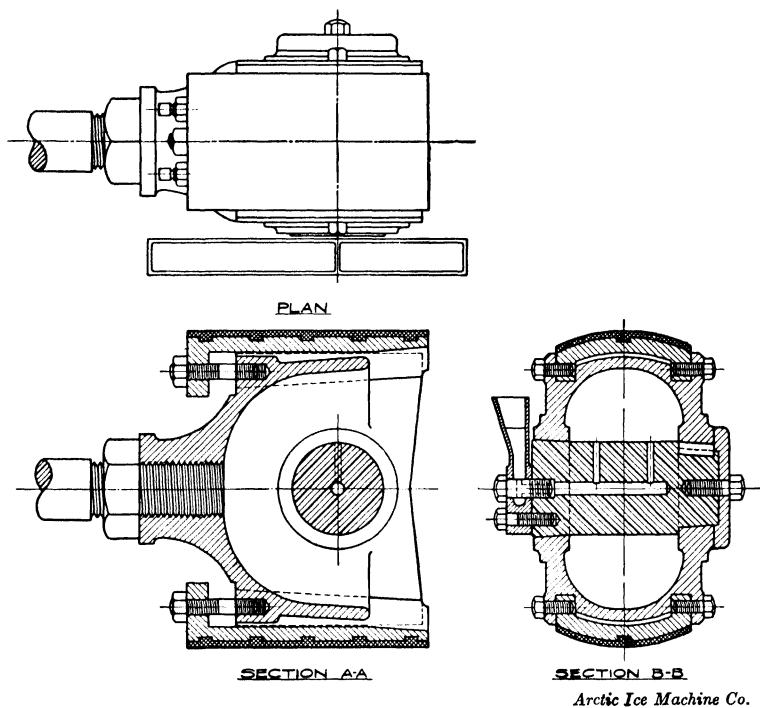


FIG. 8-17. Crosshead with tapered shoes.

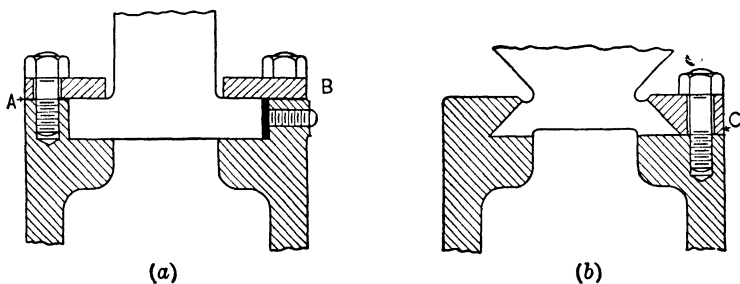


FIG. 8-18. Adjustment for wear of flat surfaces.

8-17, the contact surface between the body of the crosshead and the shoes is tapered and adjustment of the shoe is secured by means of screws. In the second, the adjustment is secured by placing shims between the shoe and the body of the crosshead.

Machine tool slides are usually designed to give complete lateral constraint, the forms most generally used being shown in Figs. 8-18 (a) and (b). The arrangement shown in Fig. 8-18(a) provides for taking up vertical wear either by removing shims from joint A or by removing metal from the bottom of the guide and then tightening the holding bolts. Horizontal wear is taken up by adjusting the wearing strip or gib by means of adjusting screws B. The type shown in Fig. 8-18(b) allows both vertical and horizontal wear to be compensated by adjusting the movable guide C. In Figs. 8-18 (a) and (b) the slider is constrained vertically as well as horizontally.

PROBLEMS — CHAPTER VIII

1. A body weighing 500 lb is to be moved along a plane by a pushing force acting at an angle of 10° with the horizontal. Assuming the coefficient of friction between body and plane to be 0.2, determine the magnitude of the force required to move the body. *Ans.* 105.26 lb.

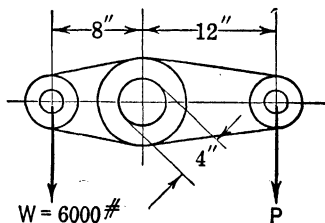
2. The main bearing of an engine is 7 in. in diameter and 11 in. long and sustains a total load of 10,000 lb. The journal rotates at 90 rpm. Assuming a coefficient of friction of 0.015, determine the total work of friction and also the work of friction per square inch of projected area in foot-pounds per minute. *Ans.* 24,900 ft-lb per min and 324 ft-lb per min per sq in.

3. An engine crosshead presses against its guide with an average force of 1500 lb. The stroke of the engine is 36 in. and its speed is 100 rpm. Assume a coefficient of friction of 0.05 and determine the foot-pounds of work lost in friction each minute.

Ans. 45,000 ft-lb per min.

4. The lever shown in the accompanying figure is to raise a load of 6000 lb. The coefficient of friction between the lever and the fulcrum pin is 0.06. Determine the operating force, P , required.

Ans. 4100 lb.



PROB. 4

5. The main bearings of an engine are 7 in. in diameter and 11 in. long and carry a unit load of 150 lb per sq in. of projected area. The shaft turns at 100 rpm. (a) Determine the total

force of friction in each bearing if the coefficient of friction is 0.015. (b) What horsepower is lost? *Ans.* 173.25 lb and 0.962.

6. The two main bearings of a triplex pump are 2 in. in diameter and sustain a load of 1600 lb each. The crankshaft rotates at 50 rpm. A dynamometer test shows that a total of 0.0635 hp is lost in friction. What is the coefficient of friction? *Ans.* 0.025.

7. A crankpin 4 in. in diameter is to be forced into a hole in the web for a distance of 4 in. The tightness of fit is such that a radial pressure of 3000 lb per sq in. is secured. Assume the coefficient of friction to be 0.3 and determine (a) the maximum force required in forcing the pin home; (b) the foot-pounds of work required to force the pin home. *Ans.* 45,400 and 7550 ft-lb.

8. The weight on each wheel of a traveling crane is 5000 lb. The journal is $4\frac{1}{2}$ in. in diameter and 8 in. long, and the wheel is 28 in. in diameter. Assume the coefficient of rolling friction to be 0.02 and the coefficient of friction in the journal to be 0.015. Determine the horsepower necessary to overcome the wheel friction of a four-wheel crane traveling at 600 ft per min.

Ans. 1.4.

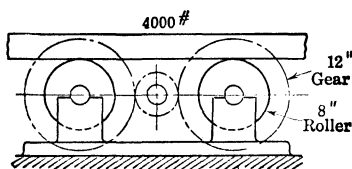
9. The dome of a small observatory weighs 4000 lb and rests on 10 cast-iron balls 4 in. in diameter running on cast iron upper and lower races. Assuming the coefficient of rolling friction to be 0.03, determine the force that must be applied to the dome to move it. *Ans.* 60 lb.

10. A load weighing 6000 lb rests on rollers 4 in. in diameter. The coefficient of rolling friction between the rollers and the load is 0.01 and that between the rollers and their support is 0.015. Determine the work done in pushing the load a distance of 50 ft. *Ans.* 1875 ft-lb.

11. Determine the force with which the brake shoe of a train must be pressed against the wheel to absorb 198,000 ft-lb of energy in 15 sec if the mean velocity of the wheel relative to the brake shoe is 30 mph and the coefficient of friction is 0.2. *Ans.* 1500 lb.

12. (a) It is found that a force of 40 lb is required to move a load of 3000 lb on rollers $1\frac{1}{2}$ in. in diameter. Both surfaces in contact with the rollers are made of the same material and the force is applied to the load. Determine the coefficient of rolling friction.

(b) What force would have been required had the coefficient of rolling friction between one surface and the roller been 0.003 and that of the other as found above? *Ans.* 0.01 in. and 26 lb.



PROB. 13

13. A load of 4000 lb rests on rollers 8 in. in diameter as shown. The journals of the rollers are 3 in. in diameter. The coefficient of rolling friction between the rollers and the load may be taken as 0.003 in. and the coefficient of sliding friction as 0.05. The rollers are driven by a pair of

12-in. diameter gears meshing as shown and attached to the journals of the rollers.

(a) Determine the force required at the pitch line of the gears to move the load. *Ans.* 26 lb.

(b) What horsepower is required to move the load at a uniform speed of 300 ft per min? *Ans.* 0.708.

14. A car has wheels 18 in. in diameter and axles 3 in. in diameter. The coefficient of sliding friction between the axles and bearings is 0.05, and the coefficient of rolling friction between the wheels and the track is 0.02. Determine the foot-pounds of work done in moving the car a distance of 600 ft if the car and load weigh 10,000 lb. A level track is assumed.

Ans. 63,320 ft-lb.

15. A hand-car with its load weighs 8000 lb; it has four wheels 16 in. in diameter pressed on axles 3 in. in diameter. The body of the car is supported on the axles by a total of four babbitted bearings 3 in. in diameter and 6 in. long. The coefficient of rolling friction between the wheels and the rails is 0.02; the coefficient of sliding friction in the babbitted bearings is 0.05. Assuming the hand-car to be on a level track, how much work in foot-pounds would be done in pushing the car 500 ft? *Ans.* 47,600 ft-lb.

16. (a) The main bearing of a center-crank engine is 7 in. long, 4 in. in diameter, and carries a total load of 3000 lb. The engine rotates at 250 rpm. Assume the coefficient of friction to be 0.015 and determine the foot-pounds of energy lost per minute in friction. *Ans.* 11,800.

(b) Consider the bearing to be of heavy construction. Determine the probable running temperature if the room temperature is 70°F.

Ans. 151.5°.

17. A $1\frac{5}{8}$ in. line shaft running 150 rpm is supported by light hanger bearings. The bearing pressure is 20 lb per sq in. Assume a coefficient of friction of 0.035 and a room temperature of 75°F and determine the probable running temperature of the bearing. *Ans.* 96.2°.

18. A $3\frac{7}{16}$ in. line shaft rotating 200 rpm is supported by heavy bearings, the unit bearing pressure being 120 lb per sq in. Assume a coefficient of friction of 0.015 and a room temperature of 80°F and determine the probable running temperature of the bearing. *Ans.* 147.2°F.

19. The bearing of a rotary condenser is 12 in. in diameter and 24 in. long and sustains a total load of 79,000 lb. The journal rotates at 720 rpm. Assume the coefficient of friction to be 0.002 and the bearing to be of heavy construction ($K = 1860$) and determine whether water cooling is necessary for a room temperature of 80°F and a running temperature of 150°F.

Ans. Yes.

20. The crankpin of a high speed stationary engine sustains a maximum load due to steam pressure of 7850 lb. Assume an allowable bearing pressure of 1100 lb per sq in. and a length equal to the diameter of the pin and determine the dimensions for the pin.

Ans. d equals l equals 2.67 in.; say $2\frac{1}{2}$ in.

21. (a) The journals of an electric motor rotating at 1150 rpm are each 3 in. in diameter and support a load of 2700 lb. Determine, for a bearing pressure of 150 lb per sq in., the length for the bearings. *Ans.* $l = 6$ in.

(b) Assuming the probable coefficient of friction to be 0.0015 and a room temperature of 70°F and that the bearing is ventilated, determine whether its running temperature is satisfactory. *Ans.* 117°F; satisfactory.

22. A steam turbine rotor is to run at 1200 rpm and is supported by bearings having a diameter of 12 in. and a length of 30 in. The bearing pressure is 60 lb per sq in. of projected area. Assume a coefficient of friction of 0.008 and determine the number of foot-pounds of energy that must be carried away per second by the bearing cooling water if the room temperature is 90°F and the maximum allowable bearing temperature is 140°F. (Consider the bearing ventilated.) *Ans.* 9580 ft-lb per sec.

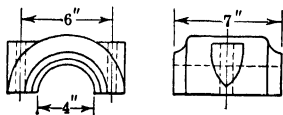
23. Each of the main bearings of a triplex pump is 4 in. long and 2 in. in diameter and sustains a load of 1600 lb. This load is directed against the cap. The distance between the center lines of the bolts on either side of the cap is $3\frac{3}{4}$ in. The cap is to be made of cast iron having an ultimate strength of 20,000 lb per sq in. in tension and 96,000 lb per sq in. in compression. Take an apparent factor of safety of 5, and determine the thickness for the cap. *Ans.* 0.56 in.; say $\frac{9}{16}$ in.

24. The crankpin of an automobile crankshaft is $2\frac{1}{8}$ in. in diameter and 2 in. long. The connecting-rod cap is to be made of a steel drop-forging having an ultimate tensile strength of 80,000 lb per sq in. The distance between the center lines of the holding bolts is $2\frac{7}{8}$ in. Take a *real* factor of safety of 4 and determine the thickness for the cap necessary to carry a load of 800 lb. *Ans.* 0.268 in.; say $\frac{1}{4}$ in.

25. The bearings of an electric motor which has a speed of 1150 rpm sustain a load of 800 lb each. The diameter of the shaft is $2\frac{1}{2}$ in. Assume a bearing pressure of 141 lb per sq in. and determine the length for the bearing. *Ans.* 2.27 in.; say $2\frac{3}{8}$ in.

26. A bearing cap is to be made of malleable cast iron with a tensile strength of 45,000 lb per sq in. The journal is 3 in. in diameter and the bearing is $4\frac{1}{2}$ in. long. The load on the cap is 1500 lb, and the bolts holding the cap in place are spaced 4 in. from center to center measured at right angles to the shaft. Assume an apparent factor of safety of 6 and find the thickness of the cap.

27. The connecting-rod cap of an engine sustains a maximum load of 40,000 lb and is made of cast steel having an ultimate strength in tension of 80,000 lb per sq in. The diameter of the crankpin is $5\frac{1}{2}$ in., the axial length of the bearing 6 in., and the distance between the holding bolts, measured at right angles to the shaft, is 8 in. Take a *real* factor of safety of 5 and determine the thickness for the cap.



PROB. 28.

28. The bearing cap shown in the figure is to be used on the connecting rod of a gas en-

gine. It is to carry a maximum load of 20,000 lb. The material is to be cast steel having an ultimate strength of 80,000 lb per sq in. Use a real factor of safety of 6 and determine the thickness for the cap.

29. A ball bearing is to be fitted to a journal which rotates at 1750 rpm. It is to sustain a purely radial load of 600 lb and its desired life is to be 1800 hr. Determine the rated capacity of the deep-groove bearing which should be specified. *Ans.* 1325 lb.

30. A ball bearing is to be fitted to a journal which rotates at 1150 rpm. It is to sustain a radial load of 600 lb and an axial load of 300 lb. It is to have a desired life of 1800 hr. Determine the rated capacity of a Series 1200-1203 SKF self-aligning bearing. *Ans.* 2595 lb.

31. An SKF deep-groove bearing is to sustain a thrust load of 1200 lb at a speed of 1600 rpm. There is no radial load. The desired life is 8000 hr. Determine the rated capacity of the required bearing. *Ans.* 6300 lb.

32. The shaft of an electric motor which rotates at 1750 rpm is to be supported by a pair of Timken roller bearings. Each bearing is to sustain a pure radial load of 950 lb and is to have a desired life of 10,000 hr. Determine the rated capacity for the bearing. *Ans.* 1984 lb.

33. A herringbone gear used in a paper machine is mounted on a pair of Timken bearings. Bearing *A* carried a radial load of 2000 lb and an axial thrust of 600 lb. Bearing *B* carries a radial load of 700 lb. The speed of the shaft is 125 rpm. The desired life is 15,000 hr. Specify the rated capacity of bearing *A*. *Ans.* 3430 lb.

34. Specify the capacity of bearing *B* in Prob. 33. *Ans.* 985 lb.

CHAPTER IX

FRICTION CLUTCHES AND BRAKES

1. Friction Clutches. When a shaft is frequently to be connected to and disconnected from a shaft which runs continuously, some form of friction clutch is used. As the name indicates, there is no positive connection between the parts of the clutch attached to one shaft and those attached to the other, sufficient friction being developed between the contact surfaces to permit the transmission of the desired power without these parts slipping relatively to each other.

The friction clutch possesses a great advantage over the jaw because it can be engaged and disengaged without serious shock when the driving shaft is rotating at a high speed. This feature renders such couplings particularly well adapted to connecting the engines of motor cars to the transmission, and the automobile industry is today the largest field of friction-clutch application. Friction clutches are also used on machine tools such as lathes and drill presses, on traction engines, hoisting machinery, and on line shafting.

2. Types of Friction Clutches. The most common types of friction clutches, using the shape of the contact surfaces as a basis of classification, are

- (a) Cone (Fig. 9-1).
- (b) Disk (Fig. 9-2).
- (c) Band (Fig. 9-3).

The cone clutch consists of the frusta of two cones, one fitting inside the other. The member *A* is attached rigidly to the driving shaft; *B* is attached to the driven shaft by means of a feather key which permits *B* to move axially, but not rotatively, relative to the driven shaft. An axial force *P* provided by a spring keeps *B*

pressed tightly against *A* except when *B* is pulled away from *A* by means of the collar *D*. The angle α must be made fairly large to prevent the clutch from sticking and requiring an application of a large force to disengage the two members. From 8 to 14° has been found the most satisfactory.

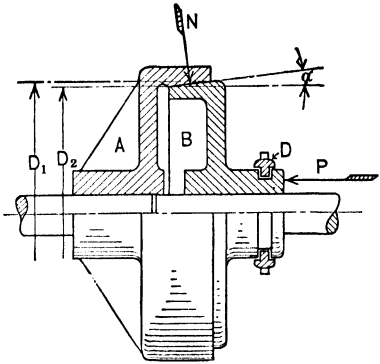
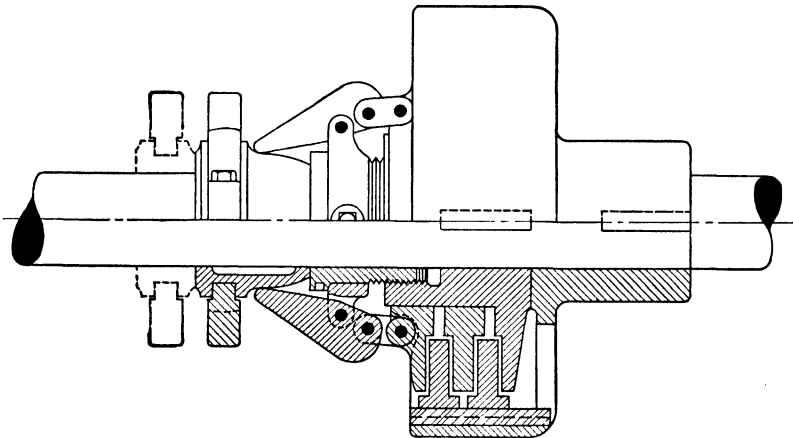


FIG. 9-1. Diagrammatic arrangement of cone clutch.

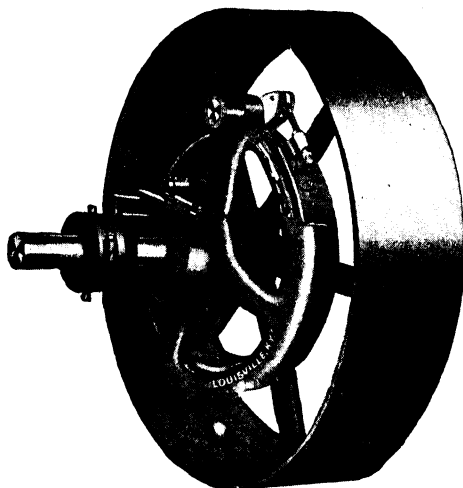
Disk clutches consist of a series of disks keyed alternately to the driving and driven shafts. They are operated in the same manner as the cone clutches. Since each pair of contact surfaces sustains the full operating force, the power which can be transmitted is directly proportional to the number of pairs of contact surfaces, other things being equal. Figure 9-2 shows a disk clutch having four pairs of contact surfaces. The disk clutch is most widely used in automobile work.



Dodge Manufacturing Corporation

FIG. 9-2. Disk clutch.

Band clutches consist of a drum keyed to one shaft and a band passing around the drum and attached to the other. By means of a suitable mechanism the band can be tightened so that it presses



W. E. Caldwell & Co.

FIG. 9-3. Band clutch.

firmly against the drum. The friction set up between the two members causes the band to rotate with the drum and so transmit power from one shaft to the other.

3. Analysis of Clutches. In the analysis of any friction clutch we must remember that the work is equal to the product of a force and the distance traversed by it. The force which transmits power in a friction clutch is the force of friction (equal to the normal pressure times the coefficient of friction), and the distance traversed by this force is the average distance moved through by the friction forces acting on the elementary areas of the contact surfaces.

In the cone clutch shown in Fig. 9-1 the axial force P drives the two cones together, thus setting up a normal force N , the hori-

zontal component of which balances P . Therefore,

$$^1 N = \frac{P}{\sin \alpha} \quad (9-1)$$

and the force of friction is

$$F = fN$$

This frictional force is distributed over the entire surface in contact, and therefore the distances traveled by the elementary forces at different points along the elements of the cone vary, that exerted at D_1 traveling farther than that exerted at D_2 because D_1 is farther from the axis of rotation; although the true average frictional radius should be determined as for the disk clutch given later, the resultant effect is nearly the same as if the entire force of friction were concentrated at the mean diameter of the contact surface, $(D_1 + D_2) \div 2 = D$. If the shafts are revolving at n revolutions per minute and D is in inches the distance traveled by the force of friction will be

$$\text{Distance} = \frac{\pi D n}{12} \text{ ft per min} \quad (9-2)$$

and the horsepower transmitted will be

$$\text{Hp} = \frac{\text{force in pounds} \times \text{feet per minute}}{33,000} = \frac{F \times \pi D n}{33,000 \times 12}$$

In the disk clutch the normal force pressing the disks together is equal to the axial force P , and the frictional radius may be taken as

$$r_f = \frac{2}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) \quad (9-3)$$

The force of friction which is the working force is therefore

$$F = fP \quad (9-4)$$

¹ The friction along the elements of the cone has been neglected. Although not all authorities agree that this should be done, the experimental results of Professor Bonte agree more closely with the calculated values if friction along the elements of the cone is neglected in the calculation.

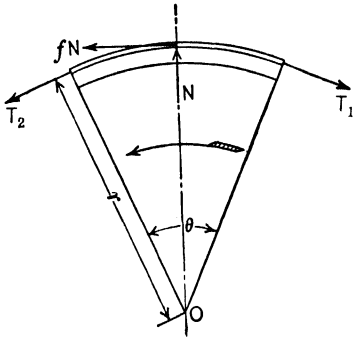


FIG. 9-4

and the distance moved through by it in feet per minute is

$$\text{Distance} = \frac{2r_f \pi n}{12} \quad (9-5)$$

Since, as pointed out above, each pair of contact surfaces sustains the full axial force P , the total power which can be transmitted will vary directly as the number of contact surfaces. Otherwise the disk clutch may be treated in the same way

as the cone clutch and the power transmitted may be found from the product of equations (9-4) and (9-5).

Let Fig. 9-4 represent an infinitesimal portion of the band of a band clutch resting on the drum. This band is in equilibrium under the action of the following forces:

T_2 , the pull in the band on the slack side

N , the radial reaction of the drum against band pressure

T_1 , the pull in the band on the tight side

fN , the force of friction between the clutch drum and the band.

The ratio between T_1 and T_2 can be shown to be

$$T_1 \div T_2 = e^{f\theta} \quad (9-6)$$

where $e = 2.718$, the base of the Napierian system of logarithms.

$T_1 - T_2$ is the net force which is available for transmitting power, and the distance it travels in feet per minute is $\pi Dn \div 12$, where D is the diameter of the clutch drum in inches. The horsepower transmitted by the band clutch is, as in the other cases, the net force in pounds times the distance it travels in feet per minute divided by 33,000 ft-lb per min per hp.

Example. A disk clutch is to be composed of five disks, giving a total of four pairs of contact surfaces. The outside diameter of the contact surfaces is to be 10 in. and the inside 6 in. Determine the pressure with which the disks must be held together if 25 hp

is to be transmitted at 600 rpm, assuming the coefficient of friction to be 0.3.

Solution.

Foot-pounds per minute to be transmitted

$$= 25 \times 33,000 = 825,000$$

Frictional radius of contact surfaces

$$= r_f = \frac{2}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) = \frac{2}{3} \left(\frac{5^3 - 3^3}{5^2 - 3^2} \right) = 4.09 \text{ in.}$$

Distance moved by driving force in feet per minute

$$= \frac{2 \times 4.09 \times \pi}{12} \times 600 = 1285 \text{ ft}$$

Driving force required (pounds)

$$= \frac{825,000 \text{ ft-lb per min}}{1285 \text{ ft per min}} = 643 \text{ lb}$$

Driving force per pair of contact surfaces

$$= \frac{643}{4} = 160.9 \text{ lb}$$

Normal force required to produce a frictional force of 160.9 lb at each contact surface

$$= \frac{160.9}{0.3} = 538 \text{ lb}$$

4. Brakes. Brakes are devices for controlling the speed of motion of parts of machines by absorbing the excess kinetic energy contained in the moving parts. They are based upon a variety of mechanical principles, the most prominent of which are frictional, hydraulic, or magnetic in nature. Only the more elementary types of friction brakes will be dealt with here.

The simplest of all friction brakes is the block brake shown diagrammatically in Fig. 9-5. It consists of a drum attached to the revolving member whose speed is to be controlled and a block

which can be pressed against the drum by means of a lever. This lever is pinned at O so that it cannot revolve with the drum. When the block is pressed against the drum a frictional force F is set up which absorbs energy from the drum. The quantity of

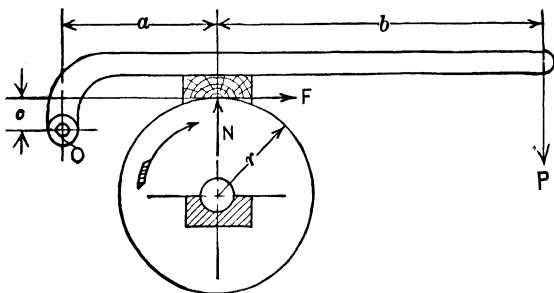


FIG. 9-5. Diagrammatic arrangement of block brake.

energy which can be absorbed will depend on the magnitude of the frictional force set up and on the rubbing velocity. Usually brakes are designed to balance a known torque on the drum shaft. In this case the force of friction F times the radius of the drum is made equal to the torque to be balanced.

In determining the operating force P necessary to balance a given torque with a brake of known dimensions, the lever and block are treated as a free body and moments are taken about the pivot point O . The force of friction required is found by dividing the torque to be balanced by the radius of the drum. The next step is to determine the force with which the block must be pressed against the drum. As pointed out in Chapter VIII, Art. 1, the coefficient of friction, f , equals $F \div N$. The materials in contact being known, f is known and the normal pressure N can be determined by solution. The lever can now be treated as a free body acted upon by the forces producing moments about O . These are F , N , and P . F and N are known, and thus P may be determined by solution. It should be noted that F will depend for its direction upon the direction of rotation of the drum, acting always in the direction in which the surface of the drum is moving relative to the block.

Example. The block brake shown in Fig. 9-5 is to balance a torque of 1800 lb-in. on the drum shaft. The drum has a diameter of 18 in., and the lengths a , b , and c are 12, 28, and 2 in. respectively. Assuming that the coefficient of friction between the block and the drum is 0.25 and that the drum rotates clockwise, determine the necessary operating force P .

Solution.

$$\text{Force of friction required (pounds)} = F = \frac{T}{r} = \frac{1800}{9} = 200 \text{ lb}$$

$$\text{Normal pressure required (pounds)} = N = \frac{F}{f} = \frac{200}{0.25} = 800 \text{ lb}$$

Taking moments about O with those counterclockwise as positive,

$$800 \times 12 - 200 \times 2 - P \times 40 = 0$$

$$P = 230 \text{ lb}$$

The block brake is not ordinarily used for heavy brake duty work because of the heavy load placed on the shaft bearing by the

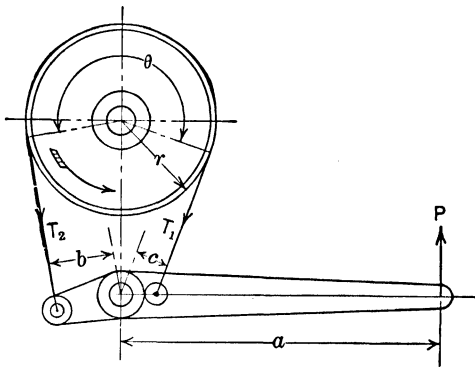


FIG. 9-6. Diagrammatic arrangement of band brake.

pressure of the block. This can be obviated by placing two blocks on opposite sides of the drum and thus balancing the radial pressure of one by that of the other.

A very powerful type of brake is the band brake, one example of which is shown in Fig. 9-6. It consists of a band usually

lined with some friction material such as wood or asbestos fabric, which passes around the drum. One end of the band is always attached to the operating lever. The other may be attached to a stationary object, such as the frame of the machine, or to another point of the operating lever as shown. In this case the pull in one side of the band assists the operating force.

The force of friction necessary to balance a given torque is found for this kind of brake as it was for the block brake, and is equal to $T_1 - T_2$, the difference between the tensions on the tight and slack sides of the band. The values of T_1 and T_2 may be determined as for the band clutch by the use of equation (9-6):

$$\frac{T_1}{T_2} = e^{f\theta}$$

Once T_1 and T_2 and their points of attachment to the lever are known, the operating force can be easily determined by taking moments about the center of the lever pivot.

Example. The band brake shown in Fig. 9-6 is to balance a torque of 1800 lb-in. on the drum shaft. The drum is 18 in. in diameter, and dimensions a , b , and c are 20, 4, and 1 in. respectively; θ is 270° . Assuming that the coefficient of friction between the band and the drum is 0.25 and that the drum rotates counter-clockwise, determine the necessary operating force P .

Solution.

Net force required at the rim of drum (pounds)

$$= \frac{T}{r} = \frac{1800}{9} = 200 = T_1 - T_2$$

Ratio

$$\frac{T_1}{T_2} = e^{f\theta} = 2.718^{0.25 \times \frac{3\pi}{2}} = 3.25$$

$$3.25T_2 - T_2 = 200$$

Therefore

$$T_2 = \frac{200}{2.25} = 88.8 \text{ lb}$$

$$T_1 = 200 + 88.8 = 288.8 \text{ lb}$$

Taking moments about the center of the lever pivot and considering clockwise moments as positive, we have

$$88.8 \times 4 - 288.8 \times 1 - P \times 20 = 0$$
$$P = 3.32 \text{ lb}$$

Inspection of the moment equation shows that, as the distance from the point of application of T_1 to the lever pivot increases, the operating force required decreases, and a point is soon reached at which the operating force is zero and the brake is "self-acting." Such a design should be avoided, because the application of such a brake will produce disastrous shocks in the mechanism whose motion is to be regulated.

The widest application of band brakes is in hoisting machinery and automobiles.

5. Materials for Clutches and Brakes. The materials used for the contact surfaces of clutches and brakes must possess the following characteristics:

- (a) High coefficient of friction.
- (b) Must resist wear.
- (c) When used for heavy duty work attended by much slipping, they must be able to withstand the high temperature produced.

Woven asbestos fabric seems best adapted to meet these requirements and is widely used in surfacing one of the engaging members in automobile clutches and brakes. Leather and wood are also used extensively where the temperatures caused by slipping are not excessive. Disk clutches frequently have plugs of cork inserted in every other disk, the area of the insertion amounting to 10 to 40 per cent of the total contact area of the disk. Steel is almost always used for the member in contact with the disk faced with the materials mentioned above. Sometimes bronze is used for one of the surfaces and steel for the other. In railway brake service, cast iron brake blocks or shoes have given excellent performance.

The coefficient of friction decreases rapidly with an increase of relative velocity of the contact surfaces. Thurston states that

TABLE 9-1

Coefficients of Friction for Brake and Clutch Materials

Mating Materials	Coefficient	Allowable Bearing Pressure (Pounds per Square Inch)	
		Brakes	Clutches
Cast iron on cast iron	0.15-0.2	50	50
Cast iron on wood	0.2 -0.25	7	40
Cast iron on brass	0.21	11	
Leather on metal (dry)	0.56		
Leather on metal (greasy)	0.23	7	8
Leather on metal (oily)	0.15		
Cork on metal (dry)	0.35		1.2
Cork on metal (greasy)	0.32		
Asbestos fabric on metal (dry)	0.35	30* (trucks)	60* (passenger cars)
Asbestos fabric on metal (oily)	0.25	30* (trucks)	60* (passenger cars)

* Good for either brakes or clutches.

for cast iron brake shoes bearing against steel tires of railway car wheels the following coefficients of friction were found:

Miles per Hour	Coefficient of Friction
5	0.360
30	0.184
60	0.062

Care must be observed, in designing brakes and friction clutches, to keep bearing pressures low. Unless this is done, extremely rapid wear and failure of the device will result. Furthermore, the amount of energy absorbed per square inch of brake surface must be limited to prevent excessive temperature of the brake.

For brakes having wooden blocks for one of the contact surfaces the amount of energy absorbed should not exceed 250 ft-lb per sq in. per min. When both contact surfaces are metal this value may be several times greater than that given for wood. Clutches are subjected to less slipping than brakes and therefore higher bearing pressures may be employed.

PROBLEMS — CHAPTER IX

1. A leather-faced cone clutch having a mean diameter of 15 in. is to transmit 20 hp at 1000 rpm. The elements of the face make an angle of 10° with the axis. Assume a coefficient of friction of 0.3 and determine the axial force with which the halves of the clutch must be pressed together.

Ans. 97 lb.

2. A leather-faced cone clutch having a mean diameter of 10 in. is to transmit 15 hp at a speed of 800 rpm. The cone angle is 12° . Assuming a coefficient of friction of 0.25 and an allowable bearing pressure of 8 lb per sq in., determine the width of face for the clutch.

Ans. $3\frac{3}{4}$ in.

3. A cone clutch having a cone angle of 9° and a mean diameter of 6 in. is to transmit 2 hp at 250 rpm. The clutch is leather-faced. Assume a coefficient of friction of 0.35 and determine the axial force with which the halves must be pressed together.

4. A disk clutch is composed of 9 steel disks alternating with 10 brass disks to give 18 pairs of contact surfaces. The outside diameter of the contact surfaces is 8 in. and the inside $6\frac{1}{2}$ in.; 25 hp is to be transmitted at 900 rpm. Assume a coefficient of friction of 0.2 and determine the axial force with which the disks must be pressed together.

Ans. 136 lb.

5. A disk clutch consisting of 2 steel disks alternating with 1 asbestos fabric-faced disk having an outside diameter of 10 in. and an inside diameter of 8 in. runs at 2000 rpm. Determine the horsepower that can be transmitted if the coefficient of friction is 0.35 and the disks are pressed together by an axial force of 250 lb.

Ans. 25.3.

6. (a) The drum of a band clutch is 18 in. in diameter, rotates at 250 rpm, and transmits 10 hp. The band is to be made of steel lined with asbestos fabric and is to be $\frac{1}{8}$ in. thick. The band makes contact with the drum for an angle of 270° . Assume the coefficient of friction to be 0.3 and determine the pulls on the tight and slack sides of the band.

Ans. 370 and 90 lb.

(b) Assuming the ultimate strength of the band material to be 60,000 lb per sq in. and taking a factor of safety of 10, determine the width for the band.

Ans. 1 in.

7. A drive shaft rotating at 1500 rpm is connected to the source of power by means of a cone clutch. The mean diameter of the clutch is 16 in. and

the cone angle is 10° . What horsepower can be transmitted if the axial force pressing the two halves together is 200 lb and the coefficient of friction is 0.35?

Ans. 76.3.

8. A band clutch has a drum diameter of 10 in. The band is made of steel having a thickness of $\frac{3}{16}$ in. and an allowable stress of 6000 lb per sq in. It is lined with asbestos fabric having a coefficient of friction of 0.4. The angle of wrap is 270° . Determine the width of band necessary to transmit 15 hp uniformly at 250 rpm.

9. A multiple disk clutch having 10 steel and 11 bronze plates is to transmit 40 hp when rotating uniformly at 2400 rpm. The outside diameter of the disks is 10 in. and the inside diameter 6 in. Take the coefficient of friction between the disks as 0.1 and determine the force with which they should be pressed together.

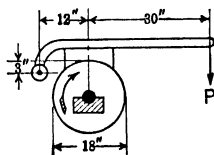
10. A cone clutch having a mean diameter of 12 in. is to transmit 10 hp when rotating at a speed of 300 rpm. The elements of the face make an angle of 8° with the axis. Assume a coefficient of friction of 0.45 and determine the axial force with which the halves must be pressed together. *Ans.* 108 lb.

11. A disk clutch is composed of three steel and two asbestos-faced disks having inside diameters of 6 in. and outside diameters of 10 in. The coefficient of friction is 0.35 and the disks are pressed together with a force of 300 lb. Find the horsepower that can be transmitted at a speed of 3000 rpm.

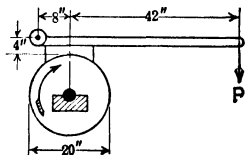
Ans. 81.

12. The block brake shown in the accompanying figure is to balance a torque of 900 lb-in. on the drum shaft. Assume a coefficient of friction of 0.2 and determine the operating force P .

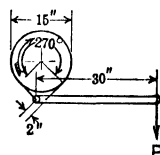
Ans. 136 lb.



PROB. 12



PROB. 14



PROB. 15

13. What would be the operating force for the brake shown in the figure for Prob. 12 if the drum rotated counterclockwise? *Ans.* 150 lb.

14. The brake shown in the figure is to balance a torque of 1200 lb-in. on the drum shaft. Assume a coefficient of friction of 0.25 and determine the operating force P . *Ans.* 86.4 lb.

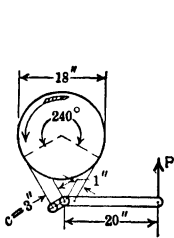
15. The band brake shown in the figure is to balance a torque of 3000 lb-in. on the drum shaft. Assume the coefficient of friction between the band and the drum to be 0.3 and determine the operating force P . *Ans.* 8.53 lb.

16. (a) The band brake shown in the figure is to balance a torque of 6300

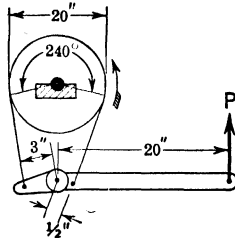
lb-in. at the drum shaft. Assume a coefficient of friction of 0.25 and determine the operating force P .

(b) What value of the dimension C will make the brake self-acting?

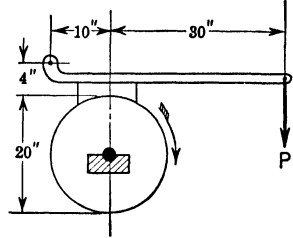
Ans. 2.8 lb; 2.85 in.



PROB. 16



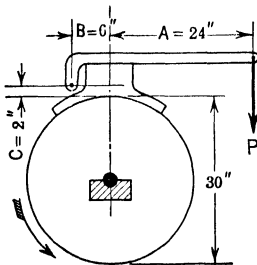
PROB. 17



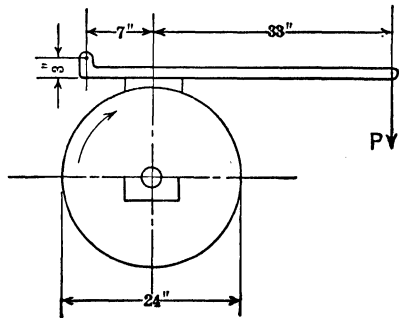
PROB. 18

17. The drum of the band brake shown in the figure is to sustain a torque of 6000 lb-in. Assuming a coefficient of friction of 0.4 and an arc of contact of 240° , determine the operating force, P , required.

18. The block brake shown in the figure is to balance a torque of 5000 lb-in. on the drum shaft. Assume the coefficient of friction between brake shoe and drum to be 0.25 and determine the operating force, P , required.



PROB. 19



PROB. 20

19. The block brake shown in the figure is to balance a torque of 15,000 lb-in. acting on the shaft. The coefficient of friction between the brake shoe and drum is 0.2. Determine the operating force, P , required. Can this brake be made self-acting? If so, state the change in dimension required.

20. The brake shown in the figure is to balance a torque of 1080 lb-in. acting on the drum shaft. Assume the coefficient of friction between the block and the drum to be 0.3 and determine the operating force, P , required.

Ans. 59.3 lb.

CHAPTER X

SHAFTS AND COUPLINGS

1. Materials. Ordinary shafting for use in line shafts for the transmission of power in shops, as well as small shafts in machine tools, is usually made from cold-rolled steel. More important shafts such as engine or marine propeller shafts are made from forged steel containing 0.35 to 0.45 per cent carbon. Where great strength is especially desired the shafts are heat treated.

2. Design of Shafting. Shafting is generally subjected to a combination of bending and torsion though occasional instances may be found where either of these may be so small as to be negligible. Thus the live axle of a motor car having a full floating rear axle is subjected to pure torsion, whereas the axle of a railway car may be considered as a shaft subjected to pure bending. Marine propeller shafts are subjected to a combination of torsion and compression; many hydraulic turbine shafts (vertical) are acted upon by torsion and tension.

The design of shafts may be based on either of two wholly separate considerations:

- (a) Strength.
- (b) Stiffness.

The process of design from the standpoint of strength is illustrated in the following example.

Assuming the force F lb to be concentrated at some radius R in., such that when rotating at the given speed it will develop the given power, we have from the definition of work given in Chapter VII, Art. 10,

$$\begin{aligned}\text{Work} &= \text{force} \times \text{distance} \\ &= \text{force} \times \text{circumference of force circle} \times \text{rpm} \\ &= (F \times 2R \times \pi \times N) \text{ in.-lb per min}\end{aligned}$$

TABLE 10-1
Cold-finished Shafting Diameters and Tolerances

Transmission Shafting Sizes Inches	Machinery Shafting Sizes Inches	Tolerances* on Diameters Inches	Transmission Shafting Sizes Inches	Machinery Shafting Sizes Inches	Tolerances* on Diameters Inches
	$\frac{1}{2}$	-0.002	$2\frac{3}{16}$	$2\frac{3}{16}$	-0.004
	$\frac{9}{16}$	-0.002		$2\frac{1}{4}$	-0.004
	$\frac{5}{8}$	-0.002		$2\frac{5}{16}$	-0.004
	$1\frac{1}{8}$	-0.002		$2\frac{3}{8}$	-0.004
	$\frac{3}{4}$	-0.002	$2\frac{7}{16}$	$2\frac{7}{16}$	-0.004
	$1\frac{3}{8}$	-0.002		$2\frac{1}{2}$	-0.004
	$\frac{7}{8}$	-0.002		$2\frac{5}{8}$	-0.004
$1\frac{5}{8}$	$1\frac{5}{8}$	-0.002		$2\frac{3}{4}$	-0.004
	1	-0.002		$2\frac{7}{8}$	-0.004
	$1\frac{1}{16}$	-0.003	$2\frac{15}{16}$	3	-0.004
	$1\frac{1}{8}$	-0.003		$3\frac{1}{8}$	-0.004
$1\frac{3}{16}$	$1\frac{3}{16}$	-0.003		$3\frac{1}{4}$	-0.004
	$1\frac{1}{4}$	-0.003		$3\frac{3}{8}$	-0.004
	$1\frac{5}{16}$	-0.003	$3\frac{7}{16}$	$3\frac{1}{2}$	-0.004
	$1\frac{3}{8}$	-0.003		$3\frac{5}{8}$	-0.004
$1\frac{7}{16}$	$1\frac{7}{16}$	-0.003		$3\frac{3}{4}$	-0.004
	$1\frac{1}{2}$	-0.003		$3\frac{7}{8}$	-0.004
	$1\frac{9}{16}$	-0.003	$3\frac{15}{16}$	4	-0.004
	$1\frac{5}{8}$	-0.003		$4\frac{1}{4}$	-0.005
$1\frac{11}{16}$	$1\frac{11}{16}$	-0.003	$4\frac{7}{16}$	$4\frac{1}{2}$	-0.005
	$1\frac{3}{4}$	-0.003		$4\frac{3}{4}$	-0.005
	$1\frac{13}{16}$	-0.003	$4\frac{15}{16}$	5	-0.005
	$1\frac{7}{8}$	-0.003		$5\frac{1}{4}$	-0.005
$1\frac{15}{16}$	$1\frac{15}{16}$	-0.003	$5\frac{7}{16}$	$5\frac{1}{2}$	-0.005
	2	-0.003		$5\frac{3}{4}$	-0.005
	$2\frac{1}{16}$	-0.004	$5\frac{15}{16}$	6	-0.005
	$2\frac{1}{8}$	-0.004			

* These tolerances are *negative* and represent the maximum allowable variation *below* the exact nominal size. For instance, the maximum diameter of the $1\frac{15}{16}$ -in. shaft is 1.938 in. and its minimum allowable diameter is 1.935 in.

Standard stock lengths for cold-finished shafting shall be: 16, 20, and 24 ft.

and

$$1 \text{ hp} = 33,000 \times 12 \text{ in.-lb per min.}$$

Therefore the horsepower developed by a force of F pounds concentrated at R in. from the center of rotation and rotating at N revolutions per minute is

$$\text{Hp} = \frac{F \times 2R \times \pi \times N}{33,000 \times 12} \quad (10-1)$$

but $F \times R =$ torque or turning moment, and equation (10-1) can be written

$$\text{Hp} = \frac{2\pi NT}{33,000 \times 12} \quad (10-2)$$

The torque acting on a shaft can be computed by using equation (10-2) if the power to be transmitted and the rotational speed are known. When the shaft is subjected to pure torsion, the relation between torque, the stress induced, and the diameter of the shaft is given by the laws of mechanics as

$$T = s_s \frac{I_p}{c} = s_s \frac{\pi d^3}{16} \quad (10-3)$$

where $s_s =$ unit stress induced by torque T

$d =$ diameter of the shaft in inches

$T =$ torque acting in pound-inches.

In the usual case of design, where the torque or horsepower is known and the diameter is desired, the induced shearing stress is taken to be equal to the ultimate shearing strength of the material divided by a suitable factor of safety, and equation (10-3) is applied. The shaft of the size thus determined will have ample strength but still may not be satisfactory because of lack of rigidity. That is to say, it may either twist or bend so much as to render it useless for the purpose for which it is intended.

Torsional stiffness may be expressed by the equation

$$\theta = \frac{32Tl}{\pi E_s d^4} \quad (10-4)$$

where θ = angle of twist in radians

l = length in inches along axis of shaft in which twist occurs

E_s = shearing or transverse modulus of elasticity of shaft material.

From equation (10-4) either the torsional deflection occurring in a known shaft or a diameter corresponding to an assumed deflection may be found when the material, torque, and length are known. Good practice limits the torsional deflection of line shafting to one degree in a length corresponding to twenty diameters of the shaft. Special considerations may, however, allow considerable variation from this standard. An examination of equations (10-3) and (10-4) shows that the stress induced by a given torque is independent of the length of the shaft, whereas the torsional deflection varies directly as the length. Long shafts should therefore always be checked for stiffness as well as strength.

Many shafts are subjected to heavy loads acting at right angles to their axes. Such shafts have to withstand considerable bending action in combination with the torsion produced by the power transmitted. The following equations apply to such cases where the *shafting is circular*:

$$M_e = \frac{M}{2} \left[1 + \sqrt{1 + \left(\frac{T}{M} \right)^2} \right] \quad (10-5)$$

$$T_e = M \sqrt{1 + \left(\frac{T}{M} \right)^2} \quad (10-6)$$

where M_e = equivalent bending moment

T_e = equivalent twisting moment

M = actual bending moment

T = actual twisting moment.

The **equivalent** bending and twisting moments may be defined as the bending and twisting moments which, acting alone, would induce the same tensile and compressive or shearing stress which the **actual** bending and twisting moments acting simultaneously produce. In other words the combined action of M and T , as

far as tension and compression are concerned, may be replaced by a single action of M_e and the shaft be designed as a member subjected to bending alone, application being made of the fundamental bending equation

$$M_e = s \frac{I}{c} = s \frac{\pi d^3}{32} \quad (10-7)$$

in which s is the allowable tensile or compressive stress. As far as shear is concerned the combined action of M and T can be replaced by a single action of T_e and the shaft can be designed as if subjected to torsion alone, application being made of equation (10-3), the fundamental torsion equation. Separate solutions for the diameter of the shaft should be made for each of these cases and the larger result chosen. The examples given below will serve to clarify and illustrate the foregoing statements.

Example. A line shaft rotating at 150 rpm is to transmit 50 hp at a uniform rate and may be considered as being subjected to torsional action only. The shafting material is to have an ultimate shearing strength of 48,000 lb per sq in. Assuming a factor of safety of 6, determine the diameter required.

Solution. From equation (10-2),

Torque required

$$T = \frac{\text{hp} \times 33,000 \times 12}{2\pi N} = \frac{50 \times 33,000 \times 12}{2\pi \times 150} = 21,000 \text{ lb-in.}$$

Working stress

$$= \frac{\text{ultimate stress}}{\text{factor of safety}} = \frac{48,000}{6} = 8000 \text{ lb per sq in.}$$

From equation (10-3),

$$d = \sqrt[3]{\frac{16T}{\pi s_s}} = \sqrt[3]{\frac{16 \times 21,000}{\pi \times 8000}} = 2.375 \text{ in.}$$

say $2\frac{7}{8}$ in. shaft diameter.

Example. Assuming the shaft to be 50 ft long, the allowable torsional deflection to be $12\frac{1}{2}^\circ$, and the transverse modulus of

elasticity of the material to be 12,000,000, determine the diameter required.

Solution.

$$\theta = \frac{12.5\pi}{180} = 0.218 \text{ radian}$$

From equation (10-4)

$$d = \sqrt[4]{\frac{32 \times 21,000 \times 50 \times 12}{\pi \times 12,000,000 \times 0.218}} = 2.65 \text{ in.}$$

say $2\frac{1}{8}$ in. shaft diameter.

In this example, a larger shaft is required to give the desired stiffness than to supply the necessary strength. Had the length of the shaft been 10 ft instead of 50 ft the reverse would have been true.

Example. Suppose the line shaft to be driven from a motor directly under it by means of a 10-in. belt running over a 60-in. pulley attached to the line shaft. The pull on the tight side of the belt is 1150 lb; on the slack side 450 lb. The weight of the pulley is 650 lb. The pulley is overhung, the distance from the central plane of the pulley to the center line of the supporting bearing being 12 in. The ultimate tensile and shearing strengths of the shaft material may be taken as 63,000 and 48,000 lb per sq in. respectively. Using a factor of safety of 6, determine the diameter for the shaft.

In this case we have to deal with a member subjected to combined bending and twisting and, as stated above, the best method of attacking the problem is to determine the equivalent bending and twisting moments and then find the diameter for the shaft subjected first to one and then to the other of these.

Solution.

Total force acting at central plane of pulley at right angles to axis of shaft

$$= 1150 + 450 + 650 = 2250 \text{ lb}$$

Bending moment at center line of supporting bearing

$$= 2250 \times 12 = 27,000 \text{ lb-in.}$$

From equation (10-5),

Equivalent bending moment

$$M_e = \frac{27,000}{2} \left[1 + \sqrt{1 + \left(\frac{21,000}{27,000} \right)^2} \right] = 30,600 \text{ lb-in.}$$

Allowable tensile stress

$$= \frac{63,000}{6} = 10,500 \text{ lb per sq in.}$$

From equation (10-7),

Diameter for strength against bending

$$= d = \sqrt[3]{\frac{32 \times 30,600}{\pi \times 10,500}} = 3.09 \text{ in.}$$

From equation (10-6), equivalent twisting moment

$$T_e = 27,000 \sqrt{1 + \left(\frac{21,000}{27,000} \right)^2} = 34,300 \text{ lb-in.}$$

$$\text{Allowable shearing stress} = \frac{48,000}{6} = 8000 \text{ lb per sq in.}$$

From equation (10-3),

Diameter for strength against twisting

$$= d = \sqrt[3]{\frac{16 \times 34,300}{\pi \times 8000}} = 2.79 \text{ in.}$$

A larger shaft is required for strength against bending than for strength against torsion and therefore a $3\frac{3}{16}$ -in. shaft should be chosen.

3. Effect of Change of Section. Shafts, in general, are not of one constant cross section, but may be comprised of several different diameters for purposes of saving material, manufacturing

expense, and weight, fillets being employed to render the transition of cross section as gradual as possible. When changes in section occur, stress concentrations are produced in the fillet

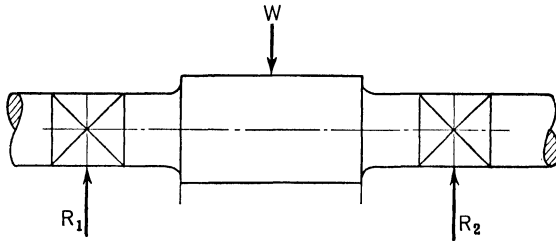


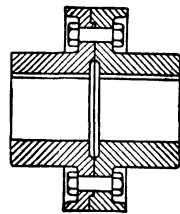
FIG. 10-1

which result in larger stress intensities than those in the adjacent portion of the member. These fillet stresses may be so large as to cause failure of a member that is otherwise safe.

When a member such as a wheel, a coupling, or an armature is pressed or shrunk on a shaft, stress concentrations occur in the shaft at the edge of the pressed-on member. As a result of this stress concentration, failure of the shaft often occurs at this section, even when the stress is apparently low. These stress concentrations can be minimized by seating the pressed-on member on an enlarged portion of the shaft as shown in Fig. 10-1.

4. Couplings. It becomes necessary at times to join short lengths of shafting into one continuous shaft. Either through suitability of design or because frequently shafts are much longer than can be manufactured or shipped, the short lengths are joined together by devices known as couplings.

There is a great variety of these devices on the market, the simplest being the flanged coupling shown in Fig. 10-2. It consists of two flanged hubs which are fitted over the ends of the shafting to be connected and secured to them by sunk keys. The actual joining of the two ends is accomplished by means of bolts passing through the holes in the flanges parallel to the axis of the shaft.



Dodge Manufacturing Co.

FIG. 10-2. Flanged coupling.

A rim is provided at the periphery of the flanges so that the danger of workmen's clothing being caught in the heads or nuts of the coupling bolts is lessened. In order to distribute the load equally on all the bolts both the bolts and the holes should be finished and a perfect fit between them secured.

It will be noticed that the flange bolts in this type of coupling are subjected to a shearing action, the shearing force on each bolt being equal to the torque on the shaft divided by the product of the radius of the bolt circle and the number of bolts. It will also be noticed that the area acted upon by this shearing force is the cross-sectional bolt-shank area at the plane of contact of the halves

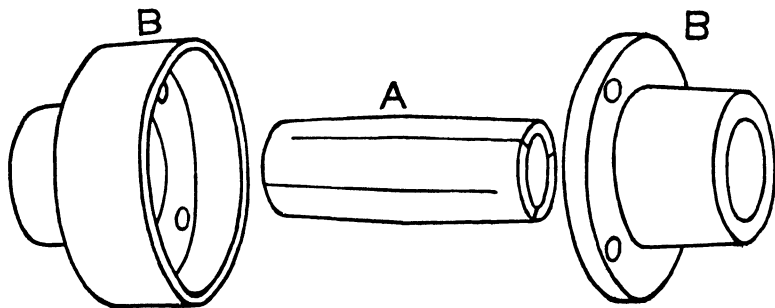


FIG. 10-3

of the coupling. The bolts are therefore designed, or at least checked, as members subjected to simple shear. For the sake of compactness the bolts should be placed as close to the hub of the coupling as possible. When this is done Professor L. D. Hayes gives the number of bolts to be used as

$$n = \frac{2}{3}D + 3 \quad (10-8)$$

where D is the diameter of the shaft in inches.

Where the service is not severe and it is desirable to join pieces of shafting without cutting keyways in them, the compression coupling, an example of which is shown in Fig. 10-3, is frequently used. It consists of a tapered segmental bushing A into which

the ends of the pieces of shafting are slipped. Two flanged hubs *B* having tapered holes in them are slipped over the bushing and drawn together by means of bolts *C*. This action wedges the bushing tightly against the ends of the shafts which are prevented from rotating relative to the bushing by the friction thus set up. The compression type of coupling has proved very satisfactory where the shaft is not subjected to heavy shocks.

Neither of the above-mentioned couplings can be used when the axes of the pieces of shafting to be joined are not in a straight line. Where the axes of the shafts are parallel the Oldham coupling, illustrated in Fig. 10-4, has given good results. It consists of

three pieces, two of which, *b* and *d*, are attached to the ends of the shafts. The third piece *c* is provided with tongues, one on each side, fitting into the corresponding grooves in the flanges of the members attached to the shafts.

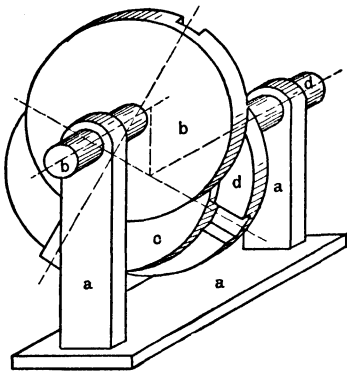


FIG. 10-4. Oldham's coupling.
(From "Kinematics" by R. J. Durley.)

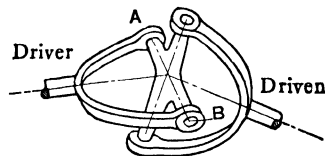


FIG. 10-5. Universal joint.

These tongues are at right angles to each other. This arrangement permits the central member *c* to slide in the direction of either of these tongues and so compensates for lack of alignment of the two shafts it connects.

When the axes of shafts intersect at an angle the shafts may be connected by a Hooke's coupling or, as it is more commonly known, a "universal joint." This mechanism is shown in Fig. 10-5.

A device of this kind is essential for connecting a driving and driven shaft where the angle between them changes in service. Such a condition is encountered in the transmission of power to

the rear axle of a motor car. Here the drive shaft connecting the engine to the rear axle does not make a fixed angle with the axis of rotation of the engine crankshaft, because of the action of the car springs.

It can be shown that a single coupling does not transmit motion with a constant angular velocity ratio, except when the shafts are in line with each other.

The velocity ratio at any instant is equal to

$$\frac{\omega_2}{\omega_1} = \frac{\cos \theta}{1 - \sin^2 \theta \sin^2 (\alpha + 90^\circ)} \quad (10-9)$$

where θ = angle between the shafts

α = angular displacement of the driving shaft from the position where the pins on the drive-shaft yoke lie in the plane of the two shafts

ω_1 = angular velocity of the driving shaft

ω_2 = angular velocity of driven shaft.

The angular velocity ratio ($\omega_2 \div \omega_1$) varies from maximum to minimum value during an angular displacement of 90° . The maximum and minimum values of this ratio are shown by the curves of Fig. 10-6. It will be observed that the speed variation increases rapidly with the shaft angle.

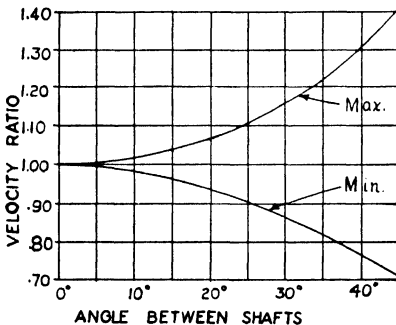


FIG. 10-6

Two shafts lying in any relative position may be connected by a pair of universal joints and an intermediate shaft so that they will have the same angular velocity at any instant. To accomplish this result, the connecting shaft is located so as to make equal angles with the main shafts, and the driving

pins on the yokes attached to the connecting shaft are placed at such an angle that each lies in the plane of the adjacent shafts

at the same instant. Under these conditions, a reduction or increase of the speed of the intermediate shaft, as compared with that of one of the main shafts, caused by the interposed coupling, will be exactly neutralized by an equal but opposite change of speed of the other main shaft as compared with that of the intermediate shaft, owing to the second coupling. The net result is that both main shafts will have the same speed at any instant. The compensating action of the couplings is due to their symmetrical arrangement with regard to the two planes containing the adjacent shafts.

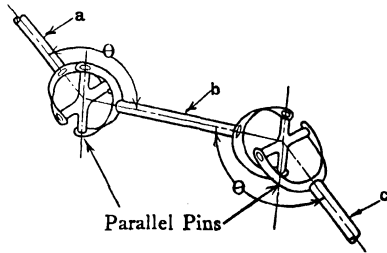
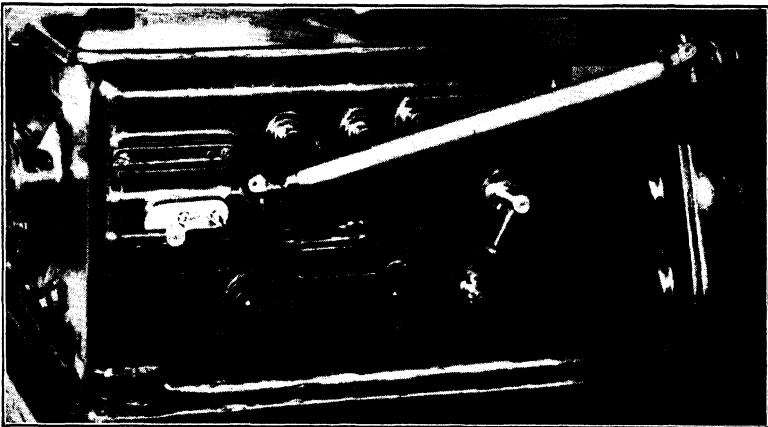


FIG. 10-7. Compensated drive using two universal joints.

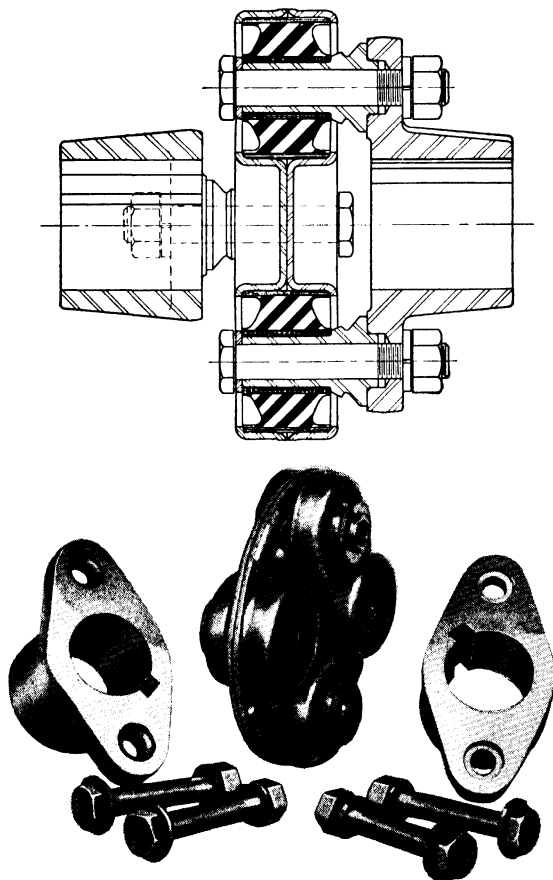
A drive for connecting parallel shafts, compensated in the manner just described, is shown in Fig. 10-7. Since the planes containing the axes of adjacent shafts are here coincident, the driving pins on the connecting shaft yokes are parallel. The com-



Brown & Sharpe Manufacturing Co.

FIG. 10-8. Universal joint application to milling machine.

compensating effect of the two couplings under these particular circumstances will be clear from inspection of the figure. If the connecting shaft is cut by a transverse plane and one-half of the



Morse Chain Co.

FIG. 10-9. Flexible coupling with rubber bushings.

arrangement is rotated about the axis of this shaft, the main shafts may be brought to any desired non-parallel position. When the ends of the connecting shaft are rejoined, compensation is

evidently still maintained, and the relative position of the couplings is that already specified as being necessary.

When the amount of misalignment to be corrected is small, as in the connection between a direct-connected motor and generator mounted on the same base, a "flexible coupling" is used. A large variety of such couplings is on the market, those shown in Figs. 10-9 and 10-10 being typical. The one shown in Fig. 10-9 shows one of these in which flexibility is secured by means of four rubber trunnion blocks set in a pressed steel housing. Opposite pairs of these blocks are bolted to the end hub members of the coupling, one pair to each hub member. They will permit angular misalignment of 3° to 6° , and parallel misalignment up to 2 per cent of the diameter. They need no protection against dirt, oil, water, or weather, and require no lubrication. The coupling shown in Fig. 10-10 consists of two spiders, *A* and *B*, one of which is fastened to each shaft, and a disk of a thin sheet of steel or fiber to which the ends of the spider are fastened. This type has proved very satisfactory for high speeds.

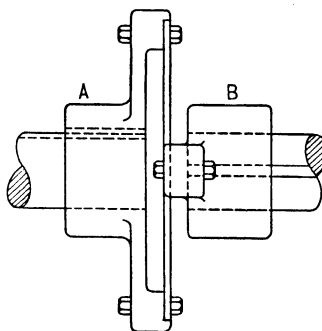


FIG. 10-10. Plate flexible shaft coupling.

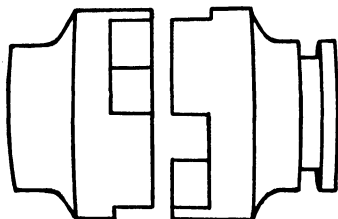


FIG. 10-11. Square jaw coupling.

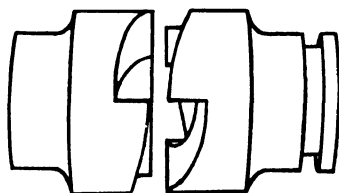


FIG. 10-12. Spiral jaw coupling.

When shafts have to be connected and disconnected frequently, a jaw clutch coupling is employed to unite the shafts. The most frequent forms of this device are the square jaw coupling shown in Fig. 10-11 and the spiral jaw coupling shown in Fig. 10-12. The

square jaw coupling has the advantage of permitting rotation in either direction but cannot be engaged while either shaft is running. It is used most frequently in line shafts. The spiral jaw coupling can be driven in one direction only, but does permit engagement when the speed of the running shaft is moderate. This type of coupling is largely employed on such machines as punch presses.

5. Keys. The purpose of a key is to prevent relative rotation between the shaft and the keyed-on member such as a pulley, coupling, or gear. The key consists of a piece of metal so placed that it is embedded partly in the shaft and partly in the keyed-on member. The most usual forms of keys are shown in Fig. 10-13. The square key consists, as its name indicates, of a piece of cold-rolled or drawn steel square in cross section, placed so that one half is embedded in the shaft and one half in the hub of the keyed-on member. The flat key is similar to the square key, but its thickness is less than its width. Both types are widely used in machine-tool and line-shaft work. They should fit the keyways in both the shaft and hub accurately on the sides but they need not bear tightly top and bottom. Where a very tight fit is desired to effect resistance to vibration which might cause the key to work out of its keyway, a taper key is used. This is similar to the square key, but is made thicker at one end than at the other. The larger end is usually fitted with a gib head to facilitate ready removal of the key when occasion demands. A taper key acts as a wedge and may be driven in too tightly. It requires accurate fitting on all four sides and is likely to throw the keyed-on member off center unless the fit between the shaft and hub is a good one. The Woodruff key permits the use of a taper keyway without requiring great accuracy, because the key may pivot in its seat and align its flat face with that of the tapered keyway. It is used only for light work and requires that the keyed-on member be slipped over it.

If the keyed-on member is to be able to slide along the shaft freely, as in the movable half of a jaw clutch coupling, a feather key is used to prevent relative rotation between them. This is a strip of metal rectangular in cross section and similar to the ordinary square key except that it is thicker than it is wide. It is usually

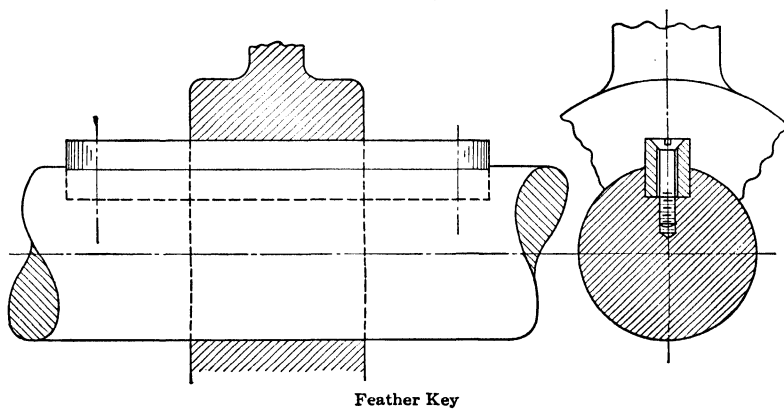
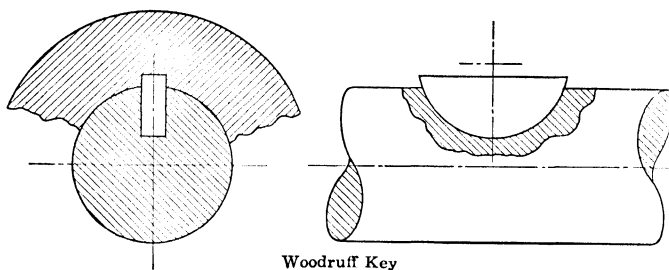
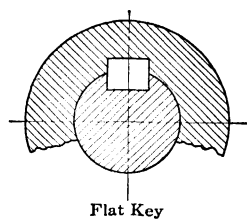
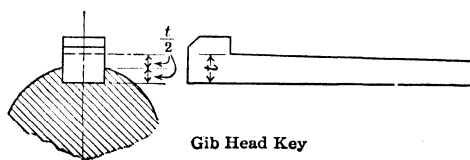


FIG. 10-13. Forms of sunk keys.

TABLE 10-2
Square and Flat Stock Keys, Dimensions and Tolerances

Shaft Diam- eter d In.	Square Stock Keys w In.	Flat Stock Keys $w \times h$ In.	Toler- ances * on Stock Keys In.	Shaft Diam- eter d In.	Square Stock Keys w In.	Flat Stock Keys $w \times h$ In.	Toler- ances * on Stock Keys In.
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8} \times \frac{3}{32}$	-0.0020	$2\frac{3}{8}$	$\frac{5}{8}$	$\frac{5}{8} \times \frac{7}{16}$	-0.0025
$\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{8} \times \frac{3}{32}$	-0.0020	$2\frac{7}{16}$	$\frac{5}{8}$	$\frac{5}{8} \times \frac{7}{16}$	-0.0025
$\frac{5}{8}$	$\frac{3}{16}$	$\frac{3}{16} \times \frac{1}{8}$	-0.0020	$2\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8} \times \frac{7}{16}$	-0.0025
$1\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16} \times \frac{1}{8}$	-0.0020	$2\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8} \times \frac{7}{16}$	-0.0025
$\frac{3}{4}$	$\frac{3}{16}$	$\frac{3}{16} \times \frac{1}{8}$	-0.0020	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{5}{8} \times \frac{7}{16}$	-0.0025
$1\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16} \times \frac{1}{8}$	-0.0020	$2\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{4} \times \frac{1}{2}$	-0.0025
$1\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{4}$	-0.0020	$2\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4} \times \frac{1}{2}$	-0.0025
$1\frac{5}{16}$	$\frac{1}{4}$	$\frac{1}{4} \times \frac{3}{16}$	-0.0020	3	$\frac{3}{4}$	$\frac{3}{4} \times \frac{1}{2}$	-0.0025
1	$\frac{1}{4}$	$\frac{1}{4} \times \frac{3}{16}$	-0.0020	$3\frac{1}{8}$	$\frac{3}{4}$	$\frac{3}{4} \times \frac{1}{2}$	-0.0025
$1\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{4} \times \frac{3}{16}$	-0.0020	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4} \times \frac{1}{2}$	-0.0025
$1\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4} \times \frac{3}{16}$	-0.0020	$3\frac{3}{8}$	$\frac{7}{8}$	$\frac{7}{8} \times \frac{5}{8}$	-0.0030
$1\frac{3}{16}$	$\frac{1}{4}$	$\frac{1}{4} \times \frac{3}{16}$	-0.0020	$3\frac{7}{16}$	$\frac{7}{8}$	$\frac{7}{8} \times \frac{5}{8}$	-0.0030
$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$3\frac{1}{2}$	$\frac{7}{8}$	$\frac{7}{8} \times \frac{5}{8}$	-0.0030
$1\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$3\frac{5}{8}$	$\frac{7}{8}$	$\frac{7}{8} \times \frac{5}{8}$	-0.0030
$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$3\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8} \times \frac{5}{8}$	-0.0030
$1\frac{7}{16}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$3\frac{7}{8}$	1	$1 \times \frac{3}{4}$	-0.0030
$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$3\frac{5}{16}$	1	$1 \times \frac{3}{4}$	-0.0030
$1\frac{9}{16}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	4	1	$1 \times \frac{3}{4}$	-0.0030
$1\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$4\frac{1}{4}$	1	$1 \times \frac{3}{4}$	-0.0030
$1\frac{11}{16}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$4\frac{7}{16}$	1	$1 \times \frac{3}{4}$	-0.0030
$1\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8} \times \frac{1}{4}$	-0.0020	$4\frac{1}{2}$	1	$1 \times \frac{3}{4}$	-0.0030
$1\frac{13}{16}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$4\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{1}{4} \times \frac{7}{8}$	-0.0030
$1\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$4\frac{5}{16}$	$1\frac{1}{4}$	$1\frac{1}{4} \times \frac{7}{8}$	-0.0030
$1\frac{15}{16}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	5	$1\frac{1}{4}$	$1\frac{1}{4} \times \frac{7}{8}$	-0.0030
2	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$5\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{4} \times \frac{7}{8}$	-0.0030
$2\frac{1}{16}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$5\frac{7}{16}$	$1\frac{1}{4}$	$1\frac{1}{4} \times \frac{7}{8}$	-0.0030
$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$5\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4} \times \frac{7}{8}$	-0.0030
$2\frac{3}{16}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$5\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{2} \times 1$	-0.0030
$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{8}$	-0.0025	$5\frac{15}{16}$	$1\frac{1}{2}$	$1\frac{1}{2} \times 1$	-0.0030
$2\frac{5}{16}$	$\frac{5}{8}$	$\frac{5}{8} \times \frac{7}{16}$	-0.0025	6	$1\frac{1}{2}$	$1\frac{1}{2} \times 1$	-0.0030

* These tolerances are *negative* and represent the maximum allowable variation *below* the exact nominal size. For instance, the standard stock square key for a 2-in. shaft has a maximum size of 0.500 by 0.500 in. and a minimum size of 0.4975 by 0.4975 in.

It is understood that these keys are to be cut from cold-finished stock and are to be used without machining.

attached to the shaft by means of countersunk screws although it is sometimes dovetailed to the shaft.

The length of all rectangular keys, except feather keys, is usually made $\frac{1}{2}$ in. greater than the hub of the keyed-on member. The length of feather keys is made equal to the length of hub of the keyed-on member plus the travel of that member.

Table 10-2 gives the tentative American standard for flat and square keys as proposed by the A.S.M.E.

The dimensions of the large ends of taper keys are the same as those of square keys. The usual taper is $\frac{1}{8}$ in. per foot of the length.

The U. S. Navy standard proportions for flat keys are given by the following empirical equations:

$$\text{Width} = \frac{3}{16}D + \frac{1}{8} \text{ in.} \quad (10-10)$$

$$\text{Thickness} = \frac{3}{32}D + \frac{1}{8} \text{ in.} \quad (10-11)$$

where D = diameter in inches of the shaft with which the key is to be used.

Table 10-3, taken from "Empirical Design" by L. D. Hayes, gives satisfactory proportions for feather keys, width and thickness in inches being given in the order stated.

TABLE 10-3

Feather Keys

Size of Shaft In.	Size of Key In.	Size of Shaft In.	Size of Key In.
1	$\frac{1}{4} \times \frac{3}{8}$	$3\frac{1}{16} - 3\frac{1}{2}$	$\frac{7}{8} \times 1$
$1\frac{1}{16} - 1\frac{1}{4}$	$\frac{5}{16} \times \frac{7}{16}$	$3\frac{9}{16} - 4$	$1 \times 1\frac{1}{4}$
$1\frac{5}{16} - 1\frac{1}{2}$	$\frac{3}{8} \times \frac{1}{2}$	$4\frac{1}{16} - 5$	$1\frac{1}{8} \times 1\frac{3}{8}$
$1\frac{9}{16} - 1\frac{3}{4}$	$\frac{7}{16} \times \frac{9}{16}$	$5\frac{1}{16} - 6$	$1\frac{3}{8} \times 1\frac{5}{8}$
$1\frac{13}{16} - 2$	$\frac{1}{2} \times \frac{5}{8}$	$6\frac{1}{16} - 7$	$1\frac{1}{2} \times 1\frac{3}{4}$
$2\frac{1}{16} - 2\frac{1}{2}$	$\frac{5}{8} \times \frac{3}{4}$	$7\frac{1}{16} - 8$	$1\frac{3}{4} \times 2$
$2\frac{9}{16} - 3$	$\frac{3}{4} \times \frac{7}{8}$	$8\frac{1}{16} - 9$	$2 \times 2\frac{1}{2}$

PROBLEMS — CHAPTER X

1. A line shaft rotating at 150 rpm is to transmit 25 hp. Assume that the shaft is subjected to torsion alone and determine its diameter so that the maximum shearing stress shall not exceed 7000 lb per sq in.

Ans. 1.97 in.; say $1\frac{5}{8}$ in.

2. A line shaft rotating at 250 rpm is to transmit 50 hp. The shaft is subjected to pure torsion. It is to be made of hot-rolled steel having an ultimate strength of 56,000 lb per sq in. in tension and 42,000 lb per sq in. in shear. Take a factor of safety of 7 and determine the diameter for the shaft.

Ans. 2.2; say $2\frac{3}{8}$ in.

3. A steel line shaft $2\frac{1}{8}$ in. in diameter rotating at 200 rpm transmits 80 hp at a uniform rate. The ultimate shearing strength of the material is 56,000 lb per sq in. Determine the stress induced in the shaft and the factor of safety.

Ans. 6750 and 8.3.

4. A certain line shaft rotating at 150 rpm is to transmit 50 hp. The torsional deflection is not to exceed 1° in a length equal to twenty diameters of the shaft. The material has an ultimate shearing strength of 56,000 lb per sq in. and a transverse modulus of elasticity of 12,000,000. Determine the diameter for the shaft, the maximum shearing stress induced, and the factor of safety.

Ans. 2.73; say $2\frac{1}{8}$ in., 5600 and 10.0.

5. The shaft of a torsion dynamometer is $\frac{3}{4}$ in. in diameter and 12 in. long. When running at 1200 rpm under a certain load it is found to have a torsional deflection of 11.7° . The transverse modulus of elasticity of the shaft material is 12,000,000. What horsepower is being transmitted and what shearing stress is induced in the shaft?

Ans. 120 and 76,000 lb per sq in.

6. An electric generator rotating at 200 rpm receives 350 hp from the driving engine. The armature is 24 in. long and is placed midway between the bearings 4 ft apart on centers. Owing to the combined weight of the armature and magnetic pull of the pole pieces the shaft is subjected to a force of 18,000 lb acting at right angles to the shaft. Consider this force uniformly distributed over the length of the armature. The shaft is to be made of steel having an ultimate tensile strength of 64,000 and an ultimate shearing strength of 56,000 lb per sq in. Use a real factor of safety of 4 and determine the diameter for the shaft.

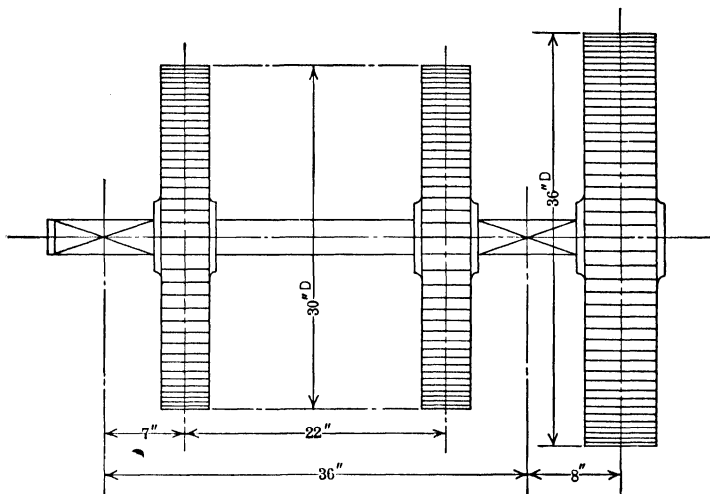
Ans. 6.10 in.; say 6 in.

7. The accompanying figure shows the head shaft of a vertical chain conveyor. The total load on the two sprockets is 4000 lb; 6 hp is required to operate the conveyor at a bucket speed of 100 ft a minute. Assume the gear reaction to act vertically downward and the shaft to be subjected to a complete reversal of stress in bending. Take the ultimate strength of the shaft material in tension and shear as 60,000 and 48,000 lb per sq in. respectively. Using a real factor of safety of 3, determine the diameter for the shaft.

Ans. 2.86 in.; say $2\frac{7}{8}$ in.

8. A line shaft is driven by a vertical belt from a motor placed directly below the center of the shaft. The belt runs over a pulley 4 ft in diameter

and weighing 500 lb. The total tension on the tight side of the belt is 2570 lb and that on the slack side is 1000 lb. The distance from the center line of the pulley, which is overhung, to the center line of the supporting bearing is 15 in. Take the maximum allowable stress in tension as 6000 lb per sq in. and in shear as 4500 lb per sq in., and determine the diameter of shaft necessary to transmit 150 hp when rotating at 250 rpm.



PROB. 7

9. The crankpin of an overhung-crank engine is acted upon by a force of 6000 lb acting at right angles to the crank. The stroke of the engine is 12 in., and the axial distance from the center line of the crankpin bearing to the center line of the supporting bearing is 10 in. The shaft is to be made of mild steel having an ultimate strength of 60,000 lb per sq in. in tension, and 48,000 lb per sq in. in shear. Use a real factor of safety of 6 and determine the shaft diameter.

10. A marine turbine developing 15,000 hp turns the shaft at 300 rpm. The propeller attached to this shaft develops a thrust of 150,000 lb. A hollow steel shaft having an outside diameter of 14 in. is to be used.

(a) Determine the inside diameter of the shaft if the maximum shearing stress based on the torsion alone is not to exceed 7500 lb per sq. in.

Ans. 9.575 in. diameter.

(b) What percentage saving in weight over a solid shaft is effected?

Ans. 37 per cent.

11. A shaft is to sustain a torque of 2,500,000 lb-in. Two designs are under consideration: (1) a solid shaft having a diameter of 13 in., and (2) a hollow shaft having an outside diameter of 14 in. and an inside diameter of $9\frac{1}{2}$ in.

(a) How do the two shafts compare on a basis of strength?

(b) How do they compare on a basis of cost, assuming the hollow shaft to cost 20 per cent more per pound than the solid one?

12. Two pieces of line shafting $1\frac{1}{8}$ in. in diameter are joined together by a flange coupling.

(a) How many bolts should the coupling have?

Ans. 4.

(b) Assume that these are placed on a bolt circle 6 in. in diameter and determine the size for the bolts, using an apparent factor of safety of 6. Shearing strength of bolt material is 54,000 lb per sq in. *Ans.* 0.39 in.; say $\frac{1}{2}$ in.

CHAPTER XI

SCREWS AND SCREW FASTENINGS

1. Purpose and Classification. Screws have two general purposes:

- (a) To act as fastenings to secure one member to another.
- (b) To transmit power.

The first of these requires a thread that is strong and that has a low efficiency. A low efficiency is necessary in order that the screw may not be easily loosened by vibration or other causes and thus allow the members joined by it to separate. The second requires the highest obtainable efficiency in order to render as small as possible the power lost in transmission.

Screw threads are classified according to their shape and the number of separate threads wound on the base cylinder. Thus we have angular threads and square threads, and each of these may be single or multiple. The National Coarse, the V, and the Acme threads are all examples of the angular type. Multiple-thread screws are similar to single-thread screws except that two or more separate threads are wound parallel to each other on the base cylinder.

2. Usual Forms of Screw Threads. Figure 11-1 illustrates the threads commonly used. The **National Coarse thread** is shown in Fig. 11-1(a). The angle included between the faces is 60° , and the top and bottom are flat, measuring one-eighth of the pitch in width. This flattening accomplishes a threefold purpose:

- (a) It gives a thread that is less easily damaged by rough handling than one having sharp edges.
- (b) It avoids sharp angles at the root, which are often starting places for cracks.
- (c) It permits a larger root diameter for a given outside diameter and hence increases the strength of the screw.

The **V-thread**, illustrated in Fig. 11-1 (b), is similar to the National Coarse except that the root and crest are sharp. It possesses no advantages over the National Coarse, and is inferior to it in strength, ruggedness, and ease of formation with a tap or die.

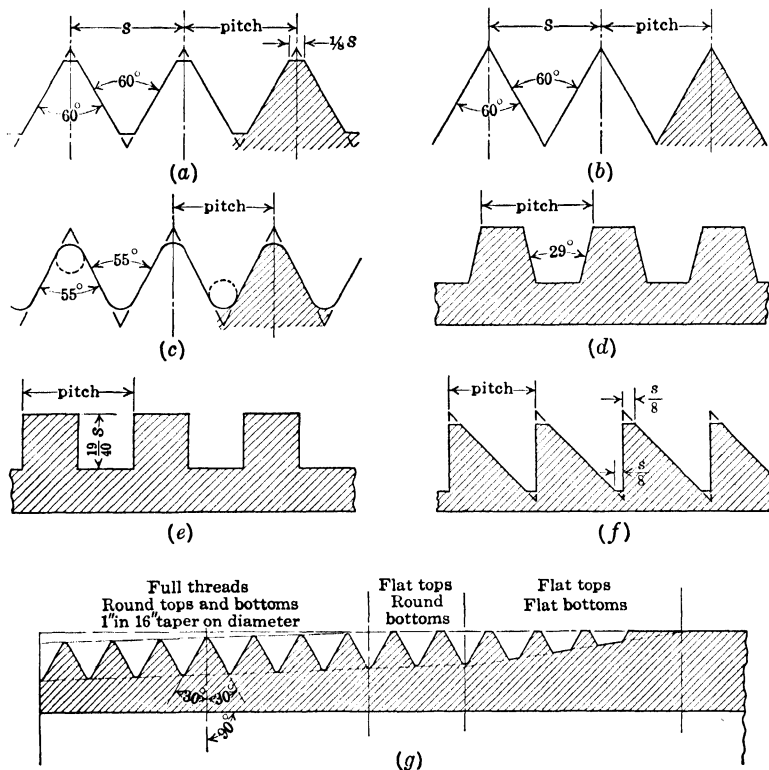


FIG. 11-1. Forms of screw threads.

The **Whitworth thread**, illustrated in Fig. 11-1 (c), has an angle of 55° between the faces. Because the threads are rounded at the top and bottom it is slightly stronger than the National Coarse, but the threading tools are more difficult and expensive to make. It is the standard thread of England.

The **Acme thread**, Fig. 11-1 (*d*), has an angle of 29° between the faces. It is easily cut and, because of the steep slope of the faces, has a high efficiency. It is superior to the square thread because wear on the engaging nut can be taken up by using a split nut. It is extensively used for light power transmission, as in the lead screws of lathes.

The **square thread**, Fig. 11-1 (*e*), has a higher efficiency than any of the angular threads. It is used for the transmission of powers greater than can be handled by the Acme thread. Wear cannot be compensated for by splitting the nut. The nuts are made of bronze or some other material softer than steel, so that the wear will come on the more easily replaced member and also because such materials run better with a steel screw than does a steel nut.

The **buttress thread**, Fig. 11-1 (*f*), is used to take loads in one direction only. It has the strength of the V-thread and the efficiency of the square thread. It is used on the breech blocks of guns and on certain types of screw jacks.

The **Briggs pipe thread**, Fig. 11-1 (*g*), is the standard adopted for pipe fittings by the manufacturers of the United States. The taper and the arrangement of thread at right angles to the axis of the pipe are features which make the joints tight against fluid pressure.

As shown in Fig. 11-1 the pitch of all threads is the axial distance between corresponding points on adjacent threads. The lead is the axial advance made by a thread in one complete turn around the base cylinder. In the single-thread screw the two are identical, but in the multiple-thread screw they are different. Thus for a triple-thread screw the lead is three times the pitch.

3. Efficiency of Screw Threads. The efficiency of a screw is the ratio of the useful work done to the work supplied to the screw. If the interval of time over which these two quantities are measured is taken as that required for one turn of the screw, the efficiency may be expressed

$$\begin{aligned}\text{Efficiency} &= \frac{\text{useful force} \times \text{distance it moves}}{\text{operating force} \times \text{distance it moves}} \\ &= \frac{W \times p}{P \times 2\pi r}\end{aligned}\quad (11-1)$$

where W = axial load on screw

p = distance the load is moved in one turn of the screw
= lead of the screw

P = operating force

r = distance from the axis of the screw to the line of action of P .

It is usually desirable to know the relation between the operating moment and the load moved. Let

d_m = mean diameter of screw thread

$$= \frac{\text{root diameter} + \text{outside diameter}}{2}$$

r_m = mean radius of thread = $\frac{d_m}{2}$

P' = a force acting at the mean radius of the thread which is great enough to overcome thread friction and the component of W along the thread

r_c = frictional radius of the collar

f_c = coefficient of friction between the collar and support

f = coefficient of friction between threads of the nut and screw.

The external torque acting on a screw is used in three ways as shown in Fig. 11-2.

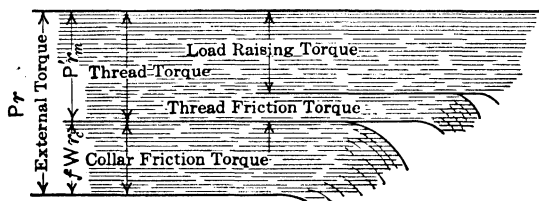


FIG. 11-2

- (a) Overcoming torque set up by the friction of the collar against the support.
- (b) Overcoming friction between the threads of the screw and nut.
- (c) Raising the load.

Items (b) and (c) may be combined under the term "thread torque." We then have

External torque = collar friction torque + thread torque

$$Pr = f_c W r_c + P' r_m \quad (11-2)$$

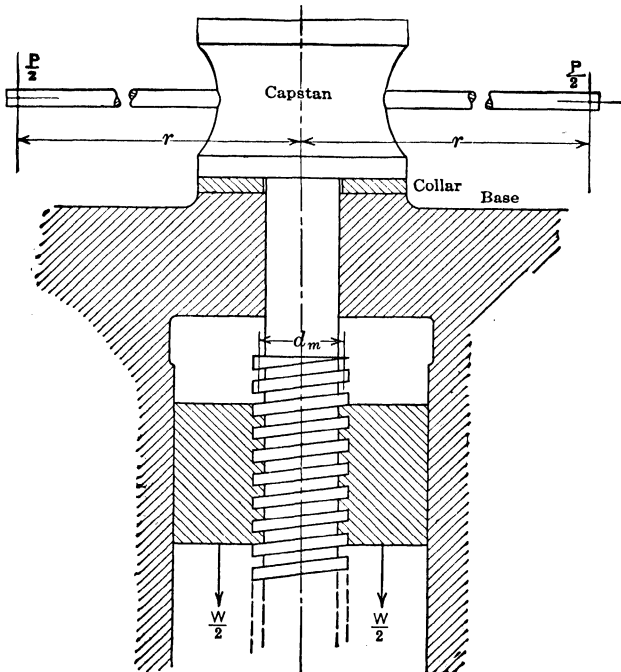


FIG. 11-3. Capstan and screw.

For the capstan and screw combination shown in Fig. 11-3 the torque acting on the section at the root of the screw thread is less than the external torque by an amount equal to the collar friction torque. This is not always so, for sometimes the screw lies between the collar and the source of energy; in such instances the entire external torque acts on the root of the screw.

The torque, Pr , that must be applied to raise the load, W , is given by the expression

$$Pr = f_c W r_c + W \left(\frac{p + f \pi d_m}{\pi d_m - f p} \right) r_m \quad (11-3)$$

No self-locking screw has an efficiency, for the screw alone, in excess of 50 per cent. When the component of the load along the thread just equals the friction, and motion down the plane impends, these are opposed and balance each other. When, however, the impending motion is up the plane, the force of friction, which always opposes motion, is reversed in direction and acts **with** instead of **against** the component of the load along the plane. The external force which produces motion must then have a component along the plane equal to $F + W \sin \alpha$, or since $F = W \sin \alpha$

$$P' \cos \alpha = 2W \sin \alpha$$

where α = helix angle of thread. The only useful work is that done in overcoming the component of W along the plane, that is, $W \sin \alpha$. Taking the distance moved through as l , we have

$$\text{Efficiency} = \frac{\text{useful work done}}{\text{work supplied}} = \frac{W \sin \alpha \times l}{2W \sin \alpha \times l} = 50 \text{ per cent}$$

Consideration of collar friction does not change the efficiency of the screw and collar combination which is merely self-sustaining, although it does increase the thread angle at which this condition obtains.

Example. The capstan and square screw-thread combination shown in Fig. 11-3 raises and lowers a gate weighing 2 tons against a maximum frictional resistance between the gate and its guides of 1000 lb induced by water pressure against the gate when in its lowest position. Determine the maximum force to be exerted at the ends of the capstan bar for raising the gate. Check the combination for self-locking quality. Also determine the number of threads required in the nut for an allowable bearing pressure in the threads of 1000 lb per sq in. of projected area. The following data are available.

Length of leverage, $r = 4$ ft.

Material of screw, steel; of nut, brass; of washer, steel; of seat, cast iron.

Diameter of washer (collar): inside, approximately 2 in.; outside, 8 in.

Screw: single, square thread; two threads per inch; shank diameter, 2 in.

Assumed coefficients of friction: at washer, 0.15; at thread, 0.13.

Solution.

$$d_m = \frac{2.5^* + 2}{2} = 2.25 \text{ in.}$$

$$r_c = \frac{2}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) = \frac{2}{3} \left(\frac{4^3 - 1^3}{4^2 - 1^2} \right) = 2.8 \text{ in.}$$

For raising, equation (11-3) applies.

Total equivalent weight: $2 \times 2000 + 1000 = 5000$ lb

$$\begin{aligned} Pr &= f_c W r_c + W r_m \left[\frac{p + f \pi d_m}{\pi d_m - f p} \right] \\ &= 0.15 \times 5000 \times 2.8 + \\ &\quad 5000 \times \frac{2.25}{2} \left[\frac{0.5 + 0.13 \times \pi \times 2.25}{\pi \times 2.25 - 0.13 \times 0.5} \right] \\ &= 2100 + 1015 = 3115 \text{ lb-in.} \end{aligned}$$

$$P = \frac{3115}{48} = 64.8 \text{ lb, or } 32.4 \text{ lb at each end of the bar}$$

Efficiency, equation (11-1):

$$e = \frac{W p}{2 \pi P r} = \frac{5000 \times 0.5}{2 \pi \times 3115} = 12.8 \text{ per cent}$$

Since the efficiency is under 50 per cent the combination is self-locking and the gate will remain in position unattended.

* The actual outside diameter of the thread is 2.475 in.

Number of threads required in the nut: N

$$\frac{\pi(D_1^2 - D_2^2)}{4} \times N \times 1000 = \frac{\pi(2.5^2 - 2^2)}{4} \times 1000 \times N = 5000$$

$$N = 2.825, \text{ say } 3.$$

It is evident from the foregoing that the allowable pressure is figured upon the projected area of the thread in the nut and not upon the actual area in contact. This standard has been widely adopted because of the relative ease secured in handling values reached experimentally.

Direct tensile stress in screw:

$$\frac{\text{Load}}{\text{Root area}} = \frac{5000}{3.1416} = 1590 \text{ lb per sq in.}$$

Direct shearing stress in screw due to raising the load:

$$T = \frac{s_s I_p}{c} = s_s \frac{\pi d^3}{16}$$

hence,

$$s_s = \frac{16T}{\pi d^3} = \frac{16 \times 1015}{\pi \times 2^3} = 646 \text{ lb per sq in.}$$

If the washer in Fig. 11-3 is replaced by a roller or ball bearing, thus reducing the collar friction torque to a negligible quantity, the required raising torque becomes 1015 lb-in. and the efficiency is raised to approximately 39 per cent.

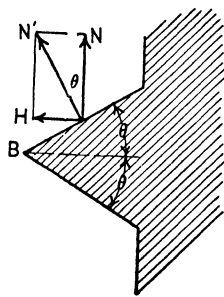


FIG. 11-4

Figure 11-4 shows a section of a V-thread taken at right angles to its elements. N is the normal force that the load W would induce in a square thread having the same helix angle. Because of the sloping sides of the angular thread, a wedging action takes place and the normal reaction of the thread against the nut becomes N' . It

will be seen that $N' = N/\cos \theta$, where θ is half the angle between the thread faces. For the National Coarse thread $N' = N/\cos 30^\circ = 1.15N$. Since the thread friction varies directly as the normal pressure, the friction of the angular thread is greater than that of the square thread. The horizontal component H is the radial force the thread exerts upon the nut, which is, therefore, in much the same condition as a boiler subjected to internal pressure. Under certain conditions this component may cause the nut to stretch and so fail.

The expression for the total external torque for raising the load becomes, for angular threads,

$$Pr = f_c W r_c + W r_m \left[\frac{p + (f \pi d_m \div \cos \theta)}{\pi d_m - (f p \div \cos \theta)} \right] \quad (11-4)$$

As for the square thread, the efficiency for the angular thread is

$$\text{Efficiency} = \frac{W p}{Pr \times 2\pi}$$

The foregoing analysis of square and angular threads has been made for the screw and capstan arrangement shown in Fig. 11-3, but it is equally true for the ordinary nut and bolt. In the latter the nut takes the place of the collar and is rotated while the screw remains stationary. The frictional radius of the nut is used for that of the collar. Otherwise the conditions are identical in the two cases.

4. Screw Fastenings. The most important screw fastenings are:

- (1) Through bolts.
- (2) Studs.
- (3) Tap bolts and cap screws.
- (4) Machine screws.
- (5) Set screws.

Through bolts. Figure 11-5 shows a bolt with its nut in position. It consists of a cylindrical bar with an integral head on one end and threads at the other for engagement with a nut. Bolt shanks may be rough or machined. If rough, a slight clearance is allowed between the hole and the bolt as indicated; if machined,

the bolt bears snugly in the hole and is often called a fit bolt. The through bolt is the best form of fastening when conditions of accessibility permit its use, since no threading is required in the parts to be held together.

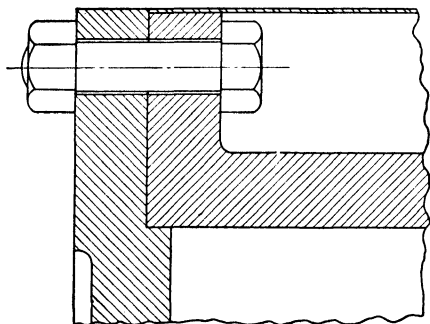


FIG. 11-5. Through bolt.

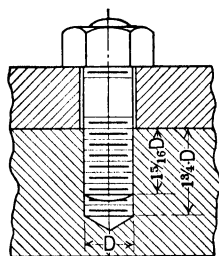


FIG. 11-6. Stud bolt.

Studs or stud bolts. A stud is a cylindrical bar threaded at both ends. It is used when only one side of the members to be fastened is accessible. A hole is drilled and tapped in one part and one end of the stud is permanently screwed into the hole with a tight fit (Fig. 11-6). The other part is drilled or punched, allowing clearance for the stud. Upon mating the parts are fastened together by attaching nuts to the free end of the stud. Parts held together by studs and nuts may be disassembled simply by removing the nuts and moving the attached member free from the stud. Clearance between the thread and the hole facilitates the removal without injury to the free end of the thread on the stud. When using studs "blind holes," i.e., those which do not go completely through the piece (Fig. 11-6), are formed if the conditions permit. Care should be exercised, especially if the parts enclose corrosive agents or are to be rendered pressure-tight. Blind holes are drilled, where possible, to a depth of $1\frac{3}{4}$ the diameter of the screw fastening for cast iron and $1\frac{1}{4}d$ for steel or wrought iron. The screw thread is entered $1\frac{5}{8}d$ and d respectively. Next to the through bolts the stud should be used when practicable.

Tap bolts and cap screws. These fastenings (Fig. 11-7, *a* and *b*) serve the same purpose as studs. An integral head replaces the nut used with the stud. The difference between tap bolts and cap screws lies in the size and form of the head. The head of the tap bolt is the same distance across flats as a nut for the same nominal dimension of shank; the head of the corresponding cap screw is smaller and rounded on top. Because of the small seating surface of its head, the cap screw is inferior in fastening qualities and is used only in cheap grades of machinery. Quantity production of a standard article resulting in a low price to the trade and the ornamental appearance are two main reasons for the existence of the cap screw. Both forms of fastenings should be used sparingly, especially with cast iron, if repeated removal is necessary, since cast iron threads disintegrate quickly with wear.

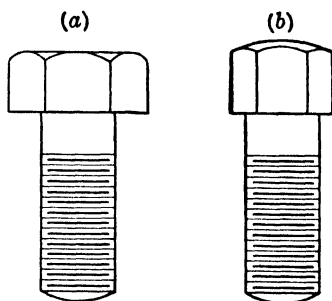


FIG. 11-7. Tap bolt (*a*) and cap screw (*b*).

Machine screws. The term “machine screws” applies to numerous forms of small screw fastenings with heads slotted so that a screwdriver instead of a wrench may be used to position them (Fig. 11-8). Their use lies in the field of small machine parts assembly where the forces are low, such as typewriters, jigs, carburetors, etc. The number of threads per inch for a given diameter has not been standardized by the manufacturers, but the American Society of Mechanical Engineers has recommended a system of standards which may lead to an agreement.

Set screws. The function of set screws is to prevent relative movement of machine parts such as collars and shafts or the hub of a pulley and the shaft. The various forms are illustrated in Fig. 11-9, although the points and heads may be interchanged to meet the requirements of the design. The square heads shown in (*a*) and (*b*) are objectionable in that they may catch in the clothing of a workman and cause serious damage; they afford, however, a

convenient-shaped head for a wrench in setting the screw up tightly. The point of (a) is cup-shaped to enable the screw point

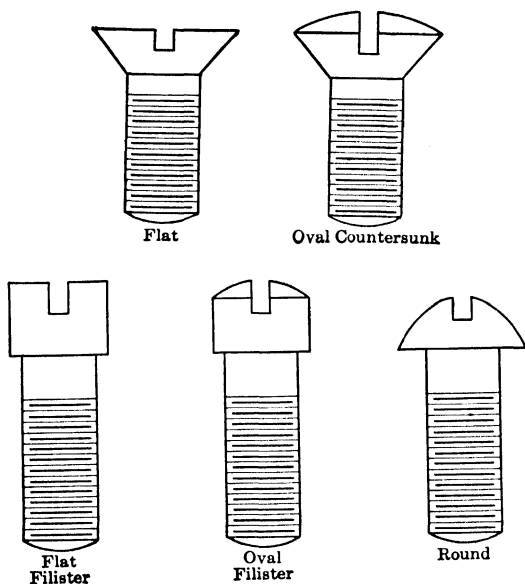


FIG. 11-8. Machine screws.

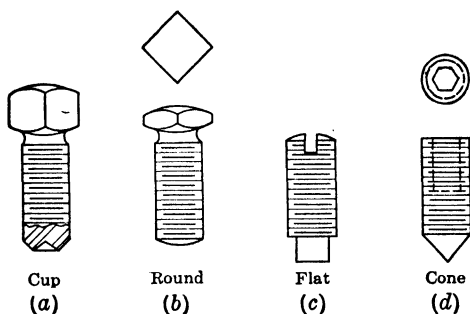


FIG. 11-9. Set screws.

to grip the shaft firmly. The subsequent ridge formed on the shaft may interfere with disassembly, so, for this reason, the points of (b), (c), or (d) are often used. The round point of (b) scores the

shaft slightly, but it proves to be one of the best fastening points, since it will not work loose readily under repeated load action. When the point shown in (d) is used, the shaft is drill-marked to accommodate the point of the set screw. Set screws are used only where the force transmitted is low. Mr. B. H. D. Pinckney¹ produces experimental data showing the safe holding power of cup or flat-pointed set screws in terms of the tangential force at the surface of the shaft. Thus, for a pulley transmitting 5 hp uniformly at 300 rpm on a 2-in. shaft, the tangential force is 1050 lb, and one $\frac{3}{4}$ -in. or two $\frac{1}{2}$ -in. set screws are required.

TABLE 11-1

Diameter of screw, inches. .	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$
Holding force, pounds.	100	168	256	366	500	658	840	1280	1830	2500	3288

5. Effect of Combined Initial and External Loads. When a mechanic sets up a nut he induces a tensile stress in the bolt which is a maximum at the root of the thread. Since this load is the first to come on the bolt it will be called the initial load. When the machine is placed in service another load comes on the bolt as a result of the forces transmitted by the connected members. This load will be known as the external load. The total load carried by the bolt lies between the initial and the initial plus external loads and depends for its value upon the ratio of the yields of the bolt and connected members. The limiting conditions are:

(a) The bolt is much more yielding than the connected members and the resultant load is either the initial or the external, whichever is greater.

(b) The connected members are much more yielding than the bolts, and the resultant load is the sum of the initial and external loads.

Although any attempt to calculate the resultant load is useless

¹ *American Machinist*, Oct. 15, 1914.

because of the difficulty in approximating the relative yields of the bolts and connected members, and also because the initial stress is uncertain, the limiting conditions given above should be kept in mind when fixing the allowable stress for bolts. Thus long bolts connecting members which make metal-to-metal contact approximate the first condition; short bolts connecting members which bear against elastic packing approximate the second condition, and a lower working stress should be assigned them.

The working loads and allowable stresses corresponding to them given in Table 11-2 will be found satisfactory for bolts in joints which have to contain fluid pressure and which may have an elastic gasket between the connected members.

TABLE 11-2
Stresses and Loads for Steel Bolts in Steam-tight Joints

Size, Diameter, in Inches	Allowable Stress Due to External Load, Pounds per Square Inch	External or Working Load, Pounds
$\frac{7}{16}$	400	37
$\frac{1}{2}$	700	88
$\frac{5}{8}$	1400	283
$\frac{3}{4}$	2000	604
$\frac{7}{8}$	2500	1,040
1	2750	1,510
$1\frac{1}{8}$	3000	2,080
$1\frac{1}{4}$	3200	2,860
$1\frac{3}{8}$	3400	3,600
$1\frac{1}{2}$	3600	4,540
$1\frac{5}{8}$	3800	5,800
$1\frac{3}{4}$	4000	7,000
$1\frac{7}{8}$	4300	8,650
2	4600	10,600
$2\frac{1}{8}$	4800	12,700
$2\frac{1}{4}$	5000	15,200

Since bolts used in joints which do not have to resist fluid pressure are, in general, less highly stressed than those just discussed, bolts smaller than $\frac{5}{8}$ in. may be relied upon to carry appre-

cial external loads. The allowable stress, based upon the external load, may also be increased as the size of the bolt diameter increases because there is less danger of a careless workman over-stressing a large bolt than a small one.

TABLE 11-3

Allowable Stresses and Loads for Mild Steel or Iron Bolts
(Ordinary Joints)

Nominal Diameter of Stud or Bolt, Inches	Stress Due to Working Load, Pounds per Square Inch	Working Load, Pounds
$\frac{1}{2}$	2000	250
$\frac{5}{8}$	2500	500
$\frac{3}{4}$	3000	900
$\frac{7}{8}$	3400	1,450
1	3900	2,150
$1\frac{1}{8}$	4300	3,000
$1\frac{1}{4}$	4700	4,200
$1\frac{3}{8}$	5100	5,400
$1\frac{1}{2}$	5500	7,100
$1\frac{5}{8}$	5800	8,500
$1\frac{3}{4}$	6300	11,000
$1\frac{7}{8}$	6600	13,100
2	7000	16,100
$2\frac{1}{4}$	7000	20,400

In addition to possible failure by breaking at the root, a screw thread may fail in the threads. Such failure is termed "stripping" of the threads. The exact nature of the failure is obscure, but bending, crushing, and shearing actions are all present. Experience has shown that if the thickness of the nut is made equal to the nominal diameter of the bolt, stripping of the thread seldom occurs; poor fitting may account for occasional failures. Where saving of space is greatly desired the height of the nut may be reduced to three-quarters of the nominal bolt diameter. Thinner nuts should not be used if they are expected to develop the full strength of the screw.

PROBLEMS — CHAPTER XI

NOTE: Use the depth of thread equal to half the pitch in the following problems on square threads.

1. A 6-in. gate valve has a gate of an area equal to 28.3 sq in. The stem is steel and is of the external-screw type, single-threaded with four square threads per inch. The gate is subjected to a maximum unbalanced pressure of 250 lb per sq in. The coefficient of friction between the gate and its seat is 0.2; between the wheel and its seat 0.12; and between the nut and the stem threads 0.15. The friction radius of the wheel is $1\frac{1}{8}$ in.

(a) Assume a factor of safety of 12 and an ultimate tensile strength of 72,000 lb per sq in. and determine the diameter for the root of the valve stem on the basis of the friction load alone.

(b) Determine the pull that must be exerted at the rim of a 16-in. hand-wheel to open the valve.

(c) Determine the efficiency of the screw and handwheel.

Ans. (a) $\frac{9}{16}$ in.; (b) 40.5 lb; (c) 17.4 per cent.

2. A square, single-threaded jack screw has two threads per inch. It is to lift 15 tons. The friction radius of the collar is 1 in. The coefficient of friction between the threads of the screw and base is 0.15 and that between the screw and collar is 0.13.

(a) Determine the diameter of the root of the screw for an allowable stress in compression of 6000 lb per sq in.

(b) Determine the pull required at the end of a 5-ft bar to raise the load.

(c) Determine the twisting moment exerted at the root of the thread.

(d) Determine the efficiency of the screw and collar.

Ans. (a) $2\frac{1}{2}$ in.; (b) 210 lb; (c) 8700 lb-in.; (d) 19 per cent.

✦ 3. A screw for the transmission of power is to propel the engaging nut at 50 ft a minute against a resistance of 5000 lb. The quadruple, square-threaded screw is to have one thread per inch. The root diameter is $1\frac{5}{8}$ in. The collar lying between the screw and the source of power has an inside diameter of $2\frac{1}{4}$ in. and an outside diameter of 5 in. Assume the coefficient of friction between the nut and threads to be 0.1, between the collar and the support to be 0.01, and determine: (a) the efficiency of the screw and collar; (b) the horsepower required to drive the screw.

Ans. (a) 78.8 per cent; (b) 9.68.

4. A screw having a square double thread is to raise a load of 20,000 lb at a velocity of 3 ft per min. The root diameter of the screw is 2 in. and the pitch is 1 in. The friction radius of the collar is $2\frac{1}{2}$ in. Taking the coefficient of friction in the collar as 0.1 and that in the threads as 0.12, determine: (a) the horsepower required to drive the screw; (b) the efficiency of the screw and collar.

5. A load of 15,000 lb is to be lifted by a double-threaded screw having two square threads per inch. The outside diameter of the screw is $1\frac{3}{4}$ in. and the frictional diameter of the collar is 4 in. The coefficient of friction in the

threads is 0.1 and that of the collar is 0.15. Determine the torque acting on the root of the threads and the efficiency of the combined screw and collar.

6. A single-threaded jack screw has a square thread with a pitch of 1 in. The root diameter is $2\frac{1}{2}$ in. The coefficient of friction in the threads is 0.15, the frictional radius of the collar is 1 in., and the coefficient of friction between the collar and seat is 0.13. The jack is to raise a load of 25,000 lb. Determine the pull required at the end of a bar 6 ft long, and the efficiency of the jack.

Ans. 180 lb; 30.7 per cent.

7. A square-threaded power screw is to transmit an axial force of 6000 lb at a speed of 50 ft per min. The screw has a root diameter of 2 in., and is fitted with quadruple threads having a pitch of $\frac{3}{4}$ in. Determine the power required to drive the screw, direct tensile and shearing stresses at the root of the threads, and the efficiency of the screw if the frictional radius of the collar is 3 in. and the coefficient of friction is 0.12 for the threads and 0.08 for the collar.

Ans. 16.7; 1910 and 2500 lb per sq in.; 53.6 per cent.

8. A 1-in. shaft is fastened to its coupling by means of two set screws placed at 120° to each other. Assuming a maximum allowable shearing stress of 6000 lb per sq in. in the shaft, determine the size of set screws required to utilize the full capacity of the shaft.

Ans. $\frac{3}{4}$ in.

9. A National Coarse thread bolt of 1-in. diameter has a root diameter of 0.84 in. and eight threads per inch. Assuming a coefficient of friction of 0.12 in the threads and 0.15 between the nut and its seat, and a friction radius of 0.8 in. for the nut, determine the pull required at the end of a 20-in. wrench to produce an axial load of 5000 lb in the bolt. What is the efficiency of the fastening under these conditions?

10. The outer half of a crankpin bearing carrying a load of 2000 lb is held in place by two bolts. Assume that each bolt carries half the load. Determine the size for the bolts and the stress on the root section due to the external load.

Ans. $\frac{7}{8}$ in.; 2380 lb per sq in.

11. (a) A 10-in. standard pipe flange has twelve bolts. Assume that each bolt carries its share of the total load and determine the size for the bolts to withstand a steam pressure of 125 lb per sq in.

(b) What is the probable setting-up and total stress in these bolts?

Ans. (a) $\frac{7}{8}$ in. (from Table 11-2); (b) 36,200 lb per sq in.

12. A pulley 5 in. in diameter is secured to a shaft 2 in. in diameter by means of a cup-pointed set screw. What size of set screw should be used if the pulley is to transmit 1 hp when rotating at 500 rpm?

Ans. $\frac{5}{16}$ in.

13. The pulley of a motor rotating at 1150 rpm is secured to a $1\frac{3}{8}$ -in. shaft by two $\frac{1}{2}$ -in. flat-pointed set screws. What horsepower can be transmitted?

Ans. 12.55.

CHAPTER XII

GEARS AND GEAR TRAINS

1. Function of Toothed Gearing. When two shafts are to be connected this may be accomplished by the use of belts, friction rolls, chains, or gears. Where the angular velocity ratio is not necessarily constant, or the drive positive, the first two may give entire satisfaction. Chain drive insures positive driving but gives an angular velocity ratio that is more or less variable. Properly

designed gears insure both positive drive and constant angular velocity ratio.

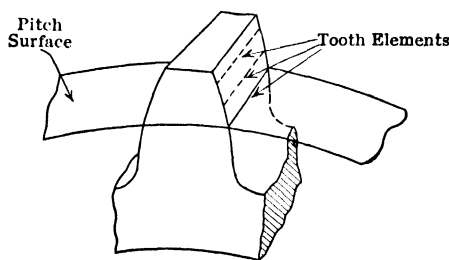


FIG. 12-1

2. Pitch Surfaces. The pitch surface of a gear wheel may be defined as an imaginary surface on which the tooth construction is based. For any gear the form and dimen-

sions of the pitch surface must be known before the teeth can be properly designed. Figure 12-1 shows the cylindrical pitch surface for a spur gear.

Tooth element. A gear tooth may be regarded as a surface swept up by a line moving through space. The line is not always straight or even of constant shape. This profile generator, in any one of its consecutive positions, is known as a tooth element. Tooth elements always connect corresponding points on tooth sections taken perpendicular to the pitch surfaces. In Fig. 12-1 are shown tooth elements for a spur gear, which are straight lines parallel to one another.

Pitch surfaces of mating gears may have either (a) pure rolling contact or (b) sliding contact. This is important, since it determines whether or not the teeth have relative sliding along the

tooth elements. When this kind of sliding action occurs it places certain limitations on the form of the elements.

When the pitch surfaces of gear wheels have line contact, the gear teeth likewise have line contact. When the pitch surfaces have point contact so also have the gear teeth, the only common exception being found in some worm gears.

3. Gear Classification. Gears may be classified according to the relative position of the axes of revolution. The axes may be (a) parallel, (b) intersecting, (c) neither parallel nor intersecting. We shall first make a brief survey of the common forms, and later discuss each in more detail.

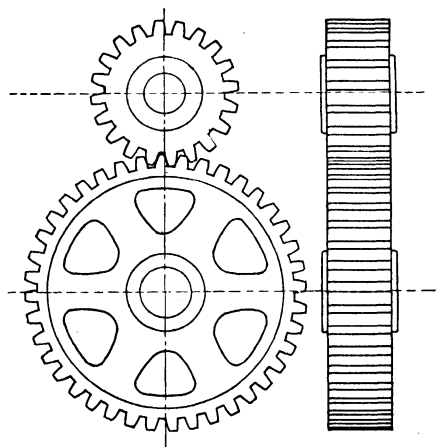


FIG. 12-2. Plain spur gears.

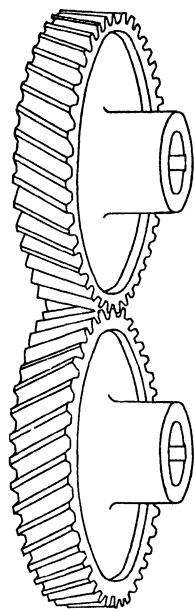


FIG. 12-3. Twisted-tooth spur gears.

(a) **Gears for connecting parallel shafts.** Here we may employ the **common spur gears** as shown in Fig. 12-2 or the **twisted-tooth spur gears** of Fig. 12-3. In both, the pitch surfaces are cylindrical

with pure rolling contact as illustrated in Fig. 12-4. Tooth elements in the common spur gears are straight lines parallel with the

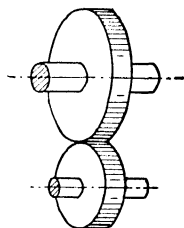


FIG. 12-4. Spur gear pitch surfaces.

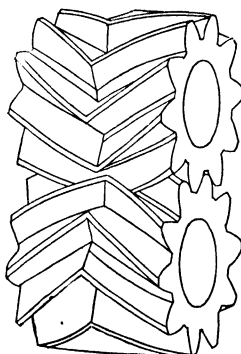


FIG. 12-5. Herringbone gears.

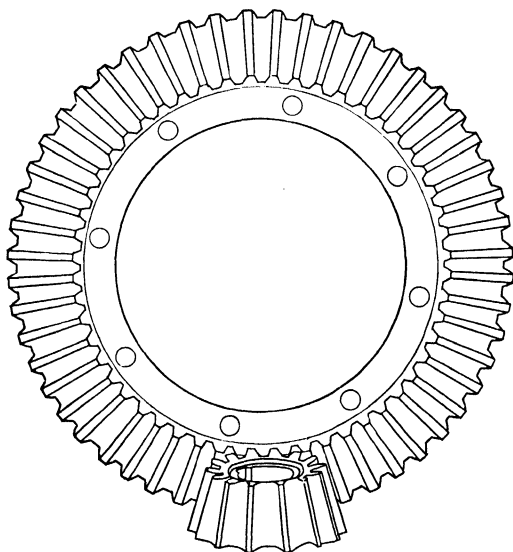


FIG. 12-6. Plain bevel gears.

axis of the gear, whereas in the twisted-tooth spur gears these elements are helices. The twisted-tooth gear generally operates

more quietly than the other type, the difference in this respect being particularly noticeable at high speed. The main disadvantage of the twisted-tooth type lies in the end thrust produced when the gear is transmitting power.

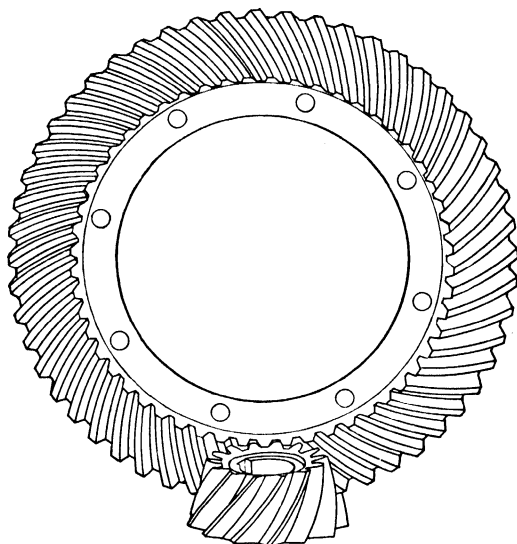


FIG. 12-7. Spiral bevel gears.

In the **herringbone gear** of Fig. 12-5 the end thrust set up by one side is balanced by an equal and opposite thrust due to the action on the other side. This gear can be regarded as composed of two twisted gears of similar dimensions, one having a right-handed and the other a left-handed helix.

(b) **Gears for intersecting shafts.** Here the **plain bevel gear**, as shown in Fig. 12-6, or the **spiral bevel** shown in Fig. 12-7 is employed. In both the pitch surfaces are cones having a common apex as shown in Fig. 12-8. In the plain bevel the tooth elements are straight lines, whereas in the spiral bevel they are conical helices.

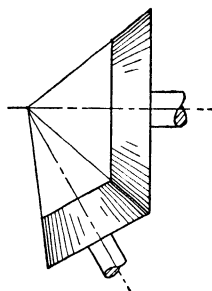


FIG. 12-8. Bevel gear pitch surfaces.

The **spiral-bevel gear** has a decided advantage over the **plain-bevel gear** as regards quietness of operation.

(c) **Gears for connecting shafts neither intersecting nor parallel.** Here **helical gears** or **skew bevels** are suitable.

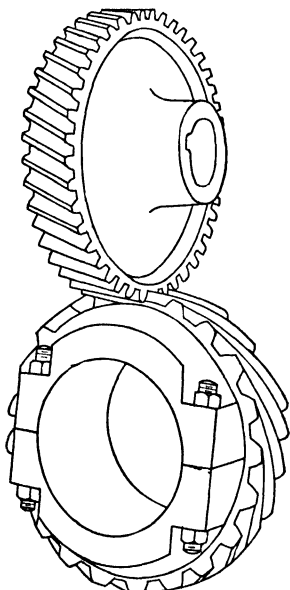


FIG. 12-9. Helical gears.

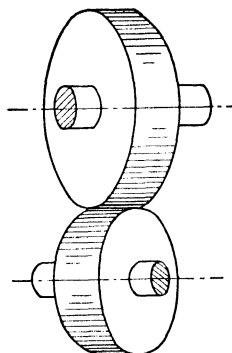


FIG. 12-10. Pitch cylinders for helical gears.

Helical gears (Fig. 12-9) are used to connect parallel as well as non-parallel shafts. In the former they are sometimes called **twisted-tooth spur gears**. Their pitch surfaces (Fig. 12-10) are cylindrical and the tooth elements are helices. Where the shafts are non-parallel, the pitch surfaces touch at a point and have sliding contact; here the teeth also make point contact and slide along the elements.

The **worm gear** shown in Fig. 12-11 is a special form of helical gear, the two members being known as the **worm** and the **worm wheel**. The worm, as compared with that which we commonly call a helical gear, has a small helix angle in proportion to its face

width, with the result that each tooth extends a long distance around the circumference. It is customary to speak of worm teeth as threads on account of the resemblance which the worm bears to a threaded bolt. Hence reference is made to a single-threaded worm, a double-threaded worm, etc., depending upon the number of teeth formed on the cylindrical surface. The worm wheel generally has the tooth surface concaved as shown in Fig. 12-11 for the purpose of obtaining line instead of point contact of the teeth.

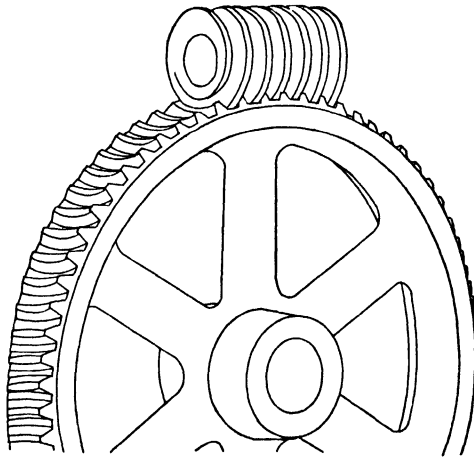


FIG. 12-11. Worm gear.

Skew bevels somewhat resemble other bevel gears in general appearance. Their pitch surfaces are not cones, however, but hyperboloids of revolution, i.e., figures swept out by the revolution of a generating line about the non-parallel axes of the gears. Contact between the two pitch surfaces thus formed takes place along this line in its position common to the two rotations. Figure 12-13 shows two such hyperboloids in contact along the line *AB*. The portions of the pitch surfaces used for skew bevels are frustums of the figures remote from the smallest sections as indicated by *X* and *Y* in the figure.

Evidently, pure rolling between the pitch surfaces is not possible and sliding takes place along the elements.

Such gears are difficult to produce and are seldom used. Standard gears of this form are not obtainable.

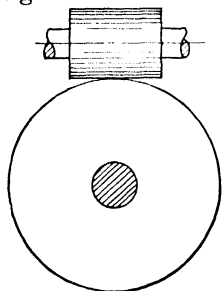


FIG. 12-12. Worm gear pitch surfaces.

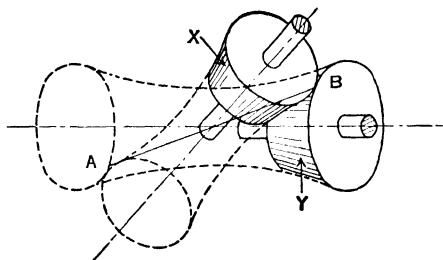


FIG. 12-13. Skew gear pitch surfaces.

4. Velocity Ratio. A general rule for pairs of toothed gears is that the **angular velocity ratio is inversely proportional to the numbers of teeth**. This law applies to all common classes of gears, such as spur, bevel, and helical gears.

When two gears are in motion it is evident that equal numbers of teeth on each gear pass any fixed point in a definite time interval, since the teeth on one gear mesh in consecutive order with the tooth spaces on the other gear. A gear having N_a teeth makes one turn while N_a teeth pass the fixed point. A meshing gear with N_b teeth will therefore make $\frac{N_a}{N_b}$ turns during the same interval.

If ω_a and ω_b are, respectively, the angular velocities of the two gears, then

$$\frac{\omega_a}{\omega_b} = \frac{1}{N_a \div N_b} = \frac{N_b}{N_a} \quad (12-1)$$

This proves the above rule.

SPUR GEARS

5. Gear Terms. Figure 12-14 illustrates many of the definitions which follow.

Pitch diameter. The diameter of the cylinder which is the pitch surface of a spur gear is known as the pitch diameter. Since the pitch cylinders of two spur gears roll together, the angular velocity ratio is the inverse ratio of the pitch diameters. A pair of mating spur gears have numbers of teeth proportional to their pitch circumferences, because both must have the same spacing of the teeth in order to obtain pure rolling of the pitch circles. Thus, for two such gears, a and b ,

$$\frac{\omega_a}{\omega_b} = \frac{D_b}{D_a} = \frac{N_b}{N_a} \quad (12-2)$$

where N , D , and ω represent, respectively, number of teeth, pitch diameter, and angular velocity.

Pitch point. That point on the line joining the centers of two gears at which the pitch circles touch is called the pitch point.

Addendum. The distance from the pitch circle to the outer end of the tooth, measured radially, is known as the addendum.

Clearance. The clearance is the amount by which the points of the teeth on one gear clear the roots of the teeth on the mating gear. This is measured along the line of centers.

Dedendum.¹ The radial distance from the pitch circle to the root circle is called the dedendum.

The **whole depth** is the sum of the addendum and dedendum.

The **working depth** is the whole depth minus the clearance.

The **face** of the tooth is that portion of the profile between the pitch circle and outer end of the tooth.

The **flank** of the tooth is that portion of the profile between the pitch circle and the root circle.

¹ As defined by the A.G.M.A. and A.S.M.E. code. By other authorities the dedendum does not include the clearance.

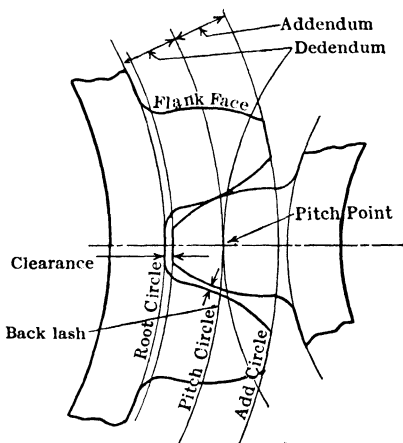


FIG. 12-14. Spur gear notation.

Backlash. The minimum distance between the non-driving side of a tooth and the adjacent side of the mating tooth is called the backlash.

Gear and pinion. When two gears mesh with each other, the larger is commonly referred to as the gear and the smaller as the pinion.

Rack. When teeth are cut along the side of a straight bar it is known as a rack. Figure 12-15 shows a pinion and rack. The pitch surface of the rack is a plane.

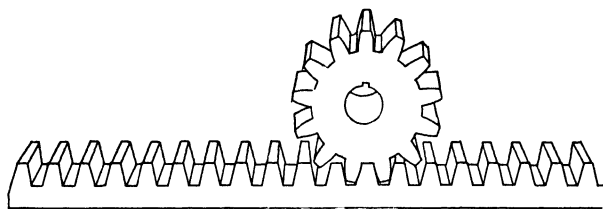


FIG. 12-15. Rack and pinion.

Angle and arc of action. The angle turned through by the driver for the period during which one of its teeth remains in contact with a mating tooth on the driven wheel is known as the angle of action of the driver. The angle turned through by the driven wheel in the same period is the angle of action of the driven wheel. The corresponding arcs on the pitch circle are called the arcs of action.

Evidently the arc of action must be greater than the circular pitch; otherwise contact between one pair of teeth would cease before the next pair made contact. In general, the longer the teeth the greater the arc of action. This consideration has been an important factor in fixing the length of standard teeth.

The **angle of approach** is the angle turned through by the gear from the instant a pair of teeth make contact to the instant at which they are in contact at the pitch point.

The **angle of recess** is the angle turned through by the gear from the instant a pair of teeth are in contact at the pitch point to the instant when contact between the same teeth ceases.

The **angle of action** is equal to the sum of the angles of approach and recess.

The **width of face** of a gear is measured on the pitch surface in a plane containing the axis of revolution. Face of tooth should not be confused with face of gear for the two are entirely different.

6. Pitch. The pitch of a gear is a measure of the size of the teeth; all tooth dimensions in standard systems are based on the pitch. Gears that are intended to run with each other must have the same pitch, as well as tooth profiles of proper form. Two common methods of stating gear pitches follow:

The **circular pitch** is the distance between corresponding points on adjacent teeth, this distance being measured along the circumference of the pitch circle. When s denotes the circular pitch,

$$s = \frac{\pi D}{N} \quad (12-3)$$

The **diametral pitch**¹ is the result obtained by dividing the number of teeth by the pitch diameter. Stated otherwise, it is the number of teeth per inch of pitch diameter. Where P is this pitch,

$$P = \frac{N}{D} \quad (12-4)$$

It should be observed that the circular pitch is a linear dimension, ordinarily expressed as so many inches. The diametral pitch, on the other hand, is merely a ratio.

Relation between circular and diametral pitch. Multiplying equation (12-3) by equation (12-4), we have

$$pP = \pi \quad (12-5)$$

The circular-pitch method of specifying tooth sizes is the older one, but the diametral-pitch method has advantages which have resulted in very general use, especially for small teeth. One

¹ The term module is sometimes used to define the pitch. The module is the pitch diameter divided by the number of teeth, or the reciprocal of the diametral pitch.

advantage is shown by the following. A gear with 19 teeth of 2 D.P. has a pitch diameter of $19 \div 2 = 9\frac{1}{2}$ in., by equation (12-4). A gear with 19 teeth of 2-in. circular pitch has a pitch diameter of $(19 \times 2)/\pi = 12.095$ in., by equation (12-3). The former calculation is easier and the result is always a rational number.

Gear teeth of 1 D.P. or smaller are commonly on the diametral-pitch system; those of 3-in. circular pitch or larger are on the circular-pitch system.

7. Requirement for Tooth Outlines. Gears are generally of circular section and give a constant angular velocity ratio of the connected shafts, though non-circular gears are employed where a variable speed ratio is desired. Whether circular or non-circular, **the teeth should be so shaped as to produce pure rolling contact of the pitch surfaces.** The pitch surfaces, therefore, comply with the laws governing bodies having pure rolling contact. Thus the

point of contact of two pitch surfaces, known as the **pitch point**, is the common instant center for the two gears.

If gear teeth are so designed as to give pure rolling of the pitch surfaces then the following law of gear teeth will result:

The common normal to the tooth surfaces at the point of contact must always pass through the pitch point.

This law is proved as follows. Figure 12-16 shows two gears making contact at *C*. The pitch point is at *P*. It is required to show that the common normal to the

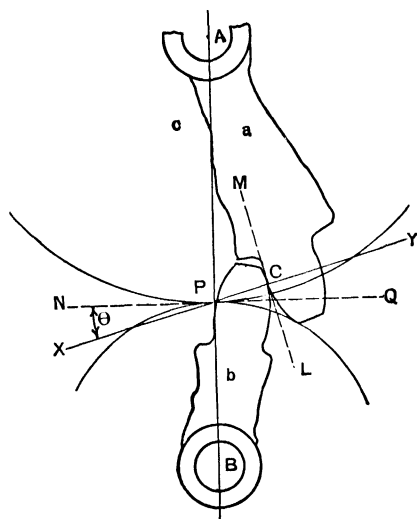


FIG. 12-16

tooth profiles at *C* will pass through *P*. Proof is made by two statements:

- (a) The mating gears are two bodies in sliding contact at *C*;

hence the relative motion at the contact point is along the common tangent LM ; otherwise the teeth would tend to either overlap or break contact. Therefore the instant center O_{ab} must lie along the common normal XY .

(b) The pitch surfaces have pure rolling contact at P . Hence P is the instant center O_{ab} .

Therefore the common normal at C passes through P .

For rolling bodies,

$$\frac{\omega_{ac}}{\omega_{bc}} = \frac{BP}{AP}$$

If the pitch surfaces are of circular section, P will occupy a fixed position on the line joining A and B . Hence BP/AP will be of constant value, and the angular velocity ratio of the gears will be constant.

If the pitch surfaces are non-circular, the value of the varying speed ratio can be calculated at any instant by means of the above equation, if the position of P is known.

8. Cycloidal Teeth. The cycloid, as used in circular gears, is a curve described by a point on a circle which rolls internally or externally on another circle. The rolling circle is known as the **describing circle**, and in forming a gear tooth outline it is rolled

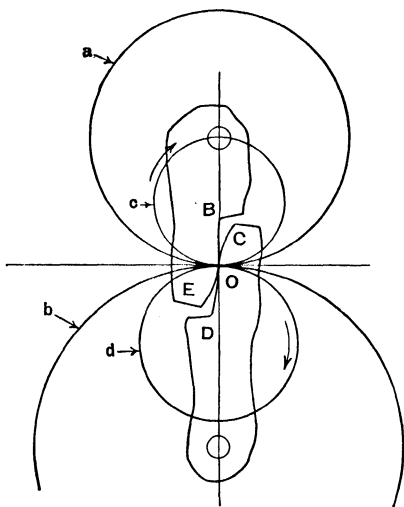


FIG. 12-17

internally and externally on the pitch circle. Internal rolling forms the flank of the tooth; external rolling forms the face. In Fig. 12-17, a and b are the pitch circles of the gears. Circle c is rolled internally on a , the point O on c describing the curve OB , which forms the flank of the tooth on the upper wheel. Circle c is then rolled externally on b , point O on c now tracing out the curve OC

which is the face of the tooth on the lower gear. Likewise, curves OD and OE are obtained by rolling circle d internally on b and externally on a . These curves form, respectively, the flank of the lower and the face of the upper gears. The two describing circles, c and d , need not be the same size, but the same circle must be used for the face of one gear and the flank of the other which works with it. In practice, to secure interchangeable gears, the same radius of describing circles is used throughout a series.

When the diameter of the describing circle is one-half the pitch diameter of the gear, the flank of the tooth becomes a radial straight line (see OB , Fig. 12-17) and the tooth is somewhat narrow at the root. If the describing circle is made larger, the tooth becomes still narrower at the root and lacks strength; also, if the describing circle is made large enough, it may be impossible to cut the gear teeth by using a milling cutter, because the space between the teeth widens out from the pitch circle toward the root. In the Brown and Sharpe standard system, the diameter of the describing circle is made equal to one-half the pitch diameter of a gear with fifteen teeth. This renders it possible to cut a twelve-tooth pinion with a milling cutter, this pinion being the smallest generally used.

9. Involute Teeth. In general, if a curve of any form is rolled on a straight line, a point in this line will describe a path known as an involute of the curve. For nearly all gears with involute teeth, the involute is formed by rolling the straight line on a circle. The only exceptions to this are gears of non-circular section. When involutes are spoken of in connection with gears, without further definition, the involute of a circle is meant. This circle is commonly called the **base circle** of the involute.

A device shown in Fig. 12-18 illustrates a method of development of conjugate involute curves. A cord CD , $C'D'$ is wrapped around two circular disks, a and b . Disk a has a transparent disk m attached to its face, and b has a similar disk n fastened to it. If a is rotated, the cord, acting as a belt, will drive b in the opposite sense. A point P on the cord will then trace out on the disk m an involute KL , and on the disk n an involute MN .

By keeping a and b in fixed position and then cutting the cord at P , the same curves may be traced out by the two ends when the

loose portions are wound on and unwound from the disks to which they are attached.

Considering the above methods for obtaining the curves, two facts are evident: (a) that the point of contact always lies along the line of the cord, i.e., along CD ; (b) that, since the tangent

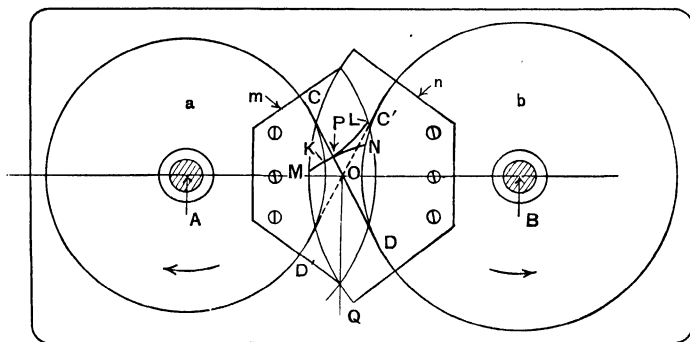


FIG. 12-18

portion of the cord is always swinging with reference to a disk about its point of tangency with the disk in question, the relative motion is always perpendicular to the tangent line. CD is therefore the common normal to the two curves at all times and it crosses the line of centers at a fixed point O . Moreover, the pressure angle is constant and equal to DOQ where OQ is a normal to the line of centers of the disks.

To summarize, it has been shown that involute profiles:

- (a) Have the essential property for correct gear tooth forms.
- (b) Have a straight-line path of contact.
- (c) Have a constant pressure angle.

Point O (Fig. 12-18), where the line CD intersects the line of centers, is the instant center of wheels a and b . Hence the angular velocity ratio $\frac{\omega_a}{\omega_b}$ is equal to $\frac{BO}{AO}$, and O is therefore the pitch point.

When the pitch point O and the pressure angle are known, the base circles may be found, since they are tangent to the line CD , whose position is fixed by these data.

10. Graphical Construction. As already pointed out, the involute can be thought of as the path traced out by the end of a

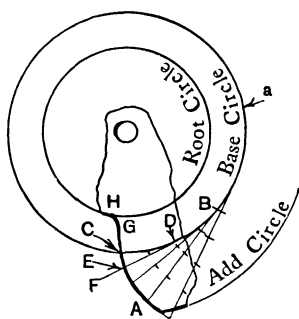


FIG. 12-19.

string which is unwound from a cylinder, the motion being, of course, in a plane perpendicular to the cylinder axis. The length of the unwound or tangent portion of the string is, in any position, equal to the length of the arc on the base circle from which it was unwrapped. Putting it another way, the tangent to the base circle from any point on an involute is equal in length to the arc from the point of tangency to the starting point of the curve. We make use of this property in constructing the curve. In Fig. 12-19 we shall suppose that a is the base circle and that we wish to draw an involute to pass through a given point A .

From A draw AB , tangent to circle a . Obtain an arc BC equal in length to AB . This can be done approximately in the drafting room by dividing AB into any number of equal parts (four are used in the diagram). One of these equal lengths is taken on the dividers, and four divisions are stepped off, starting from B around the arc, thus locating C . The point C is the inner end of the involute curve. To obtain a point between A and C , point D is taken on the arc at the second division, DF is drawn tangent at D , and two lengths are stepped off on it with the same divider setting as before, obtaining E , a third point on the involute. Other points are found in a similar manner. A smooth curve is finally drawn through the points and this forms the required involute. The curve is continued outward until the addendum circle is reached.

In completing the tooth flank, the curve must be extended inside the base circle to allow space for the mating tooth, plus a clearance. As the involute cannot penetrate the base circle, a different form is necessary for this portion. Generally radial straight line CG is used, terminating in a small circular arc GH at the junction with the root circle. Contact does not take place

on CG : its outline is determined by considerations of strength and ease of production. The purpose of the arc GH is to strengthen the tooth at the root, and it should be of as large radius as possible without causing interference with the teeth on the mating gear.

11. Practical advantages resulting from the use of involute profiles may be summarized as follows.

(a) Involute gears may be mounted with small initial inaccuracies in the center-to-center distance, or this distance may change as the result of bearing wear, tooth contact still complying with the fundamental gear tooth law.

(b) Involute gears may be used for such applications as driving rolls in steel mills, where the center-to-center distance constantly varies.

(c) The working surface of the involute rack is of the simplest possible form, a plane. This reduces to a minimum the difficulty of producing accurate conjugate teeth — a manufacturing advantage.

(d) Where the teeth are cut by formed milling cutters, the number of cutters needed in covering the range from the smallest pinion to rack is less than would be necessary for cycloidal profiles. This is due to the slow change in tooth curvature as the tooth numbers are increased.

The main disadvantage of involute teeth lies in the fact that interference is obtained with pinions having small tooth numbers. No interference difficulty is encountered in cycloidal teeth.

Involute teeth never have concave faces or flanks except in internal gears. On the other hand, cycloidal gears with more than fifteen teeth have concave flanks. Hence in the cycloidal form a concave flank normally mates with a convex face and surface contact is more nearly approximated than when using involute profiles. From this standpoint the cycloidal system would appear to be somewhat better adapted to carrying heavy loads.

In present engineering practice, the involute form of tooth has so completely displaced its rival, much used in earlier years, as to render the cycloidal form practically obsolete. Later, reference

will be made to a composite form of tooth which uses a profile composed of both involute and cycloidal curves.

12. Tooth Action. In Fig. 12-20 is shown a pair of involute gear profiles in three positions. We shall assume that the gears rotate as indicated by the arrows on the diagram. At a_1, b_1 the teeth are just beginning to make contact; at a_2, b_2 the teeth are in contact at the pitch point; and at a_3, b_3 contact is just about to cease.

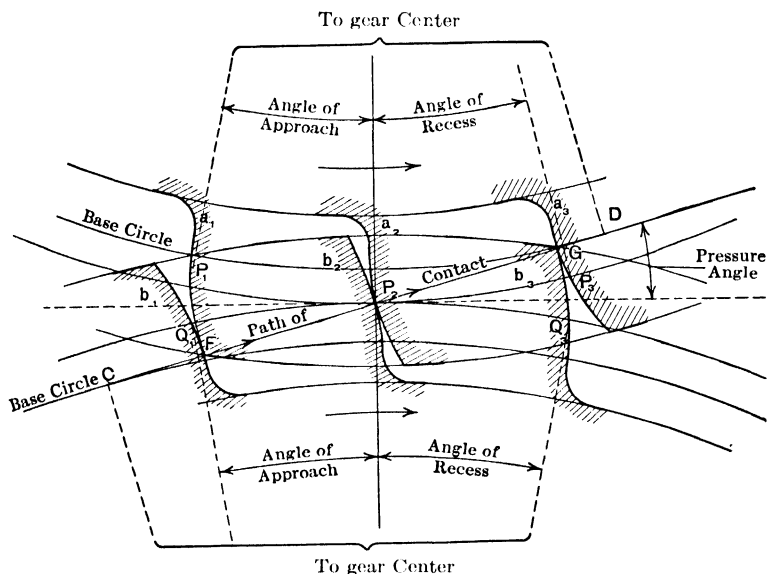


FIG. 12-20

The path of contact must lie at all times on the straight line CD , this line being the common tangent to the base circles. Contact begins at F and ends at G . P_1, P_2, P_3 on the pitch circle of the upper gear are corresponding positions of one point on this gear, and Q_1, P_2, Q_3 are similarly the three positions occupied by one point on the lower gear. Since the pitch circles have pure rolling contact, arcs $P_1P_2P_3$ and $Q_1P_2Q_3$ are of equal length. Also, by definition, P_1P_2 is the arc of approach and P_2P_3 the arc of recess for the upper gear, and arcs Q_1P_2 and P_2Q_3 have the same values

for the lower gear. The angles of approach and recess for the former are found by joining P_1 and P_3 to the center of the upper gear as indicated on the figure. A similar construction locates the angles of approach and recess for the lower gear. The pressure angle is noted in the figure.

It will be observed that points F and G , where contact begins and ends, are found at the intersections of the addendum circles with the pressure line CD . F and G are located between points C and D in this figure, but with other tooth proportions and numbers of teeth on the gears it may happen that either F or G or both F and G fall on line CD produced. This leads to interference.

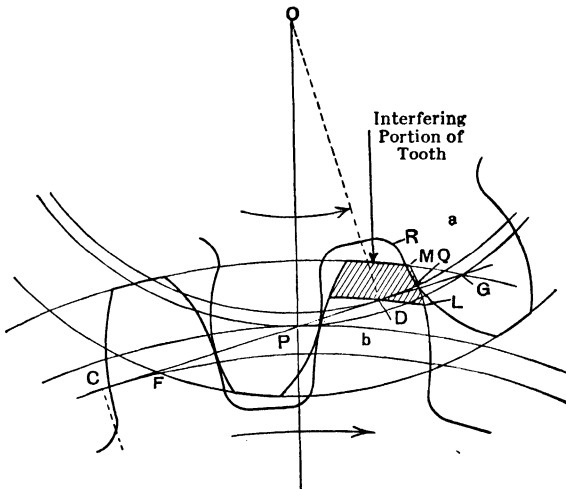


FIG. 12-21

13. Interference. Gear teeth are said to interfere when they tend to overlap or cut into the mating teeth. Under certain conditions interference will take place when the teeth have true involute profiles. Such a situation is illustrated in Fig. 12-21. Here point G , the intersection of the addendum circle of the lower gear with the contact line CD , falls on CD produced. This condition is always accompanied by interference, as may be seen from the following discussion.

Considering the two gears in Fig. 12-21 to turn in the sense indicated by the arrows, and observing a pair of teeth, a , b , initially in contact at F , we note that the contact point traces out the line FD as the gears revolve. Contact does not cease at D , but thereafter it must take place between the involute portion LM of the tooth on the lower gear and the flank QR , inside the base circle of the upper gear. This flank is not of involute form, since the involute cannot be extended inside the base circle. If QR is a radial flat surface, overlapping or "interference" will take place. Modification of the tooth forms is then necessary. The following methods are possible:

- (a) The height of the teeth may be reduced.
- (b) The pressure angle may be increased.
- (c) The radial flank of the pinion may be cut back. This is technically known as "undercutting" when any material is removed inside a radial line from the junction point of the involute and its base circle.
- (d) The face of the gear tooth may be relieved.

Methods (a) and (b) are employed in conjunction with each other in what are known as stub-tooth gears, to which reference will be made later.

Method (c) is often undesirable for two reasons. First, it weakens the pinion tooth and, second, it may result in a short arc of action, since excessive undercutting may remove the inner portion of the involute as well as the straight part of the tooth below the base circle.

Method (d) is satisfactory where manufacturing methods permit its use.

In the "composite" type of tooth, interference is avoided by use of cycloidal curves for those portions of the profile which would interfere if made of involute form.

14. Correct Modification for Interference. Where the modification for interference consists in cutting back the proper part of the tooth face on the rack or larger gear, the resulting surface should be of a form conjugate to the flat radial surface inside the base circle of the pinion. A straight line is a cycloid generated by

a rolling circle half the diameter of the directing circle. Hence the conjugate curve on the larger gear should be a cycloid generated by a rolling circle half the diameter of the pinion pitch circle when rolled externally on the pitch circle of the former. Thus in Fig. 12-21 the profile LM should be a cycloid formed by rolling a circle of diameter OP externally on the pitch circle of gear b . This curve is conjugate to the straight line QR on pinion a .

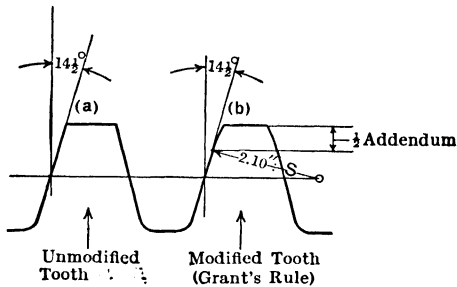


FIG. 12-22. Involute rack teeth.

15. Involute Rack Teeth. Since a rack can be regarded as a portion of a gear wheel with infinite pitch radius, the base circle for the rack involute is also of infinite radius, and the involute itself is a straight line. Rack teeth on the involute system therefore have straight working surfaces, except where modification becomes necessary to avoid interference. Figure 12-22(a) illustrates a rack tooth of this kind. The angle between the side of a tooth and the perpendicular to the pitch line is equal to the pressure angle or angle of obliquity. The amount of modification on the face of the tooth which is really required to eliminate interference depends on the size of pinion with which the rack is to mesh. For the $14\frac{1}{2}^\circ$ system George B. Grant recommends rounding off the upper half of the face by an arc of radius equal to $2.10 \text{ in.} \div \text{diametral pitch}$, or $0.67 \text{ in.} \times \text{circular pitch}$. See Fig. 12-22(b). This gives a satisfactory approximate form for cast teeth.

16. Practical Requirements. Aside from the fact that the working faces of gear teeth must comply with the fundamental law of Art. 7, there are several other requirements which have influenced the choice of standard gear tooth forms and proportions.

(a) The teeth must be capable of accurate production at low cost.

(b) The tooth form should have good wearing qualities. Low rubbing speeds and close approach to surface contact are both favorable. The latter condition is secured when mating teeth both have large radii of curvature. Tooth pressures are distributed over a wider strip of surface when large radii are used. The result is a lower intensity of pressure and less wear.

(c) The tooth form must result in good beam strength. In service, forces act on the tooth side tending to bend it like a beam. Beam strength is greatest in a short tooth with a wide section across the root.

(d) The arc of contact must be at least equal to the circular pitch; otherwise there would not be continuous contact between gears. An arc of action greater than 1.4 times the circular pitch is generally held to be good design. Below this limit, noisy action of the gears is likely unless they are very accurately cut.

(e) Interchangeability of a series of gears of the same pitch is generally desirable, although it is unnecessary with many gears which are of the special purpose type.

17. Standard Tooth Forms. In view of the requirements for tooth forms, four types of teeth have been standardized by the American Gear Manufacturers Association. These are known as:

- (a) The $14\frac{1}{2}^\circ$ composite system.
- (b) The $14\frac{1}{2}^\circ$ full-depth involute system.
- (c) The 20° full-depth involute system.
- (d) The 20° stub involute system.

Tooth proportions are given in Table 12-1.

It will be noted that the composite, the $14\frac{1}{2}^\circ$ full-depth, and the 20° full-depth systems all have teeth of the same proportions. These proportions are exactly the same as those used in the earlier Brown and Sharpe Standard system.

Each of the four standard systems has gear teeth which are conjugate to a "basic rack." The basic rack is therefore the standard or reference form in cutting teeth on gears of any size.

avoids the interference and undercutting of small pinions, which are characteristic of that system. It is the common form of tooth cut in the small shop where the formed milling cutter method is employed. It is suitable for gears with twelve or more teeth, and this range of gear sizes can be produced by means of a set of fifteen standard cutters. (See Fig. 12-24.) Each cutter can be used over a range of tooth numbers as shown in Table 12-2. Each is correct for the smallest number in the range and somewhat in error for the higher numbers. However errors introduced in this way can at least be partially corrected by slightly varying the depth of cut in the blank. Standard tables, approved by the Gear Manufacturers Association, are available¹ which show the depth of cut required for best results with various sizes of gears.

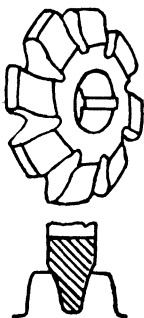


FIG. 12-24

TABLE 12-2
Composite Tooth Milling Cutters
Standard $14\frac{1}{2}^\circ$ System

Cutter Number	Tooth Numbers	Cutter Number	Tooth Numbers
1	135 to rack	$4\frac{1}{2}$	23 to 25
$1\frac{1}{2}$	80 to 134	5	21 to 22
2	55 to 79	$5\frac{1}{2}$	19 to 20
$2\frac{1}{2}$	42 to 54	6	17 to 18
3	35 to 41	$6\frac{1}{2}$	15 to 16
$3\frac{1}{2}$	30 to 34	7	14
4	26 to 29	$7\frac{1}{2}$	13
		8	12

Whereas the form of the basic rack is not so simple as that for pure involute teeth, the composite system gives a sufficiently large arc of contact and a tooth form otherwise quite satisfactory for the range of tooth numbers mentioned above. As it is difficult to

¹ See "Manual of Gear Design," Vol. 2, by Buckingham.

attain high standards of accuracy in teeth cut by formed cutters, this type is most satisfactory when loads and speeds are moderate.

$14\frac{1}{2}^\circ$ full-depth involute. This system has a basic rack with the same tooth addendum and dedendum as the composite system. The rack tooth is straight-sided, as shown in Fig. 12-25, except for a fillet arc at the root to strengthen the tooth. This system is used for generated teeth.

Interference occurs when the tooth number of equal pinions is less than twenty-three or when a rack engages a gear with less than

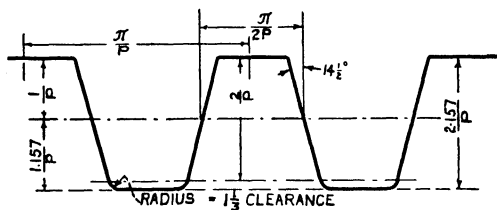


FIG. 12-25. Basic rack for $14\frac{1}{2}^\circ$ full-depth system.

thirty-two teeth. Undercutting is therefore necessary with small tooth numbers, and as a result the arc of action then becomes unsatisfactory. For example, with two twelve-toothed pinions the arc of contact is 0.034 times the circular pitch, an unusable value. The desired arc of action, namely, 1.4 times the circular pitch, is obtainable with gears having twenty, twenty-one, or twenty-two teeth, the exact value depending on the tooth number of the mating gear. This type of tooth is very satisfactory, however, when tooth numbers are large.

20° Full-depth involute system. The basic rack (Fig. 12-26) is the same as that for the $14\frac{1}{2}^\circ$ system except for the pressure angle. The use of a larger pressure angle leads to better tooth action when the tooth numbers are small. For example, an arc of action equal to 1.4 times the circular pitch is obtainable with equal gears of fourteen teeth. In this respect, this type of tooth gives the best results of any of the four standard types with low tooth numbers. It is used for generated teeth.

20° stub involute system. The basic rack (Fig. 12-27) has a tooth about 18 per cent shorter than that of the full-depth systems.

Interference difficulties are much reduced as compared with the other standard involute teeth. Thus a stub-tooth rack will mesh with a seventeen-tooth pinion without interference. A pair of twelve-tooth pinions gives an arc of action equal to 1.19 times the

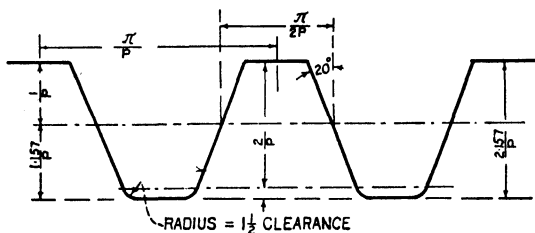


FIG. 12-26. Basic rack for 20° full-depth system.

circular pitch. This is a usable value. However, the arc of action does not increase rapidly with the increase in tooth numbers, a

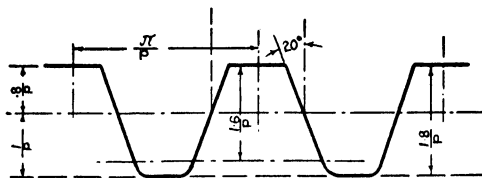


FIG. 12-27. Basic rack for 20° stub involute system.

27-30 combination having a value only 1.35 times the circular pitch. On this account accuracy of cutting is especially important if noisy action is to be avoided.

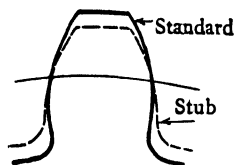


FIG. 12-28. Comparison of stub tooth and standard 14½° tooth.

Figure 12-28 shows a graphical comparison of 20° stub and 14½° full-depth teeth of the same pitch for gears of equal size. The short tooth with wide root, characteristic of the former, gives high beam strength and accounts for its suitability for use where subject to heavy shocks.

The 20° stub tooth was designed primarily for use in automobile transmissions, and it is also employed for heavy mill gearing. The teeth are cut by generating processes.

18. Approximate Gear Tooth Profiles. When gear teeth are cast instead of being cut on the gear blank, circular-arc profiles are often substituted for the theoretically correct involute or cycloidal curves. This lowers the cost of both drawings and patterns, because the tooth forms can be specified more easily by the drafting room and laid out more rapidly by the pattern maker. The errors introduced by this approximation are generally small if the centers and radii are properly chosen. In any event, cast teeth are likely to be distorted on cooling, the surfaces are somewhat rough, and extreme accuracy in making the patterns is not justified.

To save time, gear teeth are often drawn by the circular-arc method even though the teeth are to be cut to true involute form.

Many methods for the laying out of circular-arc teeth have been used. Among the better known are the Grant Odontograph and the Brown and Sharpe methods.

19. Approximate Involute Teeth: Grant Odontograph. The Grant method of constructing approximate tooth outlines is one of the best known and most useful. Table 12-3 contains the information for teeth of 15° pressure angle and standard proportions, the true involute being approximated by two circular arcs. The radius of the arc for the face of a required tooth is found by using the figures in the "face radius" column corresponding to the number of teeth on the gear. These figures are either divided by the diametral pitch or multiplied by the circular pitch, and the result is the required radius. The radius of the flank arc is found from the data in the "flank radius" column by the same method. The centers from which the arcs are drawn are located on the base circle of the true involute. Inside the base circle the tooth profile is extended along a radial line and rounded into the root circle by a fillet arc. Face and flank arcs coincide with the true involute at three points; hence this method is sometimes referred to as a three-point odontograph.

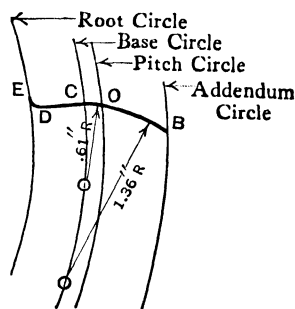


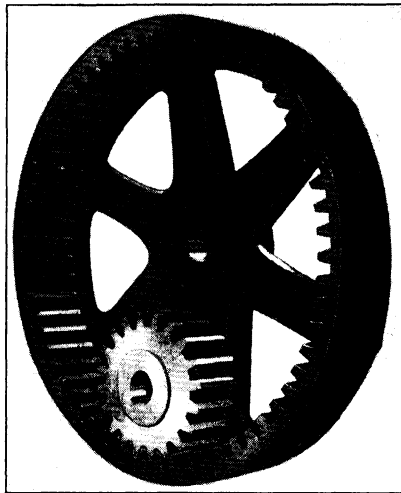
FIG. 12-29

TABLE 12-3

Grant Involute Odontograph

No. of Teeth	Divide by Diam. Pitch		Multiply by Circular Pitch	
	Face Radius, in.	Flank Radius, in.	Face Radius, in.	Flank Radius, in.
10	2.28	0.69	0.73	0.22
11	2.40	0.83	0.76	0.27
12	2.51	0.96	0.80	0.31
13	2.62	1.09	0.83	0.34
14	2.72	1.22	0.87	0.39
15	2.82	1.34	0.90	0.43
16	2.92	1.46	0.93	0.47
17	3.02	1.58	0.96	0.50
18	3.12	1.69	0.99	0.54
19	3.22	1.79	1.03	0.57
20	3.32	1.89	1.06	0.60
21	3.41	1.98	1.09	0.63
22	3.49	2.06	1.11	0.66
23	3.57	2.15	1.13	0.69
24	3.64	2.24	1.16	0.71
25	3.71	2.33	1.18	0.74
26	3.78	2.42	1.20	0.77
27	3.85	2.50	1.23	0.80
28	3.92	2.59	1.25	0.82
29	3.99	2.67	1.27	0.85
30	4.06	2.76	1.29	0.88
31	4.13	2.85	1.31	0.91
32	4.20	2.93	1.34	0.93
33	4.27	3.01	1.36	0.96
34	4.33	3.09	1.38	0.99
35	4.39	3.16	1.39	1.01
36	4.45	3.23	1.41	1.03
37-40	4.20		1.34	
41-45	4.63		1.48	
46-51	5.06		1.61	
52-60	5.74		1.83	
61-70	6.52		2.07	
71-90	7.72		2.46	
91-120	9.78		3.11	
121-180	13.38		4.26	
181-360	21.62		6.88	

Figure 12-29 illustrates the application to a gear having fourteen teeth of 2 diametral pitch. In the table, opposite 14 teeth, the face radius column shows the value 2.72, and the flank radius column 1.22. Dividing these numbers by the diametral pitch, we obtain 1.36 in. and 0.61 in. The former radius is used to draw an arc OB extending from the pitch to the addendum circle. The latter is the radius of the arc OC extending from the pitch circle inward to the base circle. A radial straight line CD and a fillet arc DE complete the tooth profile.



Stephens-Adamson Mfg. Co.

FIG. 12-30. Internal gear.

20. Internal or Annular Gears. One of these gears is illustrated in Fig. 12-30, the teeth being cut internally on a hollow cylinder or ring. Except for the clearances, an internal gear and an external gear of the same pitch, pitch diameter, and tooth proportions have the same profile, the tooth spaces on one corresponding exactly to the teeth on the other.

Internal gears are required when driving and driven members must rotate in the same sense. The drive is more compact than when external gears with the same velocity ratio are used.

HELICAL GEARS

21. Tooth Action. Sliding Velocity. Gears of this kind are used to connect non-intersecting shafts at any angle. The cylindrical pitch surfaces of the helical gears touch along a line only when the axes of the cylinders are parallel; then they are often called twisted-tooth spur gears. Otherwise the pitch surfaces touch at a point, and consequently the teeth also have point contact. Two cylinders cannot have pure rolling contact unless their axes are parallel. Thus, in helical gears connecting non-parallel shafts, the teeth slide on one another along the tooth elements. The sliding action is provided for by making the teeth of uniform cross section and the tooth elements true helices.

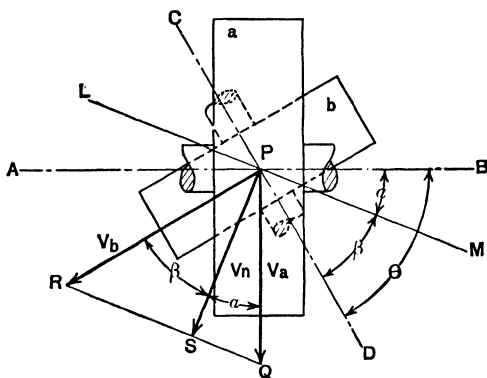


FIG. 12-31

In Fig. 12-31, a and b represent the pitch cylinders of a pair of helical gears connecting two shafts whose axes are AB and CD , at an angle θ . The pitch cylinders make contact at a point P , and the line LM is the common tangent to the helical tooth elements through P .

Point P , regarded as a point on a , has a velocity represented by a vector PQ ($= V_a$) perpendicular to AB . Point P considered as a point on b has a velocity represented by a vector PR ($= V_b$) at 90° to CD . Vectors PQ and PR must have a common component PS ($= V_n$) in a direction normal to the common tangent

ML. If this were not true the teeth would either come out of contact or interfere. The algebraic difference of the components of *PQ* and *PR* in a direction parallel to *ML*, namely, *QS* plus *SR*, represents the sliding velocity at the pitch surface along the tooth elements.

22. Angular Velocity Ratio. The angle of cut or helix angle of the teeth is the angle between a tangent to the helix at a point on the pitch surface and a line parallel to the gear axis. In Fig. 12-31, α is the angle of cut for the gear *a*, and β for the gear *b*. Evidently, from the figure,

$$\alpha + \beta = \theta$$

Also, by construction, the angle *SPQ* = α , and the angle *SPR* = β , and therefore

$$V_a \cdot \cos \alpha = SP = V_b \cdot \cos \beta \quad \text{or} \quad \frac{V_a}{V_b} = \frac{\cos \beta}{\cos \alpha}$$

Now,

$$\omega_a = V_a \div r_a \quad \text{and} \quad \omega_b = V_b \div r_b$$

Therefore, the angular velocity ratio is

$$\frac{\omega_a}{\omega_b} = \frac{V_a \cdot r_b}{V_b \cdot r_a} = \frac{r_b \cdot \cos \beta}{r_a \cdot \cos \alpha} \quad (12-6)$$

It will be noted that the velocity ratio depends not only on the pitch diameters but also on the angles of cut of the teeth. With unchanged diameters a different speed ratio can be obtained from a pair of these gears by altering the angles of cut.

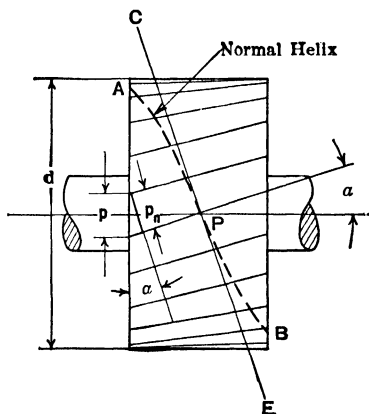


FIG 12-32

23. Normal Pitch. A helix, such as *AB*, Fig. 12-32, drawn on the pitch surface, normal to the tooth elements, is known as the normal helix. The circular pitches of a mating pair of helical

gears must be equal when these pitches are measured along the normal helix. This pitch is called the **normal circular pitch**. The **normal diametral pitch** is found by dividing the normal circular pitch into π , diametral and circular pitch therefore bearing the same relationship to one another as in common spur gears.

The normal pitch is the pitch of the cutter that must be used to produce the teeth. Thus a helical gear of 10 normal diametral pitch and a plain spur gear of 10 diametral pitch are both produced by the use of 10 D.P. cutters. It will be shown later that the same shape of cutter cannot be used in both cases if the two gears referred to have equal numbers of teeth, because the profiles on the normal plane are different.

We shall next proceed to express the relationship between normal pitch, pitch diameter, and numbers of teeth for the helical gear. In Fig. 12-32, let

p_n = normal circular pitch

p = circular pitch measured in the plane of revolution
(known as the circumferential pitch)

P_n = normal diametral pitch ($= \pi \div p_n$)

N = number of teeth

d = pitch diameter

α = angle of cut.

From the figure,

$$p = \frac{p_n}{\cos \alpha} \quad (12-7)$$

Also

$$p = \frac{\pi}{P} = \frac{\pi D}{N} \quad (12-8)$$

and

$$p_n = \frac{\pi}{P_n}$$

24. Velocity Ratio in Terms of the Number of Teeth. Equation (12-6) expresses the angular velocity ratio of helical gears in terms of the pitch diameters and angles of cut. For two gears, a and b ,

$$\frac{\omega_a}{\omega_b} = \frac{d_b \cos \beta}{d_a \cos \alpha}$$

By combining equations (12-7) and (12-8) it can be shown that

$$\frac{\omega_a}{\omega_b} = \frac{N_b}{N_a}$$

In helical as in spur gears, therefore, the angular velocities are inversely proportional to the numbers of teeth.

25. Applications of Helical Gearing. The characteristics enumerated above make helical gears smoother in action, stronger, and more efficient than ordinary spur gears. For these reasons they are especially adapted to drives required to transmit heavy loads, or to run at very high speeds. They are also desirable where quiet action is necessary. Examples of such drives are

- (a) Rolling mills.
- (b) Steam turbine reduction gears.
- (c) Automobile timing gears.

Well-made helical gears are frequently used for pitch-line velocities between 3000 and 5000 ft per min.

WORM GEARING

26. Properties and Uses. The properties of the worm gear are such as to render it suitable for many uses. Though common for connecting non-intersecting shafts at 90° , it is sometimes used for other shaft angles. When properly constructed it will operate with little noise and show good efficiency. For large speed reductions it is more compact than other types of gear drives. Under certain conditions the drive is irreversible; that is, the combination of worm and wheel can be driven only from the worm end. For this reason it has been employed in hoists, steering gears for automobiles, etc., where reversibility is not desired.

Line contact, with a consequent improvement in wearing qualities and load-carrying capacity, is secured in the gear shown in Fig. 12-33(b). Here the worm wheel surface is concaved so as to conform with the worm outline, and the active section is not confined to the central plane. This is the style generally used.

In Fig. 12-33(c) is shown a third form of worm wheel. This

presents a fairly smooth outer surface, an advantage when the wheel is sometimes turned by hand.

Figure 12-34 shows a variety of worm gear with multiple-threaded worm, used for rear-axle drives in automobiles.

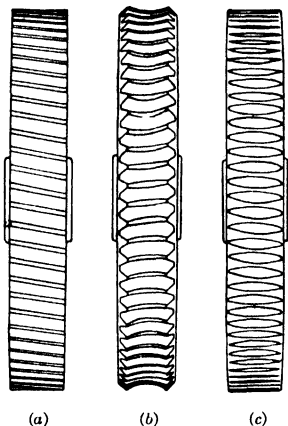


FIG. 12-33. Types of worm wheels.

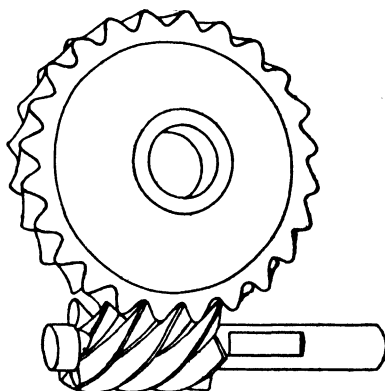


FIG. 12-34. Multiple-threaded worm and wheel.

In a special type of worm developed in England, known as the Hindley worm, the axial section of the pitch surface is curved to fit the pitch arc of the worm wheel. A large number of teeth are thus brought into a simultaneous contact. Proper action requires that accurate endwise adjustment of the worm be maintained.

27. Worm Gear Terms. Figure 12-35 illustrates the way in which worm gear dimensions are indicated.

The **pitch** of the **worm** is the distance from a point on one thread or tooth to the corresponding point on the next tooth, measured in a direction parallel to the axis of revolution. Some authorities call this the **axial pitch**. The **pitch** of the **worm wheel** is the distance from a point on one tooth to a similar point on the next tooth, measured on the pitch surface in the plane of revolution.

The **lead** is the amount the worm helix advances along the axis per turn. When a worm is "single threaded," the pitch and

lead are equal to one another; when "double threaded," the lead is twice the pitch. In general, where the worm has n threads, the lead is n times the pitch.

The **face angle** is measured as indicated in Fig. 12-35. This usually has a value between 60° and 90° .

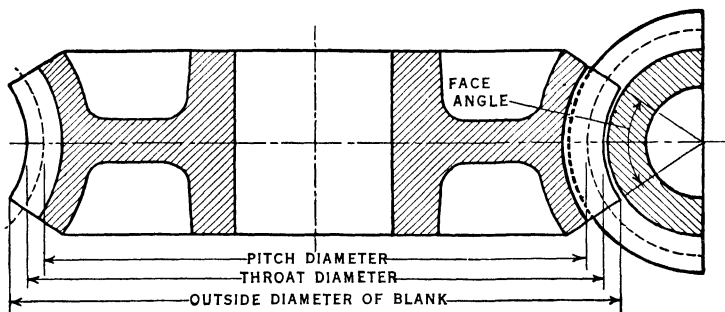


FIG. 12-35. Worm gear dimensions.

28. Velocity Ratio. When the worm has a single thread, the worm wheel is advanced one tooth per turn of the worm; hence if the worm wheel has N teeth the worm must make N revolutions per turn of the worm wheel. The velocity ratio of wheel to worm is therefore equal to $1/N$. When the worm is double threaded the worm wheel is advanced two teeth per turn of the worm, and the velocity ratio becomes $2/N$. The general expression is n/N , where n equals the number of threads on the worm. Worm gears, therefore, comply with the ordinary law for toothed gears.

The velocity ratio of a worm gear can be found directly from the lead and worm wheel pitch diameter. Per turn of the worm, a point on the pitch circumference of the wheel is advanced a distance equal to the lead. The angular movement of the worm wheel, in revolutions, for one turn of the worm, is therefore equal to

$$\frac{\text{Lead of worm thread}}{\text{Worm wheel circumference}}$$

This quantity is the angular velocity ratio of wheel to worm.

BEVEL GEARS

29. General. Bevel gears may be used to connect intersecting shafts making any angle with one another. The pitch cones of conjugate bevel gears of the ordinary type have a common apex;

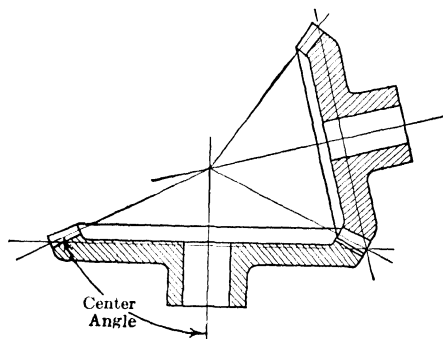


FIG. 12-36. External bevel gears.

this results in pure rolling contact of the pitch cones along their elements. The use of pitch cones not having a common apex

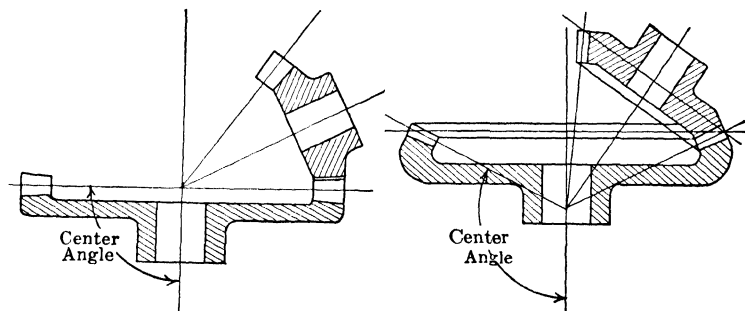


FIG. 12-37. Crown gear and pinion. FIG. 12-38. Internal bevel gear and pinion.

would increase the difficulties of design and production and the tooth forms would be less serviceable. Referring to Fig. 12-36, which shows a section through a pair of bevel gears, it will be noted that all tooth elements are straight and radiate from a

common apex. This results in similar tooth forms at all points along the cone elements.

Bevel gears may be classified according to the size of the pitch cone angle. Those having a center angle less than 90° are known as external bevels (Fig. 12-36). In a **crown gear** (Fig. 12-37) the center angle is 90° and the cone becomes a plane surface. **Internal bevels** (Fig. 12-38) have a center angle greater than 90° , the cone being inverted. Equal bevel gears connecting shafts at 90° are known as **miter gears**.

30. Bevel Gear Terms. Figure 12-39 indicates the meaning of the terms cutting angle, center angle, edge angle, angle increment, pitch diameter, etc., as applied to bevel gears.

Pitch. The circular pitch of the teeth in a bevel gear diminishes as we pass along the surface toward the apex of the pitch cone. Strictly speaking, a statement regarding the pitch of a gear of this kind should be supplemented by information as to where it is measured. In accordance with common usage, however, the circular pitch, unless otherwise specified, means the pitch measured at the outer ends of the teeth. The same statement applies to the pitch diameter, diametral pitch, addendum, dedendum, etc.

The velocity ratio of bevel gears follows the usual law for toothed gearing, the angular velocity ratio of a pair being inversely proportional to the numbers of teeth. For a given pitch, the numbers of teeth vary directly as the pitch diameters, that is, as the diameters of the bases of the pitch cones. Diameters are dependent on the cone angles. When the angle between the gear axes and the velocity ratio are known, the proper pitch cone angles may be determined by the method of Art. 4, Chapter IV, which applies to rolling pitch cones.

In view of the fact that different speed ratios require different cone angles, bevel gears must be designed in pairs for a particular ratio, and interchangeability does not exist to the same extent as in spur gears.

31. Hypoid Gears. These gears, illustrated in Fig. 12-40, possess certain of the characteristics of both the hyperboloidal and the spiral bevel gears. Like the former, they connect non-inter-

secting shafts. As in the latter, the tooth elements are of spiral shape so as to give the progressive contact which leads to quiet operation.

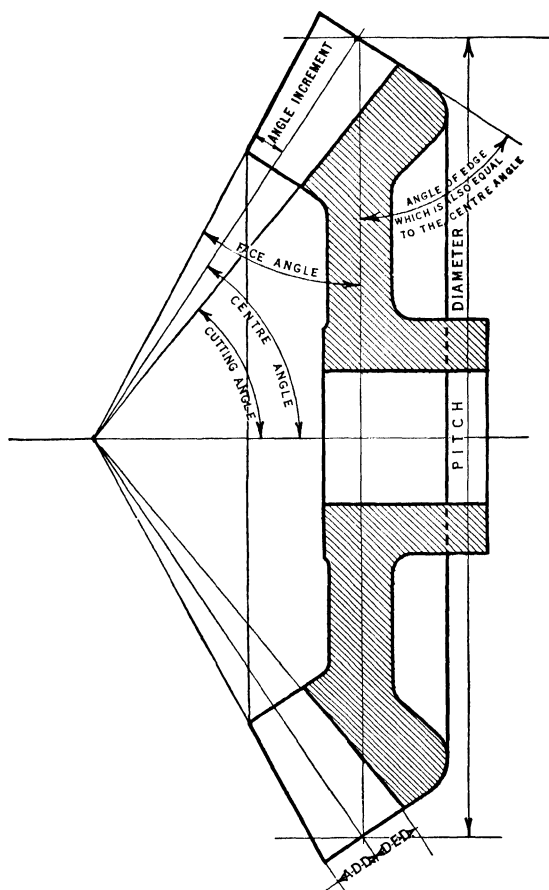


FIG. 12-39. Bevel gear notation.

Instead of using the hyperboloidal pitch surfaces of the true hyperboloidal gear, cones which approximate these surfaces are substituted. This decreases the difficulties of production. Inspection of Fig. 12-40 will indicate that the teeth slide on one

another along the tooth elements. This dictates the use of curves of a particular form for these elements; in this respect also, the hypoid gear differs from the hyperboloidal, which has sensibly straight tooth elements.

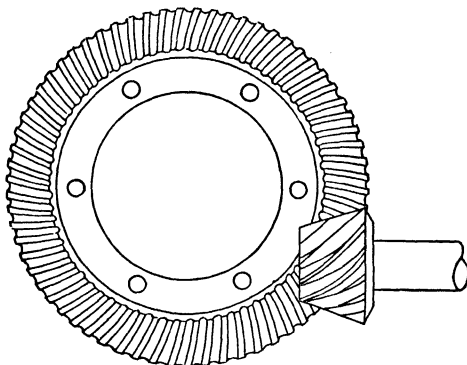


FIG. 12-40. Hypoid gear.

The hypoid gear, developed by the Gleason Gear Company, has found an important use in the automobile drive, though the fact that the shafts are offset and may be extended past one another has led to other industrial applications. The advantages for automotive use follow.

(a) The pinion axis can be placed below the gear axis, rendering possible a lower body design for the car.

(b) The circumferential pitch of the teeth on the pinion is greater than that of the teeth on the gear, owing to the difference in the helix angles which are used for the two members. This results in a pinion which is larger and stronger than that in a spiral bevel drive with the same speed ratio and the same size of gear.

(c) Surface contact is more nearly attained, thus permitting higher tooth pressures to be carried.

A considerable amount of sliding along the tooth elements is present in hypoid gearing. On this account the rubbing velocity of the teeth is higher than in other bevel gears, and hence inac-

curacy in mounting or lack of rigidity in mounting is more likely to result in failure of the teeth by scoring. To offset this tendency, special lubricants are generally recommended for hypoids. Nevertheless, the fact that this form of gearing has been adopted for truck drives by some of the larger manufacturers indicates that it is dependable and durable under heavy service.

GEAR TRAINS

32. Train Value. A mechanism which transmits motion from a driving to a driven shaft by use of two or more gear wheels is called a **gear train**. Problems involving the calculation of the velocity ratios of such trains will now be considered.

The **train value** is defined as the ratio

$$\frac{\text{Angular velocity of the last wheel (driven)}}{\text{Angular velocity of the first wheel (driver)}}$$

These velocities are measured in the ordinary gear train with reference to the fixed frame which supports the gear shafts.

A positive sign for the train value indicates that the first and last wheels turn in the same sense, whereas a negative sign is used to indicate rotation in the opposite sense.

It has been shown that the same general law for the velocity ratio applies to any pair of toothed gears, whether spur, helical, or bevel, etc. This law states that the velocity ratio for a pair of gears is the inverse ratio of the numbers of teeth. Hence the method of finding the train value in terms of the numbers of teeth is the same for all gear trains, no matter what variety or varieties of gears they contain.

33. A Simple Gear Train is one in which no two wheels are rigidly fastened to the same shaft so as to rotate at the same velocity. Figure 12-41 shows a train of this kind. Here motion is transmitted from *a* to *d* through intermediate wheels *b* and *c*.

By definition, the train value is $\frac{\omega_{dc}}{\omega_{ae}}$. The pitch circles of the gears roll together without slipping; therefore the pitch-line velocity is the same for all. It follows that wheel *d*, through contact with *c*,

will turn at the same rate as though it meshed with a . The sizes of intermediate wheels b and c , or the numbers of teeth they contain, evidently have no effect on the train value. For this reason, b and c are usually termed **idlers**.

This is somewhat of a misnomer, since these wheels transmit power as well as a and d .

If wheel c were removed from the train and d then driven from b , d would still have the same speed but in the reverse sense. Thus the **number of idlers** controls the sign of the train value.

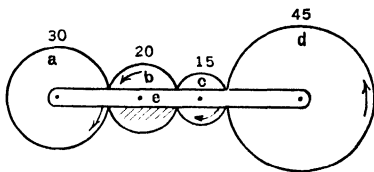


FIG. 12-41.

In view of the foregoing it is evident that the train value for a simple gear train is equal to the inverse ratio of the numbers of teeth on the first and last wheels. For the train of Fig. 12-41,

$$\frac{\omega_{dc}}{\omega_{ae}} = - \frac{N_a}{N_d} \quad (12-9)$$

where N_a and N_d are the numbers of teeth.

By inspection, wheels a and d are observed to rotate in opposite senses; this accounts for the negative sign. Substituting the numbers of teeth indicated in the figure,

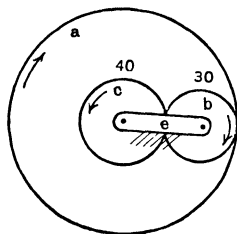


FIG. 12-42.

$$\frac{\omega_{dc}}{\omega_{ae}} = - \frac{30}{45} = - \frac{2}{3}$$

Figure 12-42 shows a simple gear train containing an annular or internal gear, driving an idler b which in turn drives wheel c . The train ratio is

$$\frac{\omega_{ce}}{\omega_{ae}} = - \frac{100}{40} = - 2\frac{1}{2}$$

The minus sign is used because the driving wheel turns clockwise when the driven wheel turns anticlockwise.

34. A Compound Gear Train is one in which at least one pair of wheels is rigidly attached to the same shaft so that both revolve

at the same angular speed. One of these trains is shown in Fig. 12-43. In this train the drive is through a , b , c , d , in order, and wheels b and c are keyed to the same shaft. To find the train ratio proceed as follows.

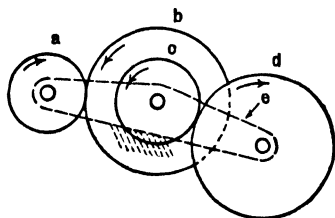


FIG. 12-43.

Considering wheels a and b ,

$$\frac{\omega_{be}}{\omega_{ae}} = \frac{N_a}{N_b} \quad (1)$$

Also, considering wheels c and d ,

$$\frac{\omega_{de}}{\omega_{ce}} = \frac{N_c}{N_d} \quad (2)$$

Multiplying (1) and (2),

$$\frac{\omega_{be}}{\omega_{ae}} \times \frac{\omega_{de}}{\omega_{ce}} = \frac{N_a \times N_c}{N_b \times N_d}$$

But,

$$\omega_b = \omega_c$$

since these wheels are keyed to the same shaft. Therefore,

$$\frac{\omega_{de}}{\omega_{ae}} = \frac{N_a \times N_c}{N_b \times N_d} \quad (12-10)$$

Calling the first wheel in each pair of meshing gears *a driver* and the second *a driven* wheel, write

$$\text{Train value} = \pm \frac{\text{product of numbers of teeth on drivers}}{\text{product of numbers of teeth on driven}}$$

The sign, as before, depends on whether rotation at the driven end of the train is the same or opposite to that at the driving end. Compound trains are often used where the speed reduction is large. In such cases a simple train with the same speed ratio might require the use of one very large gear.

35. Reverted Gear Trains. A gear train is said to be reverted when the first and last gears turn about the same axis. The gear trains in an automobile transmission which are in use on low, intermediate, or reverse are of this type. The first and last

wheels are co-axial, so that they can be coupled together when the car is in high. The back gears of the lathe form part of a reverted train. In Fig. 12-44 is shown a reverted train of four spur gears, b and c being keyed to the same shaft. The distance from center to center of the shafts is

$$R_a + R_b = R_c + R_d$$

If all wheels have teeth of the same pitch, the numbers of teeth are proportional to the pitch radii. Hence, if

$$R_a = C \times N_a$$

FIG. 12-44

then

$$R_b = C \times N_b \quad R_c = C \times N_c \quad R_d = C \times N_d$$

where C is a constant. Substituting these values in the above equation, we have

$$N_a + N_b = N_c + N_d \quad (12-11)$$

Example. A reverted gear train of four gears, arranged as shown in Fig. 12-44, has a train value of $\frac{1}{6}$. Wheel a has twenty teeth, wheel b forty teeth. Find the number of teeth for c and d , assuming that the pitch of the teeth is the same for all wheels.

The train value is

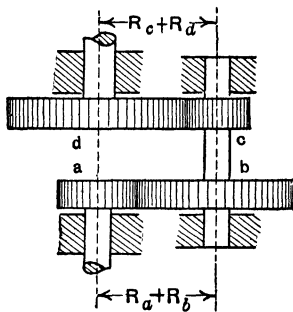
$$\frac{\omega_{de}}{\omega_{ae}} = \frac{1}{6} = \frac{N_a \times N_c}{N_b \times N_d} = \frac{20 \times N_c}{40 \times N_d}$$

Therefore,

$$\frac{N_c}{N_d} = \frac{1}{3} \quad \text{or} \quad N_c = \frac{N_d}{3} \quad (1)$$

Also, from equation (12-11),

$$\begin{aligned} N_a + N_b &= N_c + N_d \\ 20 + 40 &= N_c + N_d \end{aligned} \quad (2)$$



Equations (1) and (2) may be solved for N_c and N_d . We thus find $N_c = 15$, $N_d = 45$.

36. Sliding-gear Automobile Transmission. Figure 12-45 illustrates a common form of automobile transmission which provides

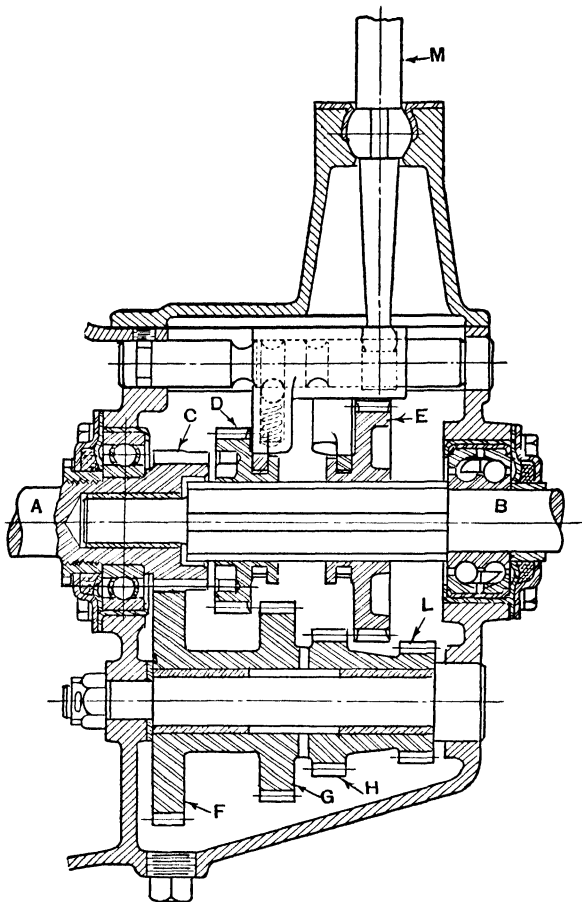


FIG. 12-45. Sliding-gear automobile transmission.

three speeds, forward, neutral, and reverse. The more important members consist of driving shaft *A* and co-axial driven shaft *B*,

sliding gears D and E which turn with B , and gears F, G, H, L , rigidly connected together and rotating on a lay shaft. The illustration shows the transmission in neutral position. Gears C and F are always in mesh, so that unit F, G, H, I is always in motion.

The gear system is controlled by lever M which slides gears D or E to right or left as desired. The transmission operates as follows:

(a) *Third Speed (direct drive)*. Gear D is moved to the left, the internal teeth on D engaging with C . Shafts A and B now rotate at the same speed.

(b) *Second Speed*. Gear D is shifted to the right, engaging gear G . The reverted gear train C, F, G, D causes B to rotate in the same sense as A but at a reduced speed.

(c) *First Speed*. Gear E is moved to the left, engaging H . The gear train C, F, H, E drives B in the same sense as A with a speed reduction of larger value than in the second speed position on account of the decrease in the ratio of tooth numbers, $N_H : N_E$ as compared with $N_G : N_D$.

(d) *Reverse*. Gear E is moved to the right, engaging an idler located behind the plane of section and meshing with L . This idler is not shown in the figure. Motion is now transmitted through C, F, L to the idler and through it to E . The addition of the idler causes B to rotate in a sense opposite to that of A .

37. Epicyclic or Planetary Gear Trains. In the ordinary gear trains already discussed, the wheels revolve about fixed axes, the frame supporting the wheels being the fixed link in the mechanism. In an epicyclic gear train, on the other hand, the **axes of certain of the wheels are in motion** and one of the gears becomes the fixed link. An ordinary gear train may be converted into an epicyclic train by fixing one of the wheels and causing the frame carrying the wheel axles to revolve. The epicyclic train of Fig. 12-46 has a stationary wheel a , and frame c revolves about the pin at A with the result that b rolls around on a .

What we often want to know about an epicyclic is the ratio of the angular velocity of the driven wheel to the angular velocity

of the frame carrying the wheel axes. In Fig. 12-46 this is $\frac{\omega_{ba}}{\omega_{ca}}$, both velocities being measured with respect to the fixed wheel. This quantity is called the **epicyclic value**.

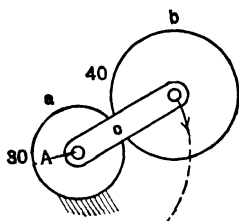


FIG. 12-46

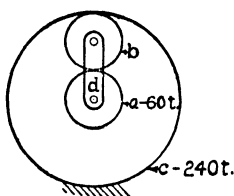


FIG. 12-47

38. Calculation of Epicyclic Value. The following procedure is useful in calculating the velocity ratio of epicyclic trains. It consists of two steps:

(1) The epicyclic train is converted into an ordinary train by locking the epicyclic arm on which certain gears are mounted and at the same time releasing the fixed gear. The gear formerly fixed is now rotated one turn clockwise and the number of turns made by other members is recorded.

(2) The gears are locked so that they can have no relative motion and the whole mechanism is rotated one turn counterclockwise. As a result each member of the train will make one turn counterclockwise.

The initial and final positions of the "fixed" gear are the same, hence the angular displacement of the other gears is the same as though the train had remained an epicyclic. From these angular displacements the velocity ratio of the train can be calculated.

Applying the above method to the epicyclic train of Fig. 12-47, (1) arm *d* is locked and gear *c* rotated one turn in a positive direction and then (2) the gears are locked and the whole mechanism rotated one turn in a negative sense. The results may be tabulated as follows:

Step	Turns			
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1	-4	-	+1	0
2	-1	-1	-1	-1
Total	-5	-	0	-1

Thus, while *d* makes -1 turn, *a* makes -5 turns and the ratio $\frac{\omega_d}{\omega_a}$ equals +5.

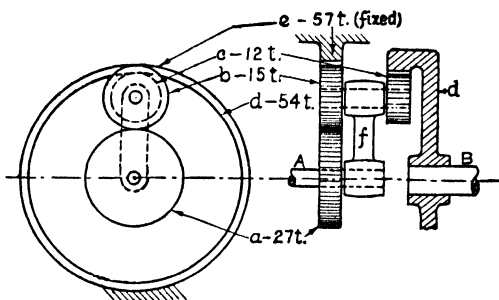


FIG. 12-48

In Fig. 12-48 is illustrated a compound epicyclic in which the epicyclic arm *f* carrying gears *b* and *c* is neither the driver nor driven link. Driver *a* engages gear *b* which in turn meshes with stationary annular wheel *e*. Gears *b* and *c* are keyed to a shaft supported by arm *f*, which is free to turn on shaft *A*. Gear *c* engages annular gear *d* which is keyed to driven shaft *B*.

The first step in finding the train ratio is to lock the arm *f* and rotate *e* one turn in a positive sense. The second step is to lock the gears and rotate the whole mechanism one turn in a negative sense, so as to return *e* to its initial position. The tabulation is as follows:

Step	Turns					
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
1	$-\frac{57}{27}$	$+\frac{57}{15}$	$+\frac{57}{15}$	$+\frac{57}{15} \times \frac{12}{34}$	+1	0
2	-1	-1	-1	-1	-1	-1
Total	$-\frac{28}{9}$	-	-	$-\frac{7}{45}$	0	-1

Thus while *a* makes $-\frac{28}{9}$ turns, *d* makes $-\frac{7}{45}$ turns. The speed ratio $\frac{\omega_d}{\omega_a} = (-\frac{7}{45}) \div (-\frac{28}{9}) = +\frac{1}{20}$. Hence while *a* makes twenty turns *d* makes one turn, in the same sense.

39. Speed Reducers. Where a reduction in the rate of revolution must be made between the prime mover and the driven machine, as for example where an electric motor is used to drive a slow speed machine, a speed reducer of the geared type shown in Fig. 12-49 is often employed as a substitute for belts, chains, or exposed gears.

The casing gives rigid support for the gears, protects them from dirt, and permits them to be operated in an oil bath. The drive is compact and efficient and requires little attention. The gear train may be composed of spur, helical, or worm gears arranged in several different ways, the design depending on the service and the ratio of reduction required. The speed reducer illustrated contains an epicyclic train, an annular gear being the fixed member.

40. Strength of Spur Gear Teeth. With perfectly designed and constructed gears the load necessary to produce the required torque on a gear would be equally divided among all the teeth in contact. Inaccuracies of cutting and mounting, however, make this condition very unlikely, and therefore it is customary to consider the entire load as being taken by one tooth. Figure 12-50 shows a tooth with the load acting at its tip, a condition which induces the maximum stress in the tooth. This load is applied along the common normal to the two conjugate teeth in contact and is shown in Fig. 12-50 as *F*. It can be resolved into the com-

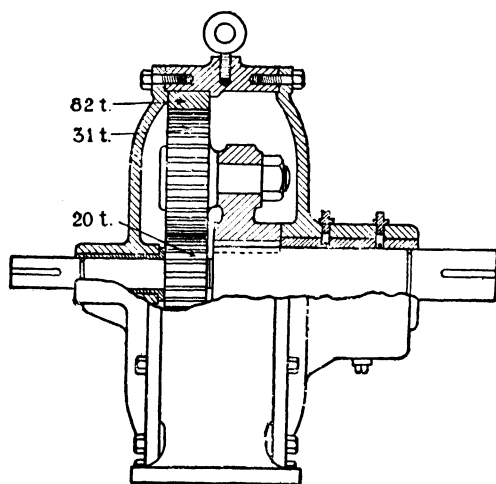


FIG. 12-49. Speed reducer with epicyclic gear train, speed ratio 1:5.1.

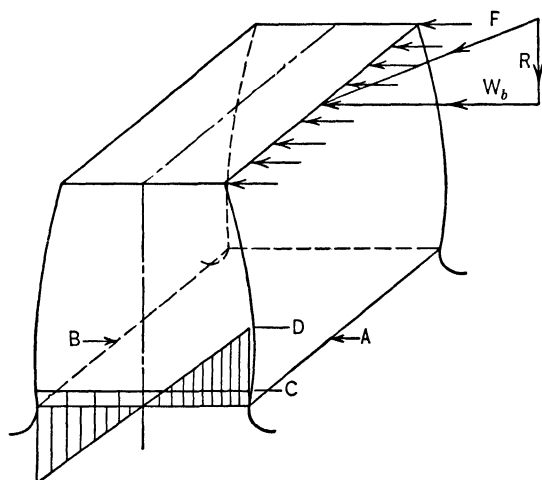


FIG. 12-50. Bending and direct compressive stresses induced in gear teeth.

ponents W_b and R , which act tangentially and radially to the periphery of the gear, respectively. The former produces bending in the tooth and induces tension at A and compression at B , whereas the latter produces compression across the entire root section of the tooth. The distribution of these stresses is shown by the ordinates to the lines D and C . The compression due to R increases the resultant stress at B , but decreases that at A . Since cast iron, for design purposes, is about $2\frac{1}{2}$ times as strong in compression as in tension, the presence of R actually strengthens a gear tooth when its material is cast iron.

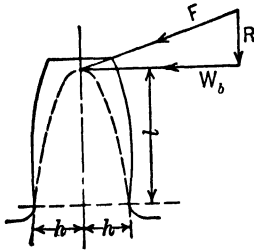


FIG. 12-51.

Mr. Wilfred Lewis has developed an equation applicable to the strength of spur gear teeth of involute and cycloidal outline wherein the tangential driving force W_b at the pitch line is determined in terms of the tooth shape and dimensions. The line of action of the normal force acting upon the gear tooth through the tip (severest loading case) is extended to meet the center line of the tooth shown in Fig. 12-51. From the intersection a parabola is drawn tangent to the sides of the tooth at the weakest section to form a beam of uniform strength equal to the effective strength of the tooth. Let

s = the flexural endurance limit in pounds per square inch

b = the breadth of the tooth in inches

p = the circular pitch

Then equating the external bending moment to the internal stress couple

$$M = W_b l = \frac{sI}{c} = \frac{sb(2h)^2}{6} = \frac{2}{3} sbh^2 = sbp \left(\frac{2}{3} \frac{h^2}{p} \right)$$

hence

$$W_b = sbp \left(\frac{2h^2}{3pl} \right) = sbp(y) \quad (12-12)$$

W_b , derived from this equation, is called the beam strength of the tooth. The factor y is a variable, dependent upon the proportions of the teeth, pressure angle, and number of teeth in the gear. Values for y for the standard $14\frac{1}{2}^\circ$ composite and the 20° stub system teeth are given in Table 12-4. Values of the allowable stress, s , are given in Table 12-5.

TABLE 12-4

Values of y

Number of Teeth	$14\frac{1}{2}^\circ$ Composite and Full Depth	20° Stub	Number of Teeth	$14\frac{1}{2}^\circ$ Composite and Full Depth	20° Stub
12	0.067	0.099	28	0.100	0.137
13	0.071	0.103	30	0.101	0.139
14	0.075	0.108	34	0.104	0.142
15	0.078	0.111	38	0.106	0.145
16	0.081	0.115	43	0.108	0.147
17	0.084	0.117	50	0.111	0.151
18	0.086	0.120	60	0.113	0.154
19	0.088	0.123	75	0.115	0.158
20	0.090	0.125	100	0.117	0.161
21	0.092	0.127	150	0.119	0.165
22	0.093	0.129	300	0.122	0.170
24	0.095	0.132	Rack	0.124	0.175
26	0.098	0.135			

The load that a gear tooth must sustain comes from two sources: first, the tangential force necessary to produce the torque to be transmitted by the gear; second, a load that results from inaccuracies in cutting the teeth. This latter load is also a direct function of the peripheral velocity of the gear. These two loads unite to make up the "dynamic" load imposed on the tooth. The value of this dynamic load is given by Buckingham⁴ as

$$W_d = \frac{0.05V(bc + W)}{0.05V + \sqrt{bc + W}} + W \quad (12-13)$$

⁴ Earle Buckingham, "Manual of Gear Design," Section Two.

where V = velocity at pitch line in feet per minute

b = width of face in inches

W = tangential force required to produce the required

$$\text{torque} = \frac{33,000 \times \text{horsepower}}{V}$$

C = a factor depending on the tooth system, the material of each gear, and the error in action. Values of C are given in Table 12-6.

TABLE 12-5

Values of s

Material	Brinell Hardness Number	Flexural Endurance Limit
Gray iron.....	160	12,000
Semi-steel.....	190	18,000
Phosphor bronze ..	100	24,000
Steel.....	150	36,000
Steel.....	200	50,000
Steel.....	240	60,000
Steel.....	280	70,000
Steel.....	320	80,000
Steel.....	360	90,000
Steel.....	400	100,000

The error of action for well-cut commercial gears may be taken as about 0.002 in. for gears of 6 diametral pitch or finer, and about 0.004 in. for gears of 2 diametral pitch. Figure 12-52 is a curve showing the maximum error of action compatible with reasonably quiet operation, for various pitch-line velocities.

As far as strength is concerned the width of face of a gear should not exceed four times the circular pitch, and three times the circular pitch is a safer limit. If the width of face exceeds these limits, the load may become concentrated on one corner of the tooth, owing to inaccuracies of cutting or mounting, and cause failure of the tooth by breaking off a corner. Wider gears are sometimes made in an attempt to reduce wear through a reduction of the

TABLE 12-6

Values of C

Material	Tooth Form	Error of Action, Inches					
		0.0005	0.001	0.002	0.003	0.004	0.005
Cast iron and cast iron.	$14\frac{1}{2}^\circ$	400	800	1600	2400	3200	4000
Cast iron and steel. . . .	$14\frac{1}{2}^\circ$	550	1100	2200	3300	4400	5500
Steel and steel.	$14\frac{1}{2}^\circ$	800	1600	3200	4800	6400	8000
Cast iron and cast iron.	20° stub	430	860	1720	2580	3440	4300
Cast iron and steel. . . .	20° stub	590	1180	2360	3540	4720	5900
Steel and steel.	20° stub	860	1720	3440	5160	6880	8600

Formula. For $14\frac{1}{2}^\circ$ tooth form $C = \frac{0.107 \times e}{\frac{1}{E_1} + \frac{1}{E_2}}$

For 20° stub tooth form $C = \frac{0.115 \times e}{\frac{1}{E_1} + \frac{1}{E_2}}$

where e = error of action in inches

E_1 and E_2 = moduli of elasticity of materials composing the gears.

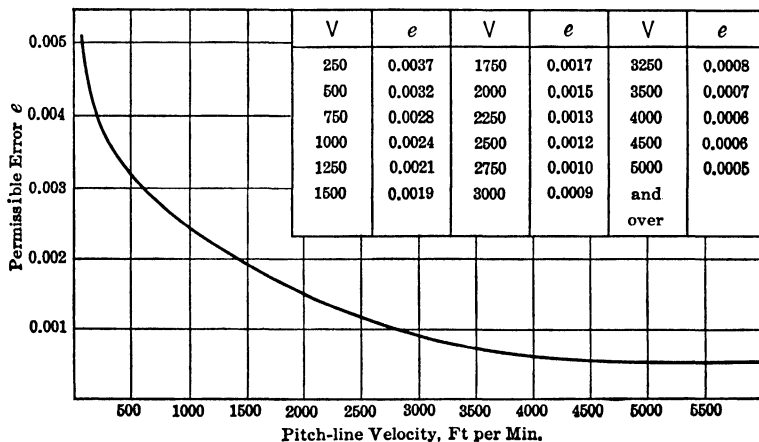


FIG. 12-52

unit bearing pressure, but the additional width should be considered as contributing nothing to the strength of the tooth.

For safety against breakage the beam strength of the tooth, W_b , as determined from equation (12-12), using the allowable stress given for the material in Table 12-5, should be somewhat greater than the dynamic load, given by equation (12-13). Buckingham recommends the following relationship:

$$\text{For steady loads } W_b = 1.25 W_d$$

$$\text{For pulsating loads } W_b = 1.35 W_d$$

$$\text{For shock loads } W_b = 1.50 W_d$$

In addition to being able to withstand the dynamic load without breaking, the teeth must be able to do so without undue wear. The load a gear tooth can carry without undue wear is fixed by the surface fatigue strength of the material and the number of applications of the load. The limiting load, on the basis of wear, is given by Buckingham as

$$W_w = DbKQ \quad (12-14)$$

where D = diameter of pinion, inches

b = width of face, inches

$$K = \text{load-stress factor} = \frac{(s_c)^2 \sin \phi}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2} \right)$$

(see Table 12-7)

$$Q = \text{ratio factor} = \frac{2N_g}{N_g + N_p} \text{ for spur gears}$$

$$= \frac{2N_g}{N_g - N_p} \text{ for internal gears}$$

ϕ = pressure angle of gear

N_g = number of teeth on gear

N_p = number of teeth on pinion.

In order that the gear may have satisfactory life the value of W_w , as given by equation (12-14), should be equal to or greater than the dynamic load W_d , as given by equation (12-13).

TABLE 12-7
Values of K

Pinion Material	Brinell Number	Gear Material	Brinell Number	s_c	$K_{14\frac{1}{2}^\circ}$	K_{20°
Steel.....	150	Steel.....	150	50,000	30	41
Steel.....	200	Steel.....	150	60,000	43	58
Steel.....	250	Steel.....	150	70,000	58	79
Steel.....	200	Steel.....	200	70,000	58	79
Steel.....	250	Steel.....	200	80,000	76	103
Steel.....	300	Steel.....	200	90,000	96	131
Steel.....	250	Steel.....	250	90,000	96	131
Steel.....	300	Steel.....	250	100,000	119	162
Steel.....	350	Steel.....	250	110,000	144	196
Steel.....	300	Steel.....	300	110,000	144	196
Steel.....	350	Steel.....	300	120,000	171	233
Steel.....	400	Steel.....	300	125,000	186	254
Steel.....	350	Steel.....	350	130,000	201	275
Steel.....	400	Steel.....	350	140,000	233	318
Steel.....	400	Steel.....	400	150,000	268	366
Steel.....	150	Cast iron.....		50,000	44	60
Steel.....	200	Cast iron.....		70,000	87	119
Steel.....	250	Cast iron.....		90,000	144	196
Steel.....	150	Ph. bronze.....		50,000	46	62
Steel.....	200	Ph. bronze.....		70,000	91	124
Steel.....	250	Ph. bronze.....		85,000	135	204
Cast iron.....		Cast iron.....		90,000	193	284

Hardened Steel (10,000,000 Repetitions)

Brinell	$K_{14\frac{1}{2}^\circ}$	K_{20°
450	421	575
500	525	718
550	647	884
600	775	1058

The discussion just given applies to metallic gears. Those made of phenolic laminated materials behave quite differently. The modulus of elasticity of these materials is so low that small errors in tooth form and spacing have little effect on the strength of the gears. Wear caused by the corner of the mating tooth seems to be the principal cause of failure. This can best be met by making the teeth of such a form as to consist entirely of addendum.

The beam strength of phenolic gears is given by the expression

$$W_b = 6000 \left(\frac{150}{200 + V} + 0.25 \right) byp \quad (12-15)$$

Phenolic gears should not be used where the starting torque is high, or where the loads fluctuate greatly or where shock loads may be encountered. Nor are they recommended for pitch-line velocities below 600 ft per minute. For best results they should be mated with steel pinions having a Brinell hardness of 400 or more. Cast iron works quite well with them, but bronze and soft steel are unsatisfactory.

Example. Two shafts 12 in. apart transmitting 25 hp are to be connected by a steel pinion having a Brinell hardness of 250 meshing with a cast iron gear. The velocity ratio is to be 3 to 1 and the smaller gear is to run at 600 rpm. The teeth are to be of $14\frac{1}{2}^\circ$ full depth involute form and are to be cut with sufficient accuracy to insure quiet action. Determine the diametral pitch of the teeth.

Solution. Let D_1 and D_2 be the diameters of the smaller and larger gears, respectively. Then

$$\frac{D_1}{D_2} = \frac{200}{600}$$

and

$$D_1 + D_2 = 2 \times 12$$

Therefore,

$$D_1 = 6 \text{ in.}$$

and

$$D_2 = 18 \text{ in.}$$

Since the 18-in. gear is made of the weaker material the design for

strength will be based on it.

$$\text{Velocity at pitch line} = \pi DN = \frac{\pi \times 6 \times 600}{12} = 942 \text{ ft per min}$$

$$\text{Tangential force, } W = \frac{25 \times 33,000}{942} = 875 \text{ lb}$$

Assume diametral pitch = 3

Number of teeth on 18-in. gear = $18 \times 3 = 54$

From Table 12-4, $y = 0.112$

From Table 12-5, $s = 12,000$

$$\text{Assume width of face, } b = 3.5p = \frac{3.5\pi}{3} = 3.68 \text{ in., say } 3\frac{3}{4} \text{ in.}$$

$$\begin{aligned} \text{Beam strength, } W_b &= sbpy = 12,000 \times 3.75 \times \frac{\pi}{3} \times 0.112 \\ &= 5280 \text{ lb} \end{aligned}$$

For quiet operation the permissible error of action is found from Fig. 12-52 to be 0.0025 in., hence $C = 2750$ (Table 12-6).

$$\begin{aligned} \text{Dynamic load } W_d &= \frac{0.05 \times V(bC + W)}{0.05 \times V + \sqrt{bC + W}} + W \\ &= \frac{0.05 \times 942(3.75 \times 2750 + 875)}{0.05 \times 942 + \sqrt{3.75 \times 2750 + 875}} + 875 \\ &= 3440 + 875 = 4315 \text{ lb} \end{aligned}$$

$$\text{Ratio } \frac{W_d}{W_b} = \frac{4315}{5280} = 1.22 \quad (\text{This is satisfactory.})$$

$$\text{Number of teeth on pinion} = 6 \times 3 = 18$$

$$Q = \frac{2N_g}{N_g + N_p} = \frac{2 \times 54}{54 + 18} = 1.5$$

Load-stress factor $K = 144$ (Table 12-7)

$$\begin{aligned} \text{Wear limit load } W_w &= DbQK \\ &= 6 \times 3.75 \times 144 \times 1.5 = 4840 \text{ lb} \end{aligned}$$

(This is satisfactory.)

41. Design of Arms, Rim, and Hub. The thickness of the rim should be such as to insure rigidity great enough to prevent the action of the teeth from being interfered with by reason of their springing out of position. Professor C. D. Albert gives as a suitable value for the thickness of the rim:

$$t = \frac{1}{P} \sqrt{\frac{N}{4n}} + 2.5 \quad (12-16)$$

where n = number of arms

N = number of teeth

P = diametral pitch.

The dimensions of the arms may be determined by considering each as a cantilever having a length equal to the pitch radius of the gear. The bending moment on each arm may be taken as equal to the torque acting on the gear divided by the number of arms. The maximum allowable stress may be taken as the ultimate strength of the material divided by a suitable factor of safety and the section at the center line of the hub may be determined. This assumes the rim to be flexible, which is in error on the side of safety. The arm section at the rim may be determined by giving the arm a suitable taper, about $\frac{1}{4}$ in. per ft on the greater dimension of the section. Gear arms are usually elliptical in cross section, the major axis of the ellipse being twice the minor, and placed in the plane of rotation.

Example. Assume the gear whose teeth were designed in Art. 40, to have six arms, and determine the dimensions of the arm cross section if the arms were extended to the center of the hub. The ultimate strength of the material is 24,000 lb per sq in., and the factor of safety is 4.

Solution.

Bending moment per arm

$$= \frac{25 \times 33,000 \times 12}{2\pi \times 200 \times 6} = 1310 \text{ lb-in.}$$

Allowable stress in the arm

$$= \frac{24,000}{4} = 6000 \text{ lb per sq in.}$$

Major axis of arm at center of wheel

$$= H = \sqrt[3]{\frac{64M}{\pi s}} = \sqrt[3]{\frac{64 \times 1310}{\pi \times 6000}} = 1.641 \text{ in.}; \text{ say } 1\frac{5}{8} \text{ in.}$$

Minor axis of arm at center of wheel

$$= \frac{H}{2} = \frac{1\frac{5}{8}}{2} = \frac{13}{16} \text{ in.}$$

Major axis at pitch line

$$= 1\frac{5}{8} - \frac{9.00}{12} \times \frac{1}{4} = 1\frac{7}{16} \text{ in.}$$

Minor axis at pitch line

$$= \frac{1\frac{7}{16}}{2} = \frac{23}{32} \text{ in.}$$

An old standard rule is to make the diameter of the hub twice that of the bore. This leads to a rather heavy hub, and the R. D. Nuttall Company recommends making the diameter of the hub 1.8 times the diameter of the bore. It also recommends making the length of the hub 1.25 times the diameter of the bore, unless the width of face exceeds this value. In such cases the length of hub is made equal to the width of face.

PROBLEMS — CHAPTER XII

1. A gear with standard $14\frac{1}{2}^\circ$ involute teeth has a pitch diameter of 6 in. and a diametral pitch of 4. Calculate the following: (a) number of teeth, (b) addendum, (c) working depth, (d) clearance, (e) root diameter, (f) outside diameter, (g) base-circle diameter.

2. Same as Prob. 1 but teeth are of standard A.G.M.A. stub tooth form.

3. Same as Prob. 1 except that gear diameter is 4 in. and diametral pitch is 5.

4. A gear has 20 teeth of 2-in. circular pitch, the teeth being of standard $14\frac{1}{2}^\circ$ involute form. Calculate (a) the pitch diameter, (b) the addendum, (c) the clearance, (d) the working depth, (e) the root diameter, (f) the outside diameter, (g) the base-circle diameter.

5. Two shafts 10 in. apart are to be connected by spur gears with external teeth, one shaft running at 400 rpm, and the other at 600 rpm. Find the pitch

diameters of the gears and the number of teeth if the diametral pitch is 4. What is the value of the circular pitch?

Ans. 12 in., 8 in., 48, 32, 0.7854 in.

6. *A* and *B* are two mating spur gears. *A* has 30 teeth of 3 diametral pitch and *B* has a pitch diameter of 15 in. The teeth are of standard $14\frac{1}{2}^\circ$ involute form. Find (a) number of teeth on *B*; (b) center-to-center distance of shafts; (c) circular pitch; (d) addendum; (e) dedendum; (f) root diameter of *A*; (g) outside diameter of *A*; (h) diameter of base circle of *A*; (i) speed ratio of *A* to *B*.

7. Spur gear *A* has 20 teeth of 2-in. circular pitch and meshes with gear *B* having 28 teeth. Find the center-to-center distance of the shafts and the speed ratio.

Ans. 15.27 in., 1.4.

8. Shaft *A* carries a spur gear with internal teeth. Its pitch diameter is 20 in. and the diametral pitch is 3. Shaft *B* carries a mating gear with twelve teeth. Find the center-to-center distance and the speed ratio of the shafts.

9. A pair of gears having teeth of 20° stub involute form, make internal contact. The shafts are 10 in. from center to center, and the angular velocity ratio is 3 : 1. The diametral pitch is 4. Find (a) pitch diameters of the gears; (b) number of teeth on each gear; (c) addendum for each gear; (d) dedendum for each gear; (e) diameter of base circle for the pinion.

10. What is an annular gear? What relationship exists between the teeth of a spur gear and those of an annular gear of an equal pitch and diameter?

11. A pair of spur gears *a* and *b* have internal contact on *b*. The number of teeth on *a* is 20, and the angular velocity ratio of *a* to *b* is 5 : 2. The center-to-center distance is $7\frac{1}{2}$ in. Teeth are $14\frac{1}{2}^\circ$ involute form. Find (a) number of teeth on *b*; (b) pitch diameter of *a* and *b*; (c) diametral pitch; (d) outside diameter of *a*; (e) height of teeth.

12. A helical gear of 6-in. pitch diameter has an angle of cut of 25° and there are 24 teeth. Find the values of the circular and diametral pitches measured both circumferentially and normal to the teeth.

Ans. $P = 4$; $p = 0.7854$ in.; $P' = 4.42$; $p' = 0.712$ in.

13. A helical gear of 10-in. pitch diameter has a normal diametral pitch of 4. The angle of cut is 20° . Find the normal circular pitch, also the diametral and circular pitches measured circumferentially.

14. A helical gear of 10-in. pitch diameter has an angle of cut of 30° , and there are 30 teeth. Find values of the circular and diametral pitches measured both circumferentially and normal to the teeth.

15. A helical gear with 26 teeth, pitch diameter 6 in., has a normal diametral pitch of 5. Find the angle of cut.

16. A helical gear of 8-in. pitch diameter has 47 teeth, the angle of cut being 21° . Find the normal circular pitch.

17. A pair of helical gears are used to connect two shafts at 60° . One gear has a pitch diameter of 8 in. and an angle of cut of 35° . Its angular velocity is 1.1 times that of the other. Find for the second gear (a) angle of cut, (b) pitch diameter.

Ans. (a) 25° ; (b) 7.94 in.

18. Two helical gears a and b are mounted on shafts intersecting at an angle of 75° . Gear a has a pitch diameter of 8 in., and b of 12 in. Gear a has an angle cut of 30° . Find the speed ratio of the gears.

19. A quadruple-threaded worm has two threads per inch. When it is mated with a certain worm wheel the angular velocity ratio is 20 : 1. Determine the pitch diameter and number of teeth on the worm wheel.

20. A worm gear consists of a worm with triple thread driving a worm wheel with 48 teeth, to which is attached a 24-in. pulley. The worm is driven at 900 rpm. Find the linear speed at the pulley face.

Ans. 353 ft. per min.

21. A worm has a lead of $\frac{1}{4}$ in. and meshes with a worm wheel of 8-in. pitch diameter. Determine the number of turns required on the worm shaft to revolve the worm wheel one turn.

22. A worm has a pitch diameter of 2 in. and is triple threaded. It meshes with a worm wheel of 12-in. pitch diameter and the speed ratio is 30 : 1. What is the axial pitch of the worm?

23. A double-threaded worm has an axial pitch of $\frac{1}{2}$ in. and meshes with a worm wheel with a pitch diameter of 6 in. Find the speed ratio.

24. In a worm gear drive the worm wheel is 12.73-in. pitch diameter and is driven by a worm which is double threaded. The axial pitch of the worm is $\frac{1}{2}$ in. If the worm rotates at 800 rpm what is the speed of the worm wheel?

25. A pair of bevel gears connecting two shafts at 90° have a speed ratio of 3 : 2. Find the center angle for both. If the larger wheel has 30 teeth of 4 diametral pitch, find the pitch diameter and the number of teeth on the smaller gear.

Ans. $33^\circ.40'$, $56^\circ.20'$, 5 in., 20.

26. A pair of bevel gears used to connect shafts at 90° have a speed ratio of 4.5 : 1. The smaller gear has a 2-in. pitch diameter, and the teeth have a diametral pitch of 5. The teeth are of standard stub involute form. Find the pitch diameter and number of teeth on the larger gear, also the center angles of both gears. What are the values of the addendum and whole depth of the teeth?

27. The back gear train of a lathe is made up as follows. Wheel a on the cone pulley with 16 teeth drives b with 72 teeth. Wheel c is rigidly connected to b and drives d , connected to the live spindle of the machine. The train value is $1 : 13\frac{1}{2}$. Find the numbers of teeth on c and d .

28. When an auto is in intermediate gear, the gear train in use is composed of the following gears, the first one being directly connected to the engine:

24 teeth

41 teeth — 34 teeth

31 teeth — 11-tooth bevel pinion

56-tooth bevel gear

(a) Find the ratio of the rear-axle speed to the engine speed.

(b) If the tires are 30 in. in diameter, find the engine speed corresponding to a car speed of 20 mph.

Ans. (a) 1 : 7.93; (b) 1775 rpm.

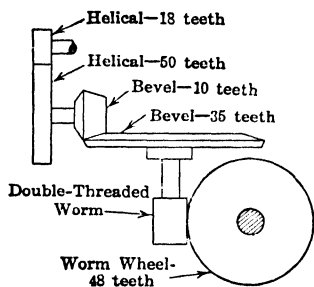
29. An epicyclic train consists of a simple train of three wheels, a , b , and c , carried on a frame e . Wheel a is fixed and e turns clockwise at 75 rpm. Wheel a has 60 teeth; b , 20 teeth; c , 30 teeth. Find the speed and sense of the motion of c .

Ans. 75 rpm counterclockwise.

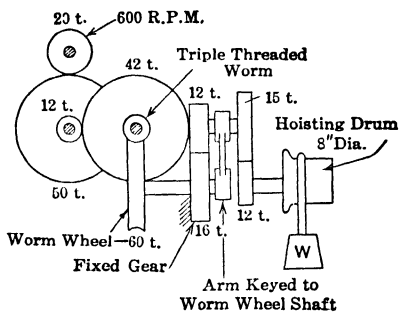
30. An epicyclic gear consists of a simple gear train of two wheels, a and b , carried on a frame e . Wheel a is fixed, b turns counterclockwise at 100 rpm. Wheel a has 40 teeth; b , 50 teeth. Find the speed and direction of motion of e .

Ans. 55.56 rpm counterclockwise.

31. An epicyclic train is composed of a fixed annular wheel a , with 150 teeth. Meshing with a is a wheel b which drives wheel d through an idler c , d being concentric with a . Wheels b and c are carried on an arm which revolves clockwise at 100 rpm. Wheel b has 25 teeth; c , 30 teeth; d , 40 teeth. Find the speed and sense of rotation of d .

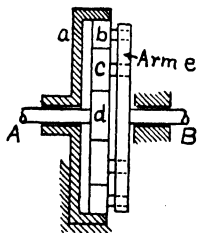


PROB. 34



PROB. 35

32. An epicyclic train contains a fixed annular wheel a , with 200 teeth. Meshing with a is wheel b with 90 teeth, which drives c with 20 teeth. Wheel c is concentric with a . Wheel b is carried on an arm e which revolves about the axis of a . Wheel c is the driver, and it rotates at 100 rpm clockwise. Find the speed of arm e .



PROB. 36

33. Make the calculation for the speed ratio of the epicyclic train shown in Fig. 12-49.

34. Find the speed ratio obtained by use of the gear train shown in the accompanying figure.

35. Determine the rate at which the weight W is raised by use of the gear train shown in the accompanying figure.

36. In the epicyclic gear train shown in the accompanying figure, shaft A is rotated at 350 rpm in a clockwise sense. Gear b has 20 teeth; c has 28 teeth; and d , 35 teeth. Gears b and c are attached to arm e . Find the number of teeth on the fixed gear a and the speed and sense of rotation of shaft B .

37. A cast iron gear 8 in. in diameter rotates at 100 rpm and meshes with one 40 in. in diameter. It is to transmit 35 hp. Assume the width of face to be three times the circular pitch, and the error of action to be 0.004 in. Determine the diametral pitch for the gear if the teeth are of $14\frac{1}{2}^\circ$ composite form.

Ans. $P = 1\frac{1}{2}$ diametral pitch; $b = 6\frac{1}{4}$ in. width of face.

38. A spur gear which has 15 teeth and rotates at 1200 rpm meshes with a gear having 30 teeth, and transmits 10 hp. Both gears are to be made of hardened alloy steel of Brinell hardness number 400, and are to be of the $14\frac{1}{2}^\circ$ full involute form. The width of face is to be about three times the circular pitch. Assume the error in action is 0.002 in. Determine the diametral pitch, width of face, and pitch diameter for the pinion.

39. A cast steel pinion having 20 teeth of 5 diametral pitch has a face width of 2 in. The Brinell hardness is 250. It meshes with a cast iron gear having 100 teeth. The teeth are of the $14\frac{1}{2}^\circ$ involute form. Determine the horsepower the pair can transmit when the pinion rotates at 250 rpm, if the error of action is 0.0025 in.

Ans. 4.5.

40. The eccentric shaft of a punch is driven by a cast iron gear having 105 teeth. The gear must transmit a maximum torque of 58,600 lb-in. when rotating at 30 rpm. It meshes with a 21-tooth steel pinion having a Brinell hardness of 250. The width of face is three times the circular pitch. Determine the diametral pitch required and the pitch diameter for the gear if of $14\frac{1}{2}^\circ$ involute type for an error of action of 0.004 in.

Ans. $P = 2\frac{1}{2}$; $D = 42$ in.

41. A cast iron spur gear having a diameter of 15 in. rotates at 480 rpm and is to transmit 20 hp. It meshes with a steel pinion having a diameter of 5 in. Assume the width of face to be three times the circular pitch and determine the diametral pitch, number of teeth, and width of face for the gear, and also the Brinell hardness for the pinion, if the teeth are of the $14\frac{1}{2}^\circ$ composite system and are to give quiet operation. The allowable stress is 3300 lb per sq inch.

Ans. $P = 3$; $N_g = 45$; $N_p = 15$; $b = 3\frac{3}{8}$; $H_b = 250$.

42. The gear described in Prob. 40 is to have six arms. Determine the dimensions for the arms and the thickness for the rim.

Ans. At hub center: major axis 3.91, say 4 in.; at pitch line: major axis, say $3\frac{1}{2}$ in.; rim thickness, say 1 in.

CHAPTER XIII

BELTS AND CHAINS

Whenever it is necessary to transmit power between shafts whose axes are too far apart to make the use of gears feasible, a wrapping connector of some sort is employed.

1. Types of Wrapping Connectors. There are two general types of wrapping connectors:

- (a) Those that drive through the medium of friction.
- (b) Those whose drive is positive.

The first type is probably the more common and includes all forms of belt and rope drives. Owing to the slip and creep of such connectors, the angular velocity ratio of the driving and driven members is not constant and is only approximately equal to the inverse ratio of the pulley diameters. The second type includes all types of conveyor and power transmission chains. There is no slip or creep with this type of connector. The drive is positive, and the angular velocity ratio of the driving and driven shafts is equal to the inverse ratio of the sprockets over which the connector runs. This property is a decided advantage in such work as driving the distributor of the ignition system of an automobile. In other cases, such as machine tool work, it is a decided disadvantage, as stalling the tool will probably result in breakage of some part of the machine. When the drive is by means of a belt the belt will slip on the pulley and prevent damage to the machine.

The individual motor drive was at one time expected to supersede the belt drive. More recent developments, however, indicate that the belt drive is still, and will continue to be, an important means of power transmission, although it does not occupy so large a place in our industrial field as formerly. Its importance is

especially prominent when belting is used with the electric motor in the so-called "group" system drive.

2. Speed Ratio in Belt Drives. We shall assume that no slip-page takes place and that the material is inextensible. Neither of these assumptions is correct in practice and therefore our calculated theoretical speed must be modified to take care of these factors.

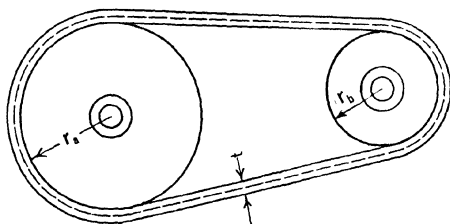


FIG. 13-1. Open belt drive.

In Fig. 13-1 we have a belt drive in which r_a , r_b are the radii of the driving and driven pulleys. Let V = belt speed. Then, neglecting belt thickness t

$$\omega_a = \frac{V}{r_a} \quad \text{and} \quad \omega_b = \frac{V}{r_b}$$

Therefore

$$\frac{\omega_b}{\omega_a} = \frac{r_a}{r_b} \quad (13-1)$$

When the speed of a driven shaft is calculated from that of the driver by the above equation, the result is known as the **theoretical speed**. The **actual speed** of the driven shaft will always be somewhat less, the difference being due to "slip," which in practice amounts to 2 to 4 per cent of the theoretical speed. Thus

$$\text{Actual rpm} = \text{theoretical rpm} \left(1 - \frac{\text{per cent slip}}{100} \right) \quad (13-2)$$

A belt of given size will transmit maximum power if run at a linear speed of 4000 to 5000 ft per min. It is therefore desirable

to design the drive so as to keep within this range, unless the rpm of the driver is low, in which event a lower belt speed may be necessary in order to avoid the use of large and expensive pulleys.

Exact speeds may be unobtainable if stock pulleys are to be used because such pulleys are made only in certain sizes, usually of even-inch diameters. A close approximation is secured by proper selection.

Example. A line shaft is to be driven at 500 rpm from an electric motor turning at 1750 rpm. The belt speed should be about 4000 ft per min. Provide for a slip of 4 per cent. Find suitable diameters of stock pulleys, which are obtainable in even-inch diameters.

Solution.

$$\text{Belt speed} = \pi D_a N_a$$

where D_a = driver diameter and N_a = rpm.

$$4000 \times 12 = \pi D_a \times 1750$$

or

$$D_a = \frac{4000 \times 12}{\pi \times 1750} = 8.75 \text{ in.}$$

Also,

$$\text{Theoretical rpm} = \frac{\text{actual rpm}}{1 - \text{slip}} = \frac{500}{1 - 0.04} = 521$$

Hence, by equation (13-1),

$$\frac{D_a}{D_b} = \frac{521}{1750} \quad \text{or} \quad D_b = D_a \times 3.36$$

If

$$D_a = \quad 7 \quad \quad 8 \quad \quad 9 \quad \quad 10 \text{ in.}$$

then

$$D_b = \begin{array}{cccc} 23.5 & 26.8 & 30.2 & 33.6 \text{ in.} \\ (23) & (27) & (30) & (34) \end{array}$$

Final choice might be influenced by relative cost of pulleys, but, considering desired belt and driven pulley speeds, the values

$D_a = 8$ in. and $D_b = 27$ in. should be selected as giving closest approximation to requirements.

3. Crossed and Open Belt Drives. The drive of Fig. 13-1, in which both pulleys turn in the same sense, is known as an open belt drive. In Fig. 13-2 is shown the crossed belt drive; here the pulleys turn in opposite senses.

Both forms give satisfactory service, though the crossed drive tends to wear out sooner on account of the rubbing action where the belt crosses itself. This is more pronounced when the belt is wide and

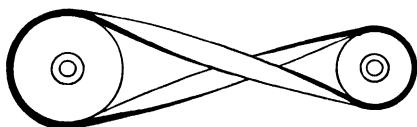


FIG. 13-2. Crossed belt drive.

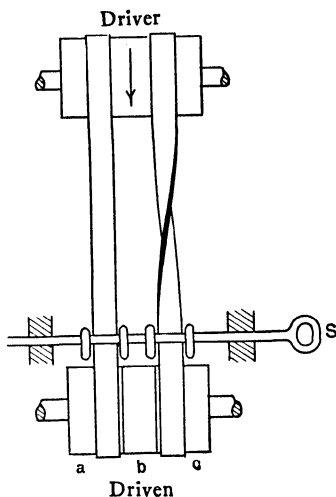


FIG. 13-3

the drive is short. The larger arcs of contact between belt and pulleys in the crossed drive are advantageous.

A combination consisting of one open and one crossed belt in conjunction with loose pulleys (see Fig. 13-3) is used as a means of driving a machine in either direction, or of stopping its motion. The belts are moved sidewise by means of a belt shifter, *S*. The belts run on loose pulleys *a* and *c* when the shifter is in its mid-position; then the driven shaft is stationary. Moving the shifter to the right throws the open belt on the fast pulley *b*, whereas moving it to the left brings the crossed belt on the pulley *b*. The motion of the driven shaft may, therefore, be clockwise or counter-clockwise, depending on the position of the shifter. The shifter should act on the slack side of the belt, near the driven pulley. This drive is quite common in connection with machine tools.

4. Crowned and Flat Pulleys. A crowned pulley has a larger diameter at the middle of the face than at the ends, whereas a

flat pulley has the same diameter throughout. When flat pulleys are used, a very slight misalignment of the shafts or a small defect in the form of the belt will cause the belt to run off the pulleys. This can be avoided by means of a belt guide which has arms to hold the belt in the required position, by flanging the pulleys, or by using crowned pulleys. Guides and flanges cause wear on the edges of the belt; hence crowned pulleys are preferable.

The reason why the crown pulley causes the belt to run centrally may be seen by reference to Fig. 13-4. Here the belt is shown in a

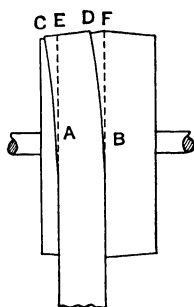


FIG. 13-4



FIG. 13-5



FIG. 13-6

position at one side of the pulley. The belt tends to stretch more on the side *BD* than on the side *AC*; it therefore assumes a curved form. As the pulley rotates, section *AB* will move along the paths indicated by the broken lines *AE* and *BF*. The belt therefore runs toward the central plane of the pulley and takes up a position where its center line coincides with the ridge of the crowned surface.

The taper on a crown pulley varies from about $\frac{3}{4}$ in. (on the diameter) per foot of width of face on narrow pulleys to $\frac{1}{4}$ in. per foot of width on wide ones. The taper may be uniform as shown in the rim section of Fig. 13-5, or the profile may be a circular arc as in Fig. 13-6.

The face of the pulley is usually a little wider than the belt in order to prevent overhang if the belt is not running exactly central.

5. Non-parallel Shaft Drives. Quarter-twist Drives. Belts are most commonly employed for connecting parallel shafts, but they may be used to connect non-parallel shafts as well. In any belt

drive the following conditions must be observed in regard to the location of the pulleys in order to keep the belt from running off.

The belt must leave each pulley in the central plane of the pulley face toward which it moves.

A drive connecting two shafts at 90° is known as a **quarter-twist** belt drive. One form is shown in Fig. 13-7. The pulleys are located to conform to the law just stated. Note the plan view of the drive, in which the pulleys intersect at *A*, the mid-point of both faces.

Evidently the belt can be run only in the direction indicated by the arrows. The point *X*, where the belt leaves the upper pulley, is in the plane *XY* passing through the middle of the lower pulley; also, point *R*, where the belt leaves the lower pulley, is in the plane *RS* passing through the middle of the upper pulley. This form of drive can also be used for connecting shafts at any angle between 0° and 180° . When the angle is 0° we have an ordinary open belt drive; at the other extreme it becomes a crossed belt drive.

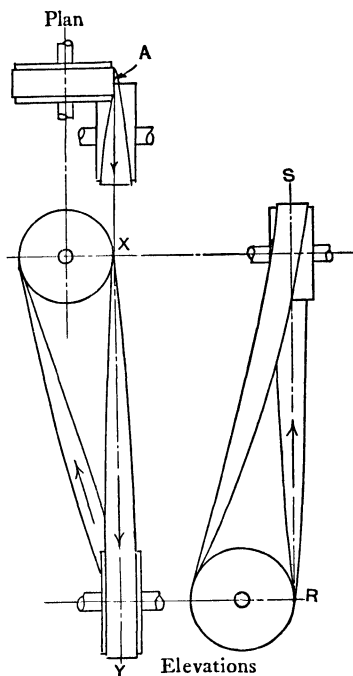


FIG. 13-7. Quarter-twist belt drive.

An alternative variety of quarter-twist drive is shown in Fig. 13-8. This has a guide pulley, or mule pulley, on the slack side of the drive. The mule pulley incidentally increases the arc of contact of the driving pulley; its main object is to bring the belt leaving this pulley into the plane passing through the middle of the driven pulley face. In this drive, a uniform tension is obtained across the belt section on the tight side, since the center of the belt is always in the plane passing through the middle of the driven

pulley. This is not true of the drive of Fig. 13-7. By tilting the forward end of the mule-pulley shaft upward to the proper angle, the tension can be made uniform across the section on the slack side also. This drive is suitable for rotation in one direction only, because reversal would put the guide pulley on the tight side of the belt.

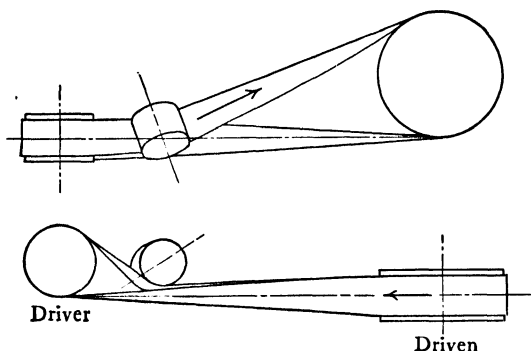


FIG. 13-8

Quarter-twist belts should be avoided wherever possible, as the wear on such belts is excessive. They give fairly good service with narrow belts on long drives.

6. Calculation of Belt Length. When the center-to-center distance of the shafts and the pulley diameters is known the approximate belt length required for the drive can be calculated.

(a) **Open belt drive.** (Rankine's Equation.)

$$\text{Belt length} = 2L + \frac{\pi}{2} (D_a + D_b) + \frac{(D_a - D_b)^2}{4L} \quad (13-3)$$

(b) **Crossed belt drive.**

$$\text{Belt length} = 2\sqrt{L^2 + \left(\frac{D_a + D_b}{2}\right)^2} + \frac{\pi}{2} (D_a + D_b) \quad (13-4)$$

D_a and D_b are the pulley diameters. L = center-to-center distance.

These equations are approximate, but are easier to use than the exact equations, and are found to give an error of less than one

per cent with a drive of usual proportions. It should be noted that **when the drive uses a crossed belt**, the belt length remains constant **provided that the sum of the pulley diameters is constant and the center-to-center distance is unaltered.**

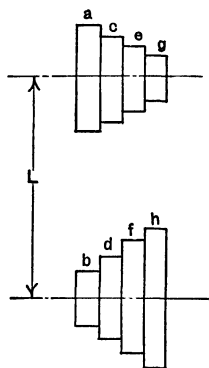


FIG. 13-9. Cone pulleys.

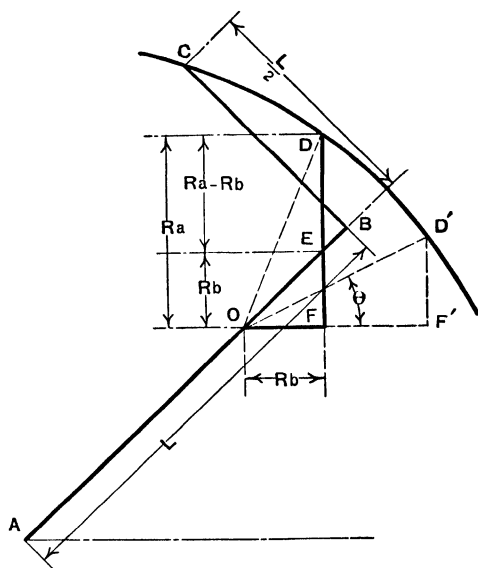


FIG. 13-10. Burmester construction.

7. Stepped-cone Pulleys. A pair of pulleys, as shown in Fig. 13-9, is frequently used where a variable speed is required on the driven shaft, a different speed ratio being obtained with each pair of steps. These pulleys must be so designed that the same length of belt is required for any pair of steps.

When using a crossed belt this condition is satisfied by having pulley radii such that

$$(R_a + R_b) = (R_c + R_d) = (R_e + R_f), \text{ etc.}$$

For an open belt the determination of the pulley sizes is more difficult, and to avoid tedious calculation several graphical methods have been devised. One of the most satisfactory of these is the Burmester method, which is illustrated in Fig. 13-10.

8. Burmester's Construction. Cone pulleys for open belts must be designed to suit a definite center-to-center distance of shafts, whereas for crossed belts the same pulleys can be used for any center-to-center distance, since then the only requirement is that the sum of the radii or diameters of each pair of steps shall be the same.

The speed of the driving shaft, the desired speeds of the driven shaft, and the center-to-center distance usually form part of the specifications of the drive. Knowing these quantities, we can find the radii of one pair of pulleys by consideration of the desired belt speed or the allowable diameter of the largest step. We shall suppose, therefore, that we are designing a pair of pulleys similar to that shown in Fig. 13-9 for an open belt, the distance L from center to center of shafts and the radii R_a and R_b for one pair of steps being known. The quantities to be determined are the radii of the other steps. The Burmester method of finding these graphically is shown in Fig. 13-10. The construction of this diagram is as follows.

Draw a line AB of length equal to L , making an angle of 45° with the horizontal. From B draw BC , equal to $\frac{1}{2}L$, perpendicular to AB . With center A and radius AC , draw an arc CDD' .

Calculate the length $R_a - R_b$, and by trial find the position of a point D on the arc CDD' such that the vertical distance DE to AB is equal to $R_a - R_b$.

Produce DE to F , making EF equal to R_b .

From F draw a horizontal line meeting AB at O .

Then $FO = R_b$ (since the angle $EOF = 45^\circ$).

Also

$$DF = R_a$$

It will be noted that

$$\tan DOF = \frac{R_a}{R_b} = \frac{\omega_b}{\omega_a}$$

If we then take any point, such as D' , on the arc and draw from it a vertical line to meet OF produced at F' , from the essential property of the diagram, the distances $D'F'$ and OF' are the radii

of a pair of steps which may be used for the cone pulley, giving a speed ratio equal to $OF'/D'F'$.

Thus, to obtain the radii R_c and R_d , for example, make θ an angle whose tangent is ω_d/ω_c . The distances $D'F'$ and OF' will then be equal to R_c and R_d , respectively. Other radii are found in the same manner.

Though not exact, this construction will generally give results sufficiently accurate for practical purposes.

9. Materials. The most generally used material for belting is oak-tanned leather, the best grade being taken from that part of the hide lying within 18 in. of the backbone of the animal. When a heavier belt is desired than can be obtained from a single thickness of the hide, two or even three thicknesses are placed one on top of the other and cemented together. These built-up belts are known as **double-** and **triple-ply** belts respectively.

The strength per square inch of cross section of built-up belts is not so great as that of single belts, a double-ply belt having about 75 per cent of the strength per square inch of a single-ply belt. If belting is to be exposed to steam or other moisture a chrome-tanned leather is used. Chrome-tanned belting is about twice as strong as oak-tanned. Double- and triple-ply beltings are less likely to flap than is single-ply belting.

Stitched canvas folded to secure the necessary number of plies, impregnated with some form of gum, is also used for belting. When gum is used the belt is generally known to the trade by the name of the gum. Thus **rubber** belting is made up of three or more layers of canvas, each coated with rubber. The layers of rubber-coated fabric are sewed together and then vulcanized. The strength of rubber belting is less than that of the best grade of oak-tanned leather, but it is about the same as that of the average grade.

Balata belting is essentially the same as rubber belting, except that balata gum is used for impregnation. It is somewhat stronger than rubber belting, but it is stiffer and does not bend so readily around the pulley as either rubber or leather belting.

Table 13-1 gives the strengths of various kinds of belting and also the weights per foot of a piece having a cross section of one square inch.

TABLE 13-1

Kind of Belt	Strength, Pounds per Square Inch	Pounds per Foot, 1 Square Inch Section
Oak leather (single).....	4000- 5,500	0.45
Oak leather (double).....	2500- 5,000	0.45
Chrome leather (single).....	7500-12,000	0.45
Cotton (solid woven).....	6850- 8,800	
Cotton (folded and stitched).....	3850- 5,150	
Rubber.....	4300	0.54

In order to make a continuous loop of belting some means must be employed for joining the ends. Many varieties of joints are in use, but with the possible exception of that made by scarfing the ends and cementing them together (see Fig.

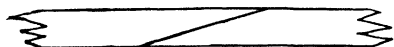


FIG. 13-11

13-11), all joints are weaker than the body of the belting. This fact must be borne in mind when fixing upon the maximum tension to be carried in a belt. Table 13-2, taken from an article by Mr. C. J. Morrison, published in *The Engineering Magazine* for July, 1916, gives the ratio of the strength of various kinds of joints to that of the belt proper.

TABLE 13-2

Kind of Joint	Relative Strength
Cement.....	1.00
Wire, machine laced.....	0.90
Cement and small rivets.....	0.89
Cement and large rivets.....	0.84
Wire, hand laced.....	0.82
Rawhide, small holes.....	0.63
Metal hooks.....	0.35

In view of the possible extreme weakness of the joint a factor of safety of 10 or 12 based on the strength of the belting should be

employed. The allowable tension in belting is frequently given in terms of the maximum pull per inch of width. F. A. Halsey gives 60, 105, and 150 lb per in. of width for single-, double-, and triple-ply belts, respectively.

Leather belts should be run with the hair side next to the pulley. In a pamphlet published by the Leather Belting Exchange of Philadelphia, Mr. R. F. Jones reports an investigation into the comparative capacity of belts run with the grain and with the flesh side next to the pulley. He concludes his report with the statement that belts run with the grain side next to the pulley will transmit from one to three times as much power as those with flesh side next to the pulley.

10. Pulleys for Belts. For many years cast iron pulleys were standard for belt drives. Recently, however, steel pulleys have become popular. They are lighter and stronger than cast iron pulleys, and are usually made in two parts; this facilitates their attachment to shafting. The manufacturers also claim for steel pulleys a higher coefficient of friction between belt and pulley and a lower windage loss. Pulleys are also made of wood or paper, and sometimes metal pulleys are leather-faced or have small disks of cork inserted in the face to increase the coefficient of friction.

— The bending of the belt as it passes around a pulley is one cause of failure of belts. This trouble increases as the size of the pulley decreases. Spooner recommends that the diameter of a pulley should not be made less than

$$D = 250t^2$$

where D = diameter of pulley in inches

t = thickness of belt in inches.

Pulleys should not be placed closer together than about 3.5 times the pulley diameter, or farther apart than 15 ft for light belts and 25 ft for heavy ones. If these limits are exceeded, trouble may be encountered in keeping the belt tight in the first case and in preventing flapping in the others.

11. Design of Belts. The relation between the tension on the tight side of a band passing around a drum and that on the slack

side is given as

$$\frac{T_1}{T_2} = e^{f\theta} \quad (13-5)$$

Although this equation was developed for use with band brakes it applies equally to belts, if modified to take into account the centrifugal force which acts on the belt as it passes around the pulley. The total centrifugal force of that portion of the belt in contact with the pulley is balanced by equal pulls in the tight and slack sides. The total tension on the tight side therefore becomes

$$T'_1 = T_1 + T_c$$

where T_c = pull induced in the tight side by the centrifugal force. The total tension on the slack side is

$$T'_2 = T_2 + T_c$$

If T_1 and T_2 in equation (13-5) are replaced by their equivalents in terms of T'_1 , T'_2 , and T_c

$$\frac{T'_1 - T_c}{T'_2 - T_c} = e^{f\theta} \quad (13-6)$$

This gives the relation between the pulls on the tight and slack sides of the belt when centrifugal force of the belt is considered. If t'_1 , t'_2 , and t_c are the corresponding tensions in pounds per square inch of belt cross section, equation (13-6) becomes

$$\frac{t'_1 - t_c}{t'_2 - t_c} = e^{f\theta} \quad (13-7)$$

where

$$t_c = \frac{wv^2}{g} \quad (13-8)$$

in which w = weight of a piece of belting having a length of 1 ft and a cross section of 1 sq in.

v = belt velocity in feet per second.

It will be noticed that the centrifugal tension is independent of the angle of wrap and the radius of the pulley. It varies directly

as the unit weight of the belting material and directly as the square of the velocity. The value of t_c as determined from equation (13-8) may now be inserted in equation (13-7).

The pull per square inch which is effective in causing rotation of the pulley is the difference between the unit tensions on the tight and slack sides, $t'_1 - t'_2$.

This value (known as the effective pull), when multiplied by the radius of the pulley, gives the torque developed per square inch of belt section; and the product of the effective pull and the velocity of the belt gives the rate of doing work per square inch of belt cross section. For belt velocities under 2000 ft per min, t_c may be taken equal to zero, since the effect of centrifugal action is so small that it may be neglected.

The coefficient of friction between a belt and pulley depends chiefly upon the kind of belt material, the pulley material, and the amount of slip between the belt and pulley. Morin gives 0.47 as a value for a leather belt running on a wooden pulley. Cromwell gives 0.45 for rubber on cast iron. The percentage of slip at which these values were determined is not stated. Mr. C. G. Barth has proposed the following formula for the coefficient of friction of leather belts on cast iron or steel pulleys in terms of the velocity of belt:

$$f = 0.54 - \frac{140}{500 + V} \quad (13-9)$$

where V = belt velocity in feet per minute.

The coefficient of friction increases with the slip of the belt on the pulley, reaching a maximum when the slip is in the region of 3 per cent. It is not advisable, however, to increase the capacity of the belts by increasing the slip, since the efficiency of the drive is thereby decreased and the life of the belt shortened.

Example. A 100-hp high speed engine running at 250 rpm is belted to an electric generator. The band wheel on the engine is 6 ft in diameter. Assuming an arc of contact on the generator pulley to be 150° , determine the necessary width for a double-ply oak-tanned leather belt $\frac{3}{8}$ in. thick.

Solution.

Velocity of belt

$$= 6 \times \pi \times 250 = 4710 \text{ ft per min or } 78.5 \text{ ft per sec}$$

Coefficient of friction

$$= 0.54 - \frac{140}{500 + V} = 0.54 - \frac{140}{500 + 4710} = 0.513 \quad (\text{eq. 13-9})$$

$$t_c = \frac{wv^2}{g} = \frac{0.45 \times 78.5^2}{32.2} = 86.5 \text{ lb per sq in.} \quad (\text{eq. 13-8})$$

Maximum allowable stress

$$t'_1 = \frac{\text{belt strength}}{\text{factor of safety}} = \frac{4200}{10} = 420 \text{ lb per sq in.}$$

Tension on slack side (t'_2)

$$\frac{t'_1 - t_c}{t'_2 - t_c} = e^{f\theta}$$

$$\frac{420 - 86.5}{t'_2 - 86.5} = 2.718^{0.513 \times \frac{150\pi}{180}} = 3.82 \quad (\text{eq. 13-7})$$

therefore $t'_2 = 174$ lb per sq in.

$$\text{Net tension} = t'_1 - t'_2 = 420 - 174 = 246 \text{ lb per sq in.}$$

Ft-lb per sq in. per min transmitted

$$= 246 \times 4710 = 1,160,000$$

Total ft-lb per min to be transmitted

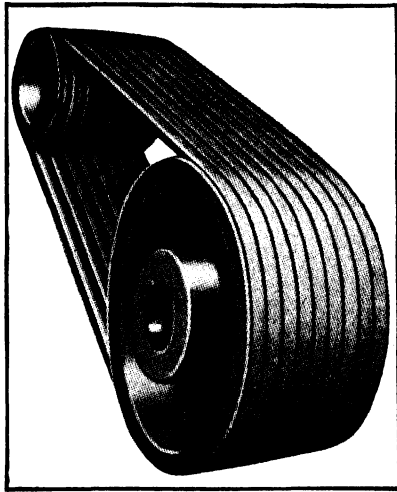
$$= 100 \times 33,000 = 3,300,000$$

Cross-sectional area of belt required

$$= \frac{3,300,000}{1,160,000} = 2.93 \text{ sq in.}$$

$$\text{Width of belt} = \frac{2.93 \times 8}{3} = 7.85, \text{ say } 8 \text{ in.}$$

12. V-belts. There has been developed recently a drive consisting of a trapezoidal cross-sectional belt made of cord or fabric and vulcanized rubber. Figure 13-12 shows such a drive. These belts run in sheaves having a varying number of grooves to suit the horsepower ratings. The ordinary automobile V fan belt is, perhaps, the most familiar form of this type of drive. Where the distance between centers of shafts is small, the V-belt is replacing the older belt drive since it permits the transmission of a larger amount of power from a pulley of a given width of face.



Dodge Mfg. Co.

FIG. 13-12. V-belt drive.

V-belt drives operate on short centers with little or no slip or noise. Although they will operate with either the tight or slack side at the top, best results are obtained when the tight side is placed uppermost. Table 13-3 gives the sizes of belt recommended by the Allis-Chalmers Manufacturing Company for their Texrope Drive transmitting various horsepowers at various rotational speeds. Table 13-4 gives the nominal power rating of single belts at various velocities, running over sheaves of various diameters.

Texropes may stretch about 4 per cent of their original length during their useful operating life. Enough adjustment should be provided to permit this amount of take-up.

As stated previously, the distance between the centers of the sheaves is usually short. This distance should be made, where possible, equal to the diameter of the larger sheave of the pair.

All the tables shown are greatly abridged, thus rendering it necessary to consult the manufacturer's catalog on Texrope Drives for more extended information.

TABLE 13-3

V-belt Selection Table

(Allis-Chalmers Manufacturing Company)

For Motor		Belt Size (Cross Section)	For Motor		Belt Size (Cross Section)
Hp	Rpm		Hp	Rpm	
$\frac{1}{2}$ to $1\frac{1}{2}$	720 to 1800	A $\frac{1}{2}$ in. $\times$ $\frac{11}{32}$ in.	20 20 to 30	1800 1800 to 600	B or C C
2 to 5	720 to 1800	A $\frac{1}{2}$ in. $\times$ $\frac{11}{32}$ in. or B $\frac{21}{32}$ in. $\times$ $\frac{7}{16}$ in.	40 40	1800 514	C D $1\frac{1}{4}$ in. $\times$ $\frac{3}{4}$ in.
$7\frac{1}{2}$ } to 1800 }	1200 to 1800 }	A or B B or C $\frac{7}{8}$ in. $\times$ $\frac{17}{32}$ in.	50 50	1800 450	C or D D
$7\frac{1}{2}$ } 900 600 }	900 600 to 720 }		75 75	1800 450	C D or E $1\frac{1}{2}$ in. $\times$ 1 in.
$7\frac{1}{2}$ }			100 100 100	1800 600 450	C D or E D or E
10 10 10 10 10	1800 1200 900 720 600		150 150 150	1200 600 360	D E E
15 15 15 15 15	1800 1200 900 720 600	B B or C B or C C C	200 200 and over	1200 720 all speeds	D E E

Example. A V-belt drive is to transmit 20 hp to a compressor. The small sheave is 8 in. in diameter and turns at 1150 rpm, while the larger sheave turns at 400 rpm. The application factor may be taken as 1.5, and the center-to-center distance as equal to the diameter of the larger sheave. Determine the size and number of belts required.

TABLE 13-4

Horsepower Ratings of Texrope Belts

(Allis-Chalmers Manufacturing Company)

" A " Section V-belts

Velocity in Feet per Minute	Pitch Diameters				
	2.6 in.	3.0 in.	3.4 in.	4.2 in.	5.0 in. and larger
1000	0.5	0.7	0.8	0.9	1.0
1200	0.6	0.8	1.0	1.1	1.2
1500	0.8	1.0	1.1	1.4	1.5
2000	1.0	1.3	1.5	1.8	2.0
2500	1.1	1.5	1.7	2.1	2.4
3000			2.0	2.4	2.7
4000				2.8	3.3
5000					3.4

" B " Section V-belts

Velocity in Feet per Minute	Pitch Diameters				
	5.0 in.	5.4 in.	5.8 in.	6.6 in.	7.0 in. and larger
1000	1.3	1.4	1.5	1.7	1.8
1200	1.5	1.6	1.7	1.9	2.0
1500	1.8	2.0	2.1	2.4	2.5
2000	2.4	2.6	2.8	3.1	3.2
2500	2.8	3.1	3.3	3.7	3.9
3000	3.2	3.5	3.8	4.3	4.5
4000	3.5	4.0	4.4	5.0	5.3
5000	3.3	3.8	4.3	5.1	5.5

TABLE 13-4 — *Continued*

“ C ” Section V-belts

Velocity in Feet per Minute	Pitch Diameters				
	7.0 in.	8.0 in.	9.0 in.	10.0 in.	12.0 in. and larger
1000	2.0	2.5	2.8	3.1	3.6
1200	2.4	2.9	3.4	3.8	4.3
1500	2.9	3.6	4.2	4.6	5.3
2000	3.7	4.7	5.5	6.1	7.0
2500	4.5	5.7	6.6	7.3	8.5
3000	5.0	6.5	7.6	8.5	9.8
4000			9.0	10.2	12.0
5000					13.2

“ D ” Section V-belts

Velocity in Feet per Minute	Pitch Diameters				
	12.0 in.	13.0 in.	14.0 in.	15.0 in.	17.0 in. and larger
1000	4.5	5.1	5.6	6.1	6.8
1200	5.3	6.1	6.7	7.3	8.2
1500	6.5	7.5	8.2	9.0	10.1
2000	8.4	9.7	10.7	11.7	13.2
2500	10.1	11.6	12.9	14.1	16.0
3000	11.4	13.3	14.9	16.3	18.6
4000		15.5	17.6	19.4	22.5
5000					24.4

TABLE 13-4 — Continued

“ E ” Section V-belts

Velocity in Feet per Minute	Pitch Diameters				
	20.0 in.	22.0 in.	24.0 in.	26.0 in.	28.0 in. and larger
1000	8.0	9.0	9.8	10.5	11.1
1200	9.5	10.7	11.7	12.6	13.3
1500	11.7	13.2	14.5	15.6	16.5
2000	15.2	17.2	18.9	20.3	21.6
2500	18.3	20.8	23.0	24.7	26.3
3000	20.9	24.0	26.5	28.6	30.5
4000	24.5	28.5	31.9	34.7	37.2
5000				37.9	40.9

Solution.

Size of belt for 20 hp and 1150 rpm = “ C ” (Table 13-3.)

Size of larger sheave

$$= \frac{8 \times 1150}{400} = 23 \text{ in. diameter}$$

Arc of contact on smaller sheave

$$= 180^\circ - 2 \sin^{-1} \frac{11.5 - 4}{23} = 142^\circ$$

Correction factor for $142^\circ = 0.89$ (Table 13-5.)

Velocity of belt

$$= \frac{\pi \times 8 \times 1150}{12} = 2400 \text{ ft per min}$$

Horsepower transmitted by one “ C ” belt running at 2400 ft per min over 8-in. diameter sheave = 5.5 (By interpolation in Table 13-4.)

Horsepower transmitted when arc is 142°

$$= 5.5 \times 0.89 = 4.9$$

Design horsepower transmitted

$$= 20 \times 1.5 = 30$$

Number of belts required

$$= \frac{30}{4.9} = 6.1, \text{ say } 6$$

TABLE 13-5

Correction Factor for Arc of Contact on Small Sheave

(Allis-Chalmers Manufacturing Company)

Arc of Contact	Correction Factor	Arc of Contact	Correction Factor	Arc of Contact	Correction Factor
180°	1.00	155°	0.93	130°	0.86
175	0.99	150	0.92	125	0.84
170	0.98	145	0.90	120	0.83
165	0.96	140	0.89	115	0.81
160	0.95	135	0.87	110	0.79

TABLE 13-6

Application Factors

(Allis-Chalmers Manufacturing Company)

Kind of Service	Service Factor
Brick and clay machinery.....	1.2 to 2.0
Conveyors.....	1.0 to 1.8
Crushing machinery.....	1.0 to 1.5
Fans and blowers.....	1.2 to 2.0
Machine tools.....	1.0 to 1.4
Pulp and paper machinery.....	1.0 to 1.8
Pumps and compressors.....	1.2 to 2.0
Textile machinery.....	1.2 to 1.8
Woodworking machinery.....	1.2 to 1.5

CHAIN DRIVES

13. Chains are made of a series of jointed metal links in a variety of forms. They may be classified in accordance with their uses

as (a) hoisting and hauling chains, (b) elevator and conveyor chains, and (c) power transmission chains.

Hoisting chains. The **coil chain** (Fig. 13-13) is the usual form of hoisting chain. It is made of iron of circular section bent to the proper form and welded at the joint. The **sprocket wheels** for this

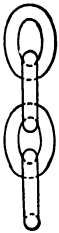


FIG. 13-13

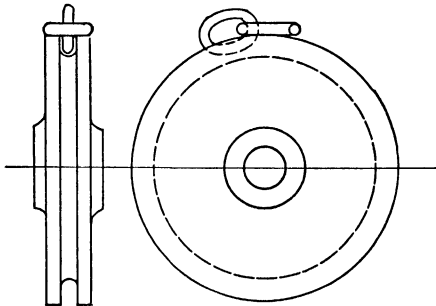


FIG. 13-14



FIG. 13-15

chain are shown in Figs. 13-14 and 13-15. The plain **grooved sheave** is suitable only as a guide and not as a means of transmitting energy to or from the chain. The **pocket sheave** (Fig. 13-15) has a central groove and depressions which conform to the profile of the links, a liberal clearance being allowed at the ends of these depressions to provide for stretching of the chain or irregularities in the pitch.

Conveyor chains. The **Ewart chain** (Fig. 13-16), which has easily detachable links, and the **pintle chain** (Fig. 13-17) are two of a large class of chains employed in mills, mines, and factories for elevators and conveyors handling a variety of materials. For such purposes, buckets, flights, etc., are connected to the chains by various kinds of attachment links. These chains are generally of malleable iron and are run on cast sprockets. This construction is suitable for rough service and slow speeds. These chains are operated at about 100 ft per min.

Power chains. As pointed out in Art. 1, power chains are used when the angular velocity ratio of driver to driven member must

be constant and the distance between shaft centers is so great as to make the use of spur gearing inconvenient. Chain drives are also used to advantage in some cases where constant angular velocity ratio is not a necessity, but where the distance between the shaft

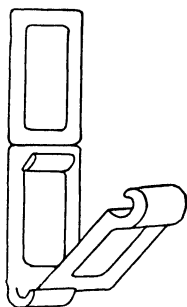


FIG. 13-16. Ewart chain.

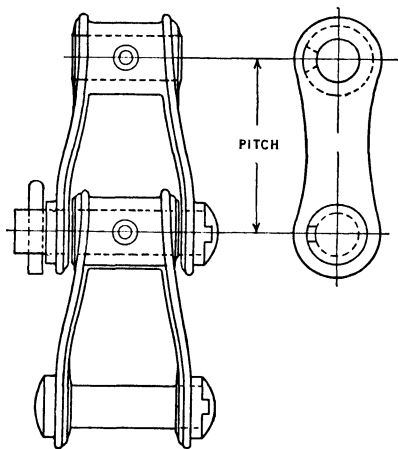


FIG. 13-17. Pintle chain.

centers is so small that belt or rope drives are unsatisfactory. Compared with belt or rope drives, power chain drives are slow speed affairs which transmit large amounts of force. The usual velocities lie between 600 and 2000 ft per min.

There are three general types of chains for power transmission:

- (a) Block chains (Fig. 13-18).
- (b) Roller chains (Fig. 13-19).
- (c) Silent chains (Figs. 13-20 and 13-21).

The side links and blocks of the block chain are held together by a pin as shown. This pin is fastened to the side links so that all motion occurring in bending of the chain takes place between the pin and the block. The contact area here is quite large and the bearing pressure consequently low. If the pin were fast in the block and the bending accomplished by moving the links over the pin, excessive wear would result because of the high bearing

pressure existing in the region of sliding. Block chains may be used for velocities up to 800 or 900 ft per min.

The roller chain has been devised in an attempt to substitute rolling for sliding contact between the tooth of the sprocket and the chain and thus increase the efficiency of power transmission.

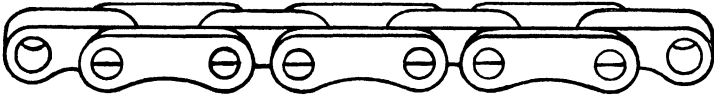


FIG. 13-18. Block chain.

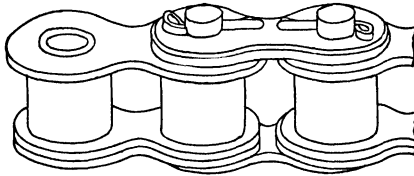
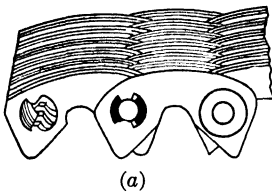
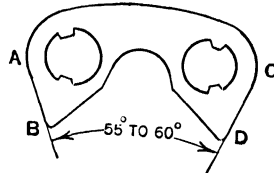


FIG. 13-19. Roller chain.



(a)

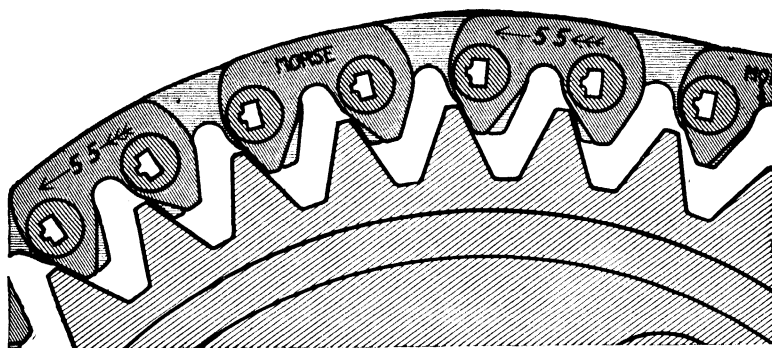


(b)

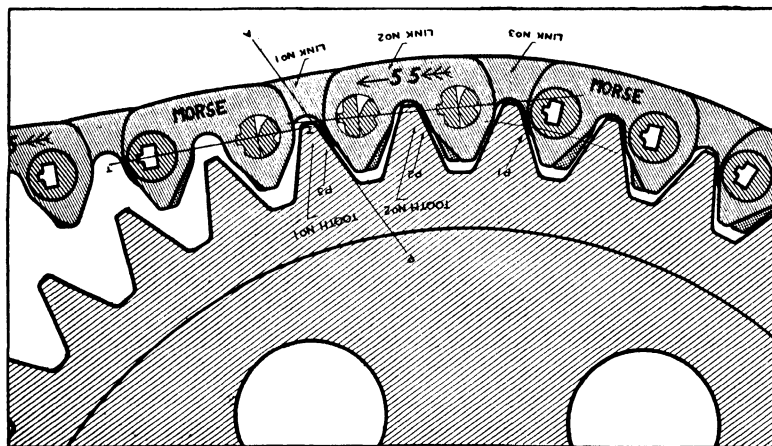
FIG. 13-20. Silent chain (a) and link (b).

The structural features of the roller chain are clearly shown in Fig. 13-19, illustrating one of the higher grades of roller chain. In the cheaper grades the bushing is omitted. This results in greater friction losses and shorter life. Roller chains give satisfactory service at velocities up to 1800 ft per min, and their efficiency varies from 93 per cent to 98 per cent.

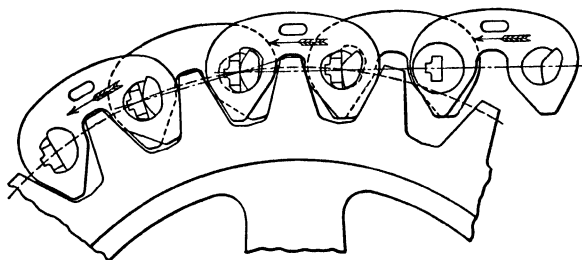
For high speed service where considerable amounts of power are to be transmitted at velocities between 1500 and 2000 ft per min, silent chains are used. The general structural features are shown in Fig. 13-20. With these chains there is no sliding motion between the tooth of the sprocket and that of the chain. Several



Position of worn chain on driven sprocket.



Position of new chain on driven sprocket.



Morse Chain Co.

FIG. 13-21. Morse silent chain.

makes of this type of chain are on the market, differing principally in the construction of the joint. The Renold chain uses a plain pin joint; the Link-Belt uses a pin joint which is provided with a pair of hardened steel bushing strips. In both these joints the relative motion is sliding. The Morse chain substitutes rolling motion for sliding. This is done by using two pins in each joint and arranging them as shown in Fig. 13-21. Efficiencies in excess of 98 per cent are claimed for the Morse chain. The absence of sliding motion in the joint also increases its life when the chain is exposed to dust or grit. Silent chains are made in sizes varying from $\frac{1}{4}$ to 5000 hp.

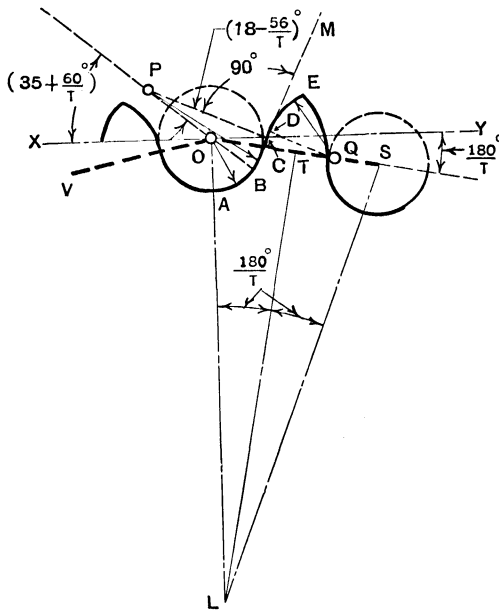


FIG. 13-22. Standard tooth for roller-chain sprocket.

14. Tooth Forms. All chains stretch somewhat in use. If the pitch of the sprocket were made to conform exactly to that of the new chain, trouble would be experienced in a short time because of the stretching of the chain and consequent increase in its pitch. To obviate difficulty from this source it was formerly customary

to make the pitch of the sprocket somewhat greater than that of the new chain. This, however, led to all the driving force being taken by one tooth at a time and this, in turn, resulted in excessive wear of the sprocket teeth.

The combined standards committee of the A.S.M.E., the S.A.E., and the A.G.M.A. has adopted a form of tooth for roller chains shown in Fig. 13-22, which does away with both these difficulties. The pitch of the teeth is made the same as that of the new chain. As the pitch of the chain increases through stretch and wear of the links the rollers ride higher and higher on the teeth, but contact is maintained between each roller and tooth included in the arc of contact. This shape of tooth has been largely responsible for recent increase in the allowable speeds and loads for roller chains.

Teeth on sprockets for silent chains have straight sides as shown in Fig. 13-21. Here again the stretch of the chain is compensated by the chain riding higher on the sprocket.

For medium and high speeds the bearing pressure of the links against the pins or bushings should not exceed 1000 lb per sq in. of projected area, although for very slow speeds bearing pressures as high as 3000 lb per sq in. are allowable.

For drives in which one of the sprockets is small the tight or driving side of the chain should be placed at the top. Unless this is done, pinching may occur where the slack accumulates.

PROBLEMS — CHAPTER XIII

1. Find the pulley diameters which must be used on motors turning at the following revolutions per minute if the belt speeds are to be 3200 ft per min.: (a) 3600 rpm; (b) 1750 rpm; (c) 900 rpm; (d) 600 rpm.

2. Split steel pulleys are available in stock sizes varying by inch increments. What diameters should be selected for a grinder turning at 4000 rpm if the motor runs at 1750 rpm and 4 per cent slip is estimated? The belt speed is to be approximately 3300 ft per min.

3. A multiple V-belt drive using "C" section belt is employed to connect a Diesel engine running at 1600 rpm to a line shaft running at 600 rpm. The manufacturer recommends a belt speed not higher than 5000 ft per min and pulley diameters not less than 9 in. Select stock pulleys (no fractional inch sizes), allowing for 3 per cent slip.

4. A belt drive is to connect a motor running at 900 rpm to a line shaft which is to turn approximately at 500 rpm. The belt speed is not to exceed 3850 ft per min. Standard pulleys are obtainable in even-inch diameters. Select pulley diameters, (a) neglecting belt thickness and slip, (b) allowing also for slippage of 5 per cent. *Ans.* (a) 15 in., 27 in. (b) 15 in., 26 in.

5. A machine which should run at 200 rpm is to be belt-driven through one countershaft running about 600 rpm by a motor running at 1800 rpm. The belt speed is not to be higher than 4000 ft per min. The largest pulley in the drive must not be over 36 in. in diameter. Select suitable pulleys of stock sizes (even-inch diameters) for the drive, allowing for a total slip of 8 per cent.

6. Calculate the belt length in an open belt drive connecting two shafts 10 ft apart, the pulleys being respectively 12 in. and 24 in. in diameter.

Ans. 24 ft $8\frac{3}{4}$ in.

7. A V-belt drive has pulleys respectively 6 in. and 18 in. in diameter. The center-to-center distance should not be less than the diameter of the larger pulley. Find a satisfactory belt length for shortest drive. Stock belts are available in lengths varying by 2-in. increments.

8. Two shafts are to be connected by stepped-cone pulleys with three pairs of steps, for a crossed belt. The driving shaft turns at 400 rpm. The driven shaft is to turn at 200, 400, and 600 rpm. The largest step on the driving pulley is 18 in. in diameter. Find the diameters of the other steps.

Ans. Driving: 18, 15, 10 in. Driven: 12, 15, 20 in.

9. A stepped-cone pulley for a lathe is to give spindle speeds of 150, 280, 540, and 1000 rpm when driving from a line shaft rotating at 400 rpm. A crossed belt is used. The largest step on the driver is 11 in. in diameter. Find the diameters of all other steps on the pulleys to the nearest $\frac{1}{8}$ in.

10. A light, single-ply oak-leather belt is to drive the cylinder of a wood jointer and transmit 5 hp. The pulley on the jointer cylinder is 4 in. in diameter and makes 3600 rpm. The arc of contact is 135° . Assume the thickness of the belt to be $\frac{3}{16}$ in., its ultimate tensile strength to be 2400 lb per sq in., and a factor of safety of 12, and determine the width for the belt.

Ans. 2.35 in., say $2\frac{1}{2}$ in.

11. The pulley on a triplex pump rotates at 275 rpm, is 18 in. in diameter, and transmits 7.5 hp. It is to be driven by a rubber belt $\frac{1}{4}$ in. thick. The ends of the belt are joined by means of a rawhide lace. The arc of contact is 180° . Assume the coefficient of friction to be 0.25 and the factor of safety based on the joint to be 6. Determine the width for the belt.

Ans. 3.15 in., say $3\frac{1}{4}$ in.

12. A 28-in. double-leather belt $\frac{3}{8}$ in. thick runs on a 26-in. diameter pulley rotating at 600 rpm. The arc of contact is 165° . Assume that the maximum allowable pull is 105 lb per in. of width and determine the horsepower the belt can transmit.

Ans. 216.

13. A 15-hp motor runs at 1150 rpm. The pulley is 6 in. in diameter and carries a heavy single-ply leather belt $\frac{1}{4}$ in. thick. The joint in the belt is

cemented. The arc of contact is 180° . Taking the maximum allowable pull in the belt as 60 lb per in. of width, determine the proper width for the belt.

Ans. 5.85 in., say 6 in.

14. A belt $\frac{1}{4}$ in. thick runs over a pulley 18 in. in diameter with a speed of 3000 ft per min. The arc of contact is 160° and the coefficient of friction is 0.45. The belt material weighs 0.40 lb per sq in. section 1 ft long, and has an ultimate tensile strength of 5000 lb per sq in. The ratio of the strength of the joint to that of the belt is 0.80. Take a factor of safety of 8, based on the strength of the joint, and determine the width necessary to transmit 30 hp.

Ans. 3.93, say 4 in.

15. A pulley 38 in. in diameter turns at 480 rpm and transmits 20 hp by means of a double-ply leather belt. Take the maximum allowable tension as 105 lb per in. of width, the thickness of the belt as $\frac{3}{8}$ in., and the weight of a piece 1 ft long and 1 sq in. in cross section as 0.45 lb. The angle of wrap is 180° . Determine the width for the belt.

16. A two-ply leather belt $\frac{3}{8}$ in. in thickness transmits 25 hp while running over a 24-in. pulley turning at 350 rpm. Take the maximum allowable pull as 105 lb per in. of width, the weight of the belt as 0.45 lb for a piece 1 ft long and 1 sq in. in cross section, the angle of wrap as 150° , and determine the width for the belt.

17. The compressor of an ice machine is to be driven by means of V-belts. The speed of the motor is 1750 rpm and the diameter of its sheave is 5.4 in. The speed of the compressor is 250 rpm. Fifteen horsepower is to be transmitted. Determine the size and number of belts required.

Ans. Size B; 7 belts.

18. The cylinder of a planer is to be driven at 3000 rpm by a motor turning at 1750 rpm and delivering 5 hp. The smaller sheave is 3 in. in diameter. The distance between the centers of the sheaves is to be not less than 15 in. Determine the size and number of V-belts.

Ans. Size A; 5 belts.

19. A motor turning at 1150 rpm delivers 50 hp to a counter shaft which is to run at 300 rpm, and which drives machine tools. The diameter of the smaller sheave is 13 in. Assume an application factor of 1.1 and determine the size and number of V-belts to be used.

Ans. Size D, number 4.

20. A single-acting reciprocating pump making 60 strokes a minute is driven from a geared speed reducer having a 10 to 1 ratio. The reducer is to be driven through a V-belt drive by a motor turning at 1750 rpm. The diameter of the smaller sheave is 9 in. Forty horsepower is required. Determine size and number of belts required.

Ans. Size C; 6 belts.

CHAPTER XIV

SPRINGS

1. General Types of Springs. There are three general types of springs:

- (a) Helical, in which the principal stress is shear due to torsion.
- (b) Flat spiral, in which the principal stresses are tension and compression due to bending.
- (c) Flat leaf, in which the principal stresses are also tension and compression due to bending.

2. Helical Springs. The helical, or as it is frequently termed coil, spring consists of a wire or rod wound in the form of a helix, and is intended to sustain a load applied along the axis of the helix. This load may be applied in such a manner as to cause the spring either to lengthen or shorten as it deflects under the action of the load. The valve springs of an automobile engine are perhaps the most familiar examples of a helical spring carrying a load tending to shorten it; the spring in the ordinary spring balance is an example of a helical spring carrying a load which lengthens it. The straining action, the nature of the stresses set up, and the methods of design are the same in each case.

3. Design of Helical Springs for Strength and Stiffness. If an axial load is applied to a helical spring the distance from the line of action of that load to the center of the supporting wire is equal to the mean radius of the coil. The twisting moment on the **wire** is therefore $P \times r$, where P is the load and r the mean radius of the **coil**. Equating this to the resisting moment for the wire,

$$Pr = s_s \frac{I_p}{c} \quad (14-1)$$

where s_s = induced shearing stress

I_p = polar moment of inertia of the wire

c = distance from neutral axis of wire to outermost fiber.

If the wire is circular in cross section, equation (14-1) becomes

$$Pr = s_s \frac{\pi d^3}{16} \quad \text{or} \quad PD = s_s \frac{\pi d^3}{8} \quad (14-2)$$

where d = diameter of wire in inches

D = mean diameter of coil in inches.

Equation (14-2) neglects the effect of curvature, bending, and direct shear. All these serve to increase the stress in the wire, especially when the ratio mean diameter of coil to diameter of wire is small. A. M. Wahl¹ has shown that this increase may be included by writing equation (14-2) as

$$PD = \frac{s_s}{K} \times \frac{\pi d^3}{8} \quad (14-3)$$

$$\text{where } K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \cdot \left(C = \frac{D}{d} \text{ for round wire.} \right)$$

These equations express the relations between dimensions of the spring, the load imposed on, and the stress induced in the spring. They do not involve, in any way, the deflection caused by the load.

Since a helical spring may be considered as a bar subjected to torsion caused by a force P acting at a distance r from the axis of the bar, the deflection of the spring may be taken as equal to the distance traveled by the force in twisting the bar through an angle θ .

On the basis of this assumption the deflection of the spring becomes

$$\Delta = \frac{8PD^3n}{E_s d^4} \quad (14-4)$$

¹ A.S.M.E. Transaction A-35, 1935.

where D = mean diameter of coils

n = number of active turns

E_s = transverse modulus of elasticity

d = diameter of wire.

Wahl states that, although this equation also neglects the factors named earlier, it gives results that are not more than 2 per cent in error. For most cases, where the ratio of spring diameter to wire diameter exceeds four, the error is less than 1 per cent.

The expressions given above hold for compression springs and for tension springs exclusive of the hook at the ends as shown in Fig. 14-1. Where this hook is formed by turning up a coil, Wahl recommends that the additional deflection be cared for by adding half a turn to the number of active turns, n . Unless great accuracy is required this can be neglected.

Springs made from rectangular wire are stronger and stiffer than springs of round wire occupying an equal space.

Where compactness is especially desirable, therefore, rectangular wire should be used.

4. General Considerations. Helical springs are made almost exclusively of steel, bronze, or brass. If steel is used two general methods of manufacture are employed. The spring may be formed from wire in the annealed condition, and the desired strength and high elastic limit secured by subsequently tempering the spring. A second method is to wind the spring from hard-drawn, high carbon piano wire. This method does not require heat treatment after the spring is formed, but it is not satisfactory for springs requiring wire of large diameter. Brass and bronze cannot be hardened by heat treatment, and therefore all springs made from them are produced by winding the wire without subsequent heat treatment.

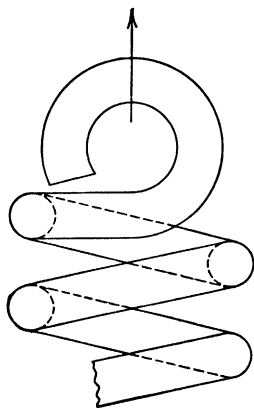


FIG. 14-1

It has been found that the unit strength of spring wire increases as the size of the wire decreases. It is therefore customary to allow higher working stresses for small wires than for large. Table 14-1 shows a fair average range of allowable working stresses for various sizes of wire.

TABLE 14-1

Material and Size	Allowable Stress, Pounds per Square Inch
Piano wire.....	120,000
Tempered steel, $\frac{1}{8}$ in. diameter.....	100,000
Tempered steel, $\frac{1}{4}$ in. diameter.....	80,000
Tempered steel, $\frac{3}{8}$ in. diameter.....	70,000
Tempered steel, $\frac{1}{2}$ in. diameter.....	60,000
Phosphor bronze.....	25,000
Brass.....	15,000

These stresses are for springs subjected to conditions approximating static loads. If the spring is in continual operation, as is the valve spring of a gas engine, fatigue must be allowed for and the above stresses reduced by at least 50 per cent.

When the spring is used to sustain a tensile load it is customary to bend (see Fig. 14-1) a hook in either end of the wire so that the spring can be attached easily to the load and anchorage. It has been found that the most usual region of failure of such springs is at the base of the hook where the stress due to bending is frequently greater than that due to torsion in the body of the spring. In order to reduce the danger of failure from this cause, hooks should be made with large radii and should be checked for bending. Better still, hooks can be obviated by using one of the devices shown in Fig. 14-2.

When springs are used in compression, the end turns are either ground, as in Fig. 14-3, or flattened, as in Fig. 14-4, in order to fit securely the seat for the spring. The designer must remember that whatever portion of the spring is in contact with the seat is inactive and that the number of turns appearing in equations for

deflection are **active** turns. The inactive portion of the spring usually amounts to three-quarters of a turn at each end.

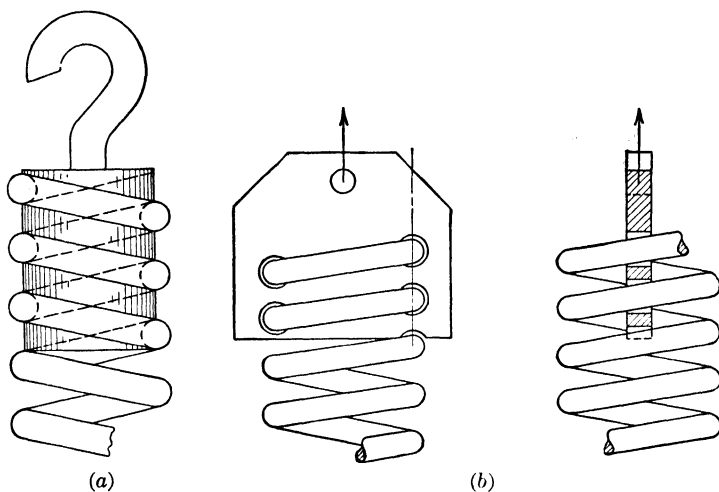


FIG. 14-2. Plug and plate attachments for helical springs.

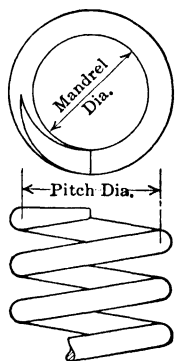


FIG. 14-3. Helical spring with ground ends.

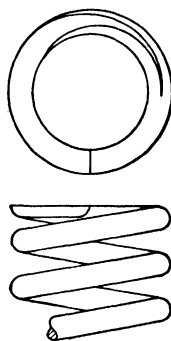


FIG. 14-4. Helical spring with flattened ends.

It is customary in specifying springs to state the free height of the spring, that is, the axial length of the spring when under no load. Figure 14-5(a) shows the free height; Fig. 14-5(b) the

assembled height when the spring is given an initial deflection; Fig. 14-5(c) the height when under full load; and Fig. 14-5(d) the solid height. Obviously the free height is the sum of the solid height, total clearance between the coils under full load, and the total deflection.

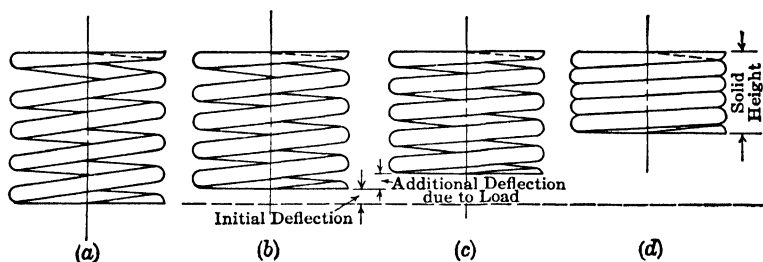


FIG. 14-5

Example. A gas engine valve spring is to have a mean diameter of $1\frac{1}{2}$ in. The maximum load it will have to sustain is 100 lb with a corresponding deflection of $\frac{1}{2}$ in. The spring is to be made of tempered steel wire. Since the material is to be subjected to repeated loading and fatigue must be considered, a low working stress of 40,000 lb per sq in. will be used. Determine the size for the wire and the number of coils to be used.

Solution. Assume $\frac{D}{d} = 6$, then

$$K = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.15 + 0.1025 = 1.2525$$

$$d = \sqrt[3]{\frac{8 \times 100 \times 1.5 \times 1.2525}{\pi \times 40,000}} = 0.228 \text{ in. diameter}$$

(Use Washburn & Moen No. 4 wire — 0.2253 in. in diameter.)

$$\text{Actual value of } \frac{D}{d} = \frac{1.5}{0.2253} = 6.62$$

(The assumption was close enough.)

$$n = \frac{\Delta E_s d^4}{8PD^3} = \frac{0.5 \times 12,000,000 \times (0.2253)^4}{8 \times 100 \times (1.5)^3} \quad (\text{eq. 14-4})$$

$$= 5.6, \text{ say } 5\frac{1}{2} \text{ active turns}$$

Since this is a compression spring a total of $1\frac{1}{2}$ inactive turns should be allowed, making the total number of turns

$$5\frac{1}{2} + 1\frac{1}{2} = 7 \text{ turns}$$

5. Flat Springs. Semi-elliptical flat springs may be considered as the equivalent of a trapezoidal plate like that shown in Fig. 14-6. For the sake of securing compactness this plate is cut into strips.

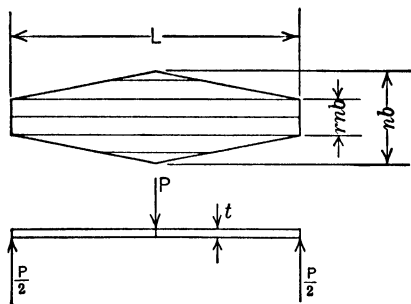


FIG. 14-6

When the strips are piled one above another the familiar semi-elliptical spring is formed. The behavior of the spring under load is identical with that of the equivalent trapezoidal plate, and the following equations have been developed on this basis by Professor Peddle.

Semi-elliptical springs are supported at the ends and loaded at the middle. The trapezoidal plate equivalent is shown in Fig. 14-6. The spring therefore is a simple beam of width nb at the center and thickness t , in which

$$\begin{aligned} n &= \text{total number of leaves} \\ b &= \text{width of leaf} \\ t &= \text{thickness of leaf} \end{aligned}$$

the bending moment at the center of the spring is $\frac{PL}{4}$ which equals

$$\frac{sI}{c} = \frac{sbt^2}{c} \quad (14-5)$$

The **deflection** of a trapezoidal plate carrying a concentrated load at the center is given by the following expression:

$$\Delta = \frac{L^2 sk}{2tE} \quad (14-6)$$

where L = length of plate in inches

s = maximum stress induced in pounds per square inch

t = thickness of plate in inches

E = modulus of elasticity of the material

$$k = \frac{1}{(1-r)^3} \left[\frac{1-r^2}{2} - 2r(1-r) + r^2 \log_e \frac{1}{r} \right]$$

$$r = \frac{\text{number of full-length leaves}}{\text{total number of leaves}}.$$

In order to make the laminated leaf spring correspond as closely as possible with the trapezoidal plate the ends of all but the leaves running the entire length of the spring should be pointed and thinned.

The bundle of leaves is held together by either a band shrunk around them at the center or by a bolt passing through the center. In the first case the band exerts a stiffening and strengthening effect, and this should be allowed for by subtracting the width of the band from the overall length of the spring to obtain the effective length. In the second case the diameter of the bolt hole must be subtracted from the width of the leaf when making the calculation for strength. The deflection will not be affected materially by the presence of the bolt hole.

The full elliptical spring is simply two semi-elliptical springs combined with their concave sides toward each other. The stress induced by a given load is exactly the same as that induced in a semi-elliptical spring, but the deflection will be twice as great.

The allowable working stress for steel laminated springs may be taken as from 60,000 to 80,000 lb per sq in. for springs subjected to more or less static loads. F. A. Halsey gives allowable stresses of 50,000 lb per sq in. for alloy and 45,000 lb per sq in. for carbon-steel automobile springs subjected to severe shock and fatigue.

Example. A semi-elliptical, laminated, automobile spring to carry a load of 625 lb is to consist of seven leaves $2\frac{1}{4}$ in. wide, two of the leaves extending the full length of the spring. The spring is to be 43 in. in length and is to be attached to the axle by two U-bolts 3 in. apart. These bolts clamp the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. The leaves are to be made of silico-manganese steel. Assuming an allowable stress of 50,000 lb per sq in. determine the thickness for the leaves and the deflection.

Solution.

Equivalent length = $43 - 3 = 40$ in.

Bending moment at center of equivalent trapezoidal plate

$$= \frac{625 \times 40}{4} = 6250 \text{ lb-in.} = \frac{sI}{c} = \frac{sbn t^2}{6}$$

Thickness of plate

$$= t = \sqrt{\frac{6M}{sbn}} = \sqrt{\frac{6 \times 6250}{50,000 \times (7 \times 2\frac{1}{4})}} = 0.219, \text{ say } \frac{7}{32} \text{ in.}$$

Deflection

$$\Delta = \frac{L^2 sk}{2tE} \quad (\text{eq. 14-6})$$

$$k = \frac{1}{(1-r)^3} \left[\frac{1-r^2}{2} - 2 \times r(1-r) + r^2 \log_e \frac{1}{r} \right] \text{ where } r = \frac{2}{7}$$

$$k = \frac{1}{(1 - 0.286)^3} \left[\frac{1 - 0.286^2}{2} - 2 \times 0.286(1 - 0.286) + 0.286^2 \log_e 3.5 \right] = 0.435$$

$$\Delta = \frac{40 \times 40 \times 50,000 \times 0.435}{2 \times \frac{7}{32} \times 30,000,000} = 2.66 \text{ in.}$$

PROBLEMS — CHAPTER XIV

1. The spring for a spring balance is to elongate 4 in. when subjected to a load of 50 lb. Assume that the mean radius of the coils is to be $3\frac{1}{2}$ times the diameter of the wire and that the maximum stress to be induced is 80,000 lb per sq in. Determine the diameter for the wire, the diameter for the coil, and the number of coils required. ($E_s = 11,500,000$.)

Ans. $d = 0.1163$; $D = \frac{3}{16}$ in.; $n = 39.5$.

2. An engine indicator has a plunger $\frac{3}{4}$ in. in diameter. The indicator spring attached to this plunger is to be compressed $\frac{1}{2}$ in. when the steam pressure acting on the plunger is 100 lb per sq in. Assume a stress in the wire of 80,000 lb per sq in. and assume that the mean radius of the coils is three times the diameter of the wire. Determine the diameter for the wire, the diameter for the coils, and the number of turns required. ($E_s = 11,500,000$.)

Ans. $d = 0.1030$; $D = 0.618$ in.; $n = 7.7$.

3. The counter recoil spring of a 5-in. gun is to sustain a maximum load of 6540 lb. This is $2\frac{1}{2}$ times the load when assembled with the gun in firing position. The travel of the gun during the recoil is 12 in. The spring is to be made of round wire. The mean diameter of the coils is $5\frac{1}{4}$ in. When deflected to the maximum amount the clearance between the coils is to be $\frac{1}{4}$ in. Assume the maximum allowable stress in the spring to be 100,000 lb per sq in. and the transverse modulus of elasticity to be 11,500,000. Determine the diameter of the wire, the number of active turns, and the free height of the spring.

4. A helical spring of round steel wire $\frac{1}{4}$ in. in diameter is compressed 6.4 in. by a load of 100 lb. The mean radius of the coils is 1 in. Assume the transverse modulus of elasticity of the wire material to be 11,500,000. Determine the number of coils, the stress in the wire, and the force necessary to compress the spring 1 in.

Ans. $n = 45$; 38,700; 15.6 lb.

5. The exhaust spring of a gas engine is to have a mean radius of coils of $\frac{1}{2}$ in., and a deflection of 1 in. under a load of 50 lb. Assume the maximum stress in the wire to be 64,000 lb per sq in. and the transverse modulus of elasticity to be 11,500,000. Determine the diameter for the wire and the number of turns in the spring.

Ans. $d = 0.1345$ in.; $n = 9\frac{1}{2}$.

6. A body weighing 1500 lb and moving horizontally at 180 ft per min is to be brought to rest by a spring whose maximum deflection is to be 4 in. Assume the mean diameter of the coils to be 6 in. and the maximum stress in the wire to be 60,000 lb per sq in. The transverse modulus of elasticity of the wire material is 11,500,000. Determine the diameter for the wire and the number of turns in the spring. *Ans.* 0.725 in., say $\frac{3}{4}$ in.; 6.65, say 7.

7. A helical spring of round steel wire is to sustain the impact of a weight of 700 lb falling through a height of 3 ft. The mean radius of the coils is to be four times the diameter of the wire. The maximum deflection of the spring is to be 14 in. Take the allowable stress as 60,000 lb per sq in. and the modulus of elasticity as 11,500,000 and determine the diameter for the wire and the number of turns required.

Ans. 1.415, say $1\frac{7}{8}$ in.; and 11.3, say $11\frac{1}{2}$ turns.

8. A helical spring made of round steel wire is to absorb 300 ft-lb of energy while being compressed 6 in. The coils shall have a mean diameter six times the diameter of the wire, and the maximum induced stress is to be not more than 60,000 lb per sq in. Take E_s as 11,500,000 and determine the size for the wire and the number of turns required.

9. The valve spring of an engine is to have a mean diameter of $1\frac{1}{2}$ in. The spring is assembled so that the initial load is 80 lb. The maximum lift of the valve is $\frac{1}{2}$ in. and the corresponding load on the spring is 110 lb. Assume a maximum working stress of 60,000 lb per sq in. in the wire and determine the size of wire required, the number of active turns, and the initial deflection. ($E_s = 11,500,000$.)

10. A flat spring 56 in. long is to be made up of eight leaves $2\frac{1}{2}$ in. wide; one of which extends the full length of the spring. It is to have a deflection of 5 in. when subjected to a load of 800 lb. The leaves are held together at the center by a band 4 in. wide. Taking the modulus of elasticity as 30,000,000 determine the thickness for the leaves and the maximum stress induced.

Ans. 0.234 in., say $\frac{1}{4}$ in.; 49,820.

11. A semi-elliptical laminated spring 36 in. long is to have six leaves $2\frac{1}{4}$ in. wide, one of which extends the full length of the spring. The leaves are held together at the center by the equivalent of a band 2 in. wide. Assume a maximum working stress of 70,000 lb per sq in. and a modulus of elasticity of 30,000,000 for the spring material. Determine the thickness for the leaves and the deflection for a load of 1000 lb.

Ans. $t = 0.232$ in., say $\frac{1}{4}$ in.; 2.4 in.

12. (a) A semi-elliptical laminated spring 36 in. long and $2\frac{1}{4}$ in. wide is held together at the center by the equivalent of a band 2 in. wide. Assume a maximum working stress of 70,000 lb per sq in. and a thickness of each leaf of $\frac{3}{16}$ in. Determine the number of leaves for the spring to carry a load of 1000 lb.

Ans. 9.2, say 9.

(b) Assuming that two of these leaves extend the full length of the spring,

determine the deflection of the spring if the modulus of elasticity of the spring material is 30,000,000. *Ans.* 3.10 in.

13. A flat-leaf spring having eight leaves, two of which are full length, has an effective length of 40 in. The leaves are $2\frac{1}{2}$ in. wide. Taking the maximum allowable stress as 70,000 lb per sq in. and the modulus of elasticity as 30,000,000, determine the thickness for the leaves to accommodate a load of 1400 lb, and the deflection caused by the load.

14. A flat, laminated spring, 60 in. long, is made up of seven leaves, one of which extends the entire length of the spring. These leaves are $\frac{1}{4}$ in. in thickness, 3 in. in width, and are held together at the middle by a band 3 in. wide. Assume a maximum induced stress of 60,000 lb per sq in. and determine the load the spring can carry; also the maximum deflection caused by the load. ($E = 30,000,000$.)

CHAPTER XV

FLYWHEELS

1. Function of Flywheels. Whenever the supply of energy does not vary in exactly the same way as does the demand for energy, some device for its redistribution must be interposed between the source of energy and the place where it is used. A flywheel is one type of such a device. When attached to a prime mover, such as a steam engine or gas engine which generates power in its cylinder at a variable rate, it enables the withdrawal of power at a constant rate equal to the average rate of production. When the flywheel is attached to a machine such as a punch which uses power at a variable rate, the selection of the driving device may be based on the average rate of power consumption. This often results in a great reduction in the size and cost of the driving device.

2. Capacity of Flywheels. The energy stored in a flywheel is held there in the form of kinetic energy. The total quantity of kinetic energy a moving body contains is given by the expression

$$\text{Kinetic energy} = K = \frac{Wv^2}{2g} \quad (15-1)$$

where W = weight of moving body in pounds

v = velocity in feet per second of the point which is the center of mass of the body

g = acceleration due to gravity = 32.2.

Since the mass of any given body is constant a change in kinetic energy can be secured by changing its velocity. Supposing this change to take place, the expression for the change in kinetic energy is

$$\Delta K = K_1 - K_2 = \frac{Wv_1^2}{2g} - \frac{Wv_2^2}{2g} = \frac{W}{2g} (v_1^2 - v_2^2) \quad (15-2)$$

where v_1 = initial velocity in feet per second

v_2 = final velocity in feet per second.

When the moving body is a rotating wheel the point whose velocity is considered is one located at a distance from the axis of rotation equal to the radius of gyration. For flywheels with heavy rims only 10 to 15 per cent of the kinetic energy resides in the arms and hub and their effect therefore may be neglected in most cases. The radius of gyration for a ring having an internal radius of R_1 and an external radius of R_2 is

$$\rho = \sqrt{\frac{1}{2}(R_2^2 + R_1^2)}$$

For the proportions encountered in ordinary flywheel practice, however, the radius of gyration may be taken as the mean radius of the rim.

Where great accuracy is required the weight of the arms and hub may be taken account of by considering their effect as equivalent to one-eighth of the weight of the rim concentrated at the mean radius of the rim. Making this assumption, equation (15-2) becomes

$$\Delta K = \frac{1.125W}{2g} (v_1^2 - v_2^2) \quad (15-3)$$

where W = weight of the rim in pounds.

The **coefficient of regulation** may be defined as the ratio of the difference between the maximum and minimum velocities to the nominal or rated velocity. It may be expressed algebraically as

$$\delta = \frac{v_1 - v_2}{v} \quad (15-4)$$

where v = nominal or rated velocity.

In designing flywheels for engines it is usual to consider that the maximum velocity exceeds the nominal by as much as the minimum falls short of it. In this case $v_1 + v_2 = 2v$. From equation (15-4) we have $v_1 - v_2 = \delta v$. Multiplying the right- and left-hand sides of these equations, we have

$$v_1^2 - v_2^2 = 2\delta v^2 \quad (15-5)$$

This value of $v_1^2 - v_2^2$ in terms of v^2 and δ may often be used to advantage in equations (15-2) and (15-3) when the nominal velocity and the coefficient of regulation are known. It should be remembered, however, that for such machines as punches and shears the rated velocity is the same as the maximum and equation (15-5) is therefore not applicable.

The value of δ varies greatly, depending upon the kind of work the machine is to do. It is greatest in machines such as punches, where it is limited only by the danger of having the driving belt slip off the pulley. This will happen if the speed variation exceeds 15 per cent. The permissible variation is least for engines driving alternating current generators connected in parallel. For this service δ should not exceed 0.3 per cent and may have to be held to one-tenth of this value. Table 15-1 gives average values of δ for various classes of work.

TABLE 15-1
Coefficient of Regulation for Various Machines

Class of Work	Coefficient
Crushers.....	0.2
Punches and shears.....	0.1 to 0.15
Pumps.....	0.03 to 0.05
Machine shop drives.....	0.025 to 0.028
Weaving and papermaking.....	0.025
Spinning machinery, coarse yarn	0.017 to 0.025
Spinning machinery, fine yarn..	0.01 to 0.02
Direct current generators.....	0.0065
Alternating current generators..	0.0003 to 0.003

The amount of energy, ΔK , for which storage must be provided, is usually determined graphically. If the instantaneous values of the driving force or tangential effort required during one energy cycle are plotted against the distance moved through by the crank-pin in one energy cycle there results a curve the area under which represents the total energy required for one energy cycle. Figure 15-1 shows such a curve drawn for a single-cylinder double-acting

steam engine; the energy cycle corresponds to one revolution of the crank. If, now, a line is drawn parallel to the base line and at a height above it equal to the mean ordinate of the curve, this line may be taken to represent the energy demand of a constant load which would just use up the output of the engine for the energy cycle. It is seen at once that at certain times the supply of energy is greater and at others less than the demand from the load. The function of the flywheel is, as stated, to absorb the excesses and make them available when the supply is deficient, otherwise the engine would stop as soon as the energy supply failed to meet the demand. It is also obvious that the largest amount of energy the flywheel will be called upon to handle is represented by the largest area included between the energy supply or tangential

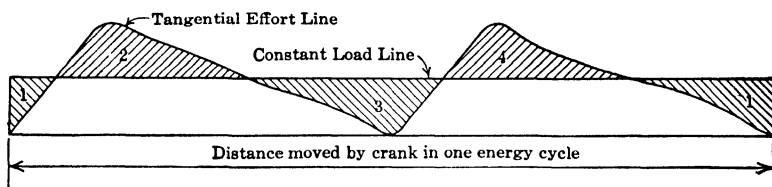


FIG. 15-1. Energy cycle for steam engine.

effort line and the load line. This largest area then becomes the basis of the flywheel design.

The energy represented per square inch of diagram area is the product of the force scale and the space scale. Thus, if the diagram shown in Fig. 15-1 is drawn to a force scale of 1 in. = 50 lb per sq in., and a space scale of 1 in. = $\frac{1}{2}$ ft, each square inch of area under the curve represents $50 \times \frac{1}{2} = 25$ ft-lb of energy per square inch of piston area. The maximum area included between the energy supply curve and the demand line can easily be measured by a planimeter, and the value of ΔK secured by multiplying it by the energy scale. Usually the revolutions per minute have been fixed upon beforehand, so the value of δ is selected with reference to the kind of service for which the engine is intended; the diameter of the flywheel is assumed and the weight is computed by using equation (15-3).

Example. A steam engine having a piston 20 in. in diameter runs at 100 rpm. From the tangential effort diagram shown in Fig. 15-1 the area designated by 2, representing the maximum *excess* or deficiency of energy, is found to be 1.5 sq in. The diagram is drawn to a force scale of 1 in. = 50 lb per sq in. of piston area and a space scale of 1 in. = 3 in. Assuming a mean diameter of 10 ft for the flywheel, determine the weight of the rim required if the engine is to run a direct current generator.

Solution.

Energy scale of diagram

$$= 50 \times \frac{1}{4} = 12.5 \text{ ft-lb per square inch of diagram per square inch of piston area}$$

Area of piston

$$= \frac{\pi \overline{20}^2}{4} = 314 \text{ sq in.}$$

Maximum amount of energy to be handled by the flywheel at one time

$$= \Delta K = 12.5 \times 314 \times 1.5 = 5890 \text{ ft-lb}$$

Value of δ chosen

$$= 0.0065 \quad (\text{Table 15-1})$$

Mean velocity of rim

$$= \frac{\pi \times 10 \times 100}{60} = 52.5 \text{ ft per sec}$$

Combining equations (15-3) and (15-5), we have

$$\Delta K = \frac{1.125W}{2g} \times 2\delta v^2$$

or

$$W = \frac{\Delta K g}{1.125 \delta v^2} = \frac{5890 \times 32.2}{1.125 \times 0.0065 \times (52.5)^2} = 9400 \text{ lb}$$

3. Stresses in Flywheels. The stresses present in flywheels come under three general classes:

- (a) Centrifugal stresses.
- (b) Stresses due to change of speed and load.
- (c) Cooling stresses in the foundry.

The first of these may be estimated with considerable accuracy; the second with only fair accuracy; and the third, which may be fully as large as either of the others, cannot be estimated with any degree of accuracy. The last named can be cared for only by careful proportioning and arrangement of the flywheel parts and by the use of a large factor of safety.

In investigating the centrifugal stresses there are two distinct limiting conditions between which the flywheel in action may be assumed to lie.

- (a) Absolutely flexible arms which do not exert a restraining effect on the rim.
- (b) Rigid arms which completely restrain the rim at the places of attachment.

It can be shown that for (a) the induced stress is

$$s_1 = \frac{12wv^2}{g} \quad (15-6)$$

and for (b) the induced stress is

$$s_2 = \frac{144\pi^2 wv^2 D}{gn^2 t} \quad (15-7)$$

where w = weight in pounds of 1 cu in. of rim material

v = mean velocity of rim in feet per second

g = acceleration of gravity = 32.2

D = mean diameter of rim in feet

n = number of arms

t = thickness of rim in inches.

In actual flywheels the arms stretch somewhat, but not enough to permit full radial expansion of the rim. The stress actually induced is the sum of a certain portion of that due to expansion

and a certain portion of that due to bending. Professor Lanza has concluded that in the average flywheel the arms permit about three-fourths of the expansion perfectly flexible arms would allow. Under this condition the resultant induced stress in the rim would be

$$s = \frac{3}{4} s_1 + \frac{1}{4} s_2 = \frac{9wv^2}{g} \left(1 + \frac{4\pi^2 D}{tn^2} \right) \quad (15-8)$$

Example. A cast iron flywheel with six arms has a mean diameter of 13 ft and rotates at 120 rpm. The rim has a width of 10 in. and a thickness of 12 in. Determine the stress induced in the rim, considering first that the arms are perfectly flexible and second that they allow three-fourths full radial expansion.

Solution. Assuming full radial expansion possible, equation (15-6) applies.

Weight of 1 cu in. of cast iron = 0.26 lb

Mean velocity of rim

$$v = \frac{\pi \times 13 \times 120}{60} = 81.9 \text{ ft per sec}$$

$$s_1 = \frac{12wv^2}{g} = \frac{12 \times 0.26 \times 81.9^2}{32.2} = 650 \text{ lb per sq in.}$$

Assuming three-fourths full radial expansion equation (15-8) applies.

$$\begin{aligned} s &= \frac{9wv^2}{g} \left(1 + \frac{4\pi^2 D}{tn^2} \right) = \frac{9 \times 0.26 \times 81.9^2}{32.2} \left(1 + \frac{4\pi^2 \times 13}{12 \times 6^2} \right) \\ &= 1110 \text{ lb per sq in.} \end{aligned}$$

Usually the dimensions and weight of the wheel are fixed by practical considerations, such as the space available and the mass required to redistribute the required amount of energy. The stresses induced are determined merely as a check to ascertain the safety of the wheel.

Flywheel arms act as cantilevers loaded from two sources:

- (a) The pull of the belt, if the flywheel functions as a band wheel.
- (b) The inertia of the rim which opposes variations in speed.

The first of these is readily estimated with a considerable degree of accuracy, but the second, depending as it does upon the rate at which the speed variation takes place, is indefinite. A usual assumption is that the load applied at the rim is sufficient to produce a torque at the shaft equal to the maximum driving torque. This being the case, the force at the mean radius of the rim is

$$F = \frac{T}{R \times 12}$$

where T = maximum torque on the shaft in pound-inches. Hence the load per arm is

$$\frac{F}{n} = \frac{2T}{D \times 12n}$$

Supposing the arm to extend to the center of the flywheel shaft, its length is $(D/2) \times 12$ in. The maximum bending moment, if the arm is considered as a free cantilever, then is

$$M_1 = \frac{2T}{D \times 12 \times n} \times \frac{D}{2} \times 12 = \frac{T}{n} \quad (15-9)$$

This assumption presupposes that the rim is flexible enough to allow the arm to bend as shown in Fig. 15-2, but the condition rarely exists. In many cases the rim is so heavy as to be, for all practical purposes, rigid. In this event the arm, where it joins the rim, must remain normal to the rim, and as a whole must bend somewhat as shown in Fig. 15-3. It is, therefore, equivalent to a guided cantilever. The maximum bending moment for this case is one-half that for the free cantilever and is

$$M_2 = \frac{T}{2n} \quad (15-10)$$

This bending moment occurs at each end of the arm.

In designing the arm the dimensions at the center line of the hub are made such as to enable the arm to withstand the maximum moment M_1 which can occur there. The dimensions at the rim are determined from M_2 .

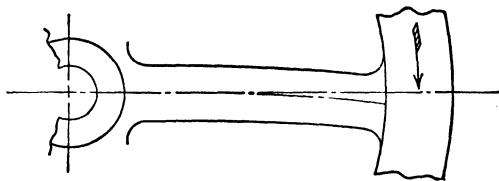


FIG. 15-2. Arm as a free cantilever.

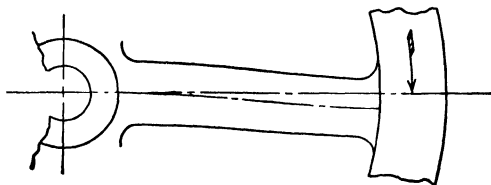


FIG. 15-3. Arm as a guided cantilever.

Another method for finding the dimensions at the rim is to find the dimensions of the arm at the center line of the hub as outlined above and taper the arm from the hub to the rim about one-quarter inch per foot of length. This taper applies to the greater dimension of the arm. The taper of the lesser dimension is half this amount.

Flywheel arms are almost invariably made with an elliptical cross section, the major axis being twice the minor. They are placed in the wheel so that the major axis lies in the plane of rotation. Thus windage is minimized and the arm acts to maximum advantage relative to strength and stiffness.

In fixing the allowable stress to use when designing a flywheel arm it should be borne in mind that a direct tensile stress due to the radial expansion of the rim is present, and that the sum of this stress and that due to bending should not exceed the ultimate strength of the material divided by a suitable factor of safety.

If the length of arm is taken as equal to the mean radius of the rim the stress in the arm can be found as follows. Let

r = mean radius of the rim in inches

Δ = increase in radius caused by radial expansion of the rim under the action of centrifugal force

Then the unit stretch of the arm is $\frac{\Delta}{r}$ and that of the rim is

$$\frac{2\pi(r + \Delta) - 2\pi r}{2\pi r} = \frac{\Delta}{r}$$

The unit deformations of the arm and rim are, therefore, the same; and since the material of the arm and rim are the same, the stresses induced in each by rim expansion are the same. The value of this stress when the rim is fully expanded is given by equation (15-6).

Example. The cast iron flywheel of a Corliss engine is 10 ft in diameter and has six arms. The rim width is 6 in. and depth 8 in. The speed of the engine is 100 rpm. Determine the stress induced in the rim, assuming that the arms permit three-fourths full radial expansion.

Solution.

$$\text{Velocity of rim} = \frac{\pi \times 10 \times 100}{60} = 52.4 \text{ ft per sec}$$

From equation (15-8)

$$s = \frac{9wv^2}{g} \left(1 + \frac{4\pi^2 D}{tn^2} \right) = \frac{9 \times 0.26 \times 52.4^2}{32.2} \left(1 + \frac{4\pi^2 \times 10}{8 \times 6^2} \right)$$

$$= 470 \text{ lb per sq in.}$$

Example. Assuming that the arms for the wheel described above are to be elliptical in section, with the minor axis one-half the major, and taking the maximum allowable stress as 2000 lb per sq in., determine the dimensions of the cross section of the arms at the center of the hub and at the middle of the rim. The

maximum driving torque as found from the tangential effort diagram is 142,000 lb-in.

Solution.

Bending moment on arm at center of hub

$$= \frac{142,000}{6} = 23,700 \text{ lb-in.}$$

Section modulus, I/c , for an ellipse having a major axis twice the minor = $\frac{\pi h^3}{64}$.

The maximum allowable resultant stress of 2000 lb per sq in. is made up of the stress due to bending and the stress due to partial radial expansion of the rim. This latter stress is the same as that induced in the rim, and, for three-fourths full radial expansion, is

$$\frac{3}{4} \times \frac{12uv^2}{g} = \frac{3}{4} \times \frac{12 \times 0.26 \times 52.4^2}{32.2} = 199 \text{ lb per sq in.}$$

The maximum stress available to take care of the bending is therefore

$$2000 - 199 = 1801, \text{ say } 1800 \text{ lb per sq in.}$$

$$M_1 = s \frac{I}{c} = s \frac{\pi h_1^3}{64}$$

therefore

$$h_1 = \sqrt[3]{\frac{64M_1}{\pi s}} = \sqrt[3]{\frac{64 \times 23,700}{\pi \times 1800}} = 6.45 \text{ in., say } 6\frac{1}{2} \text{ in.}$$

and the width is $\frac{6\frac{1}{2}}{2} = 3\frac{1}{4} \text{ in.}$

Determining the dimensions of the arm at the center of the rim on the assumption that the arm acts as a guided cantilever:

$$M_2 = \frac{M_1}{2} = \frac{23,700}{2} = 11,850 \text{ lb-in.}$$

and

$$h_2 = \sqrt[3]{\frac{64 \times 11,850}{\pi \times 1800}} = 5.1, \text{ say } 5 \text{ in.}$$

and the width is $\frac{5}{2} = 2\frac{1}{2}$ in.

If these dimensions are determined by assuming the arm to taper in its longer axis $\frac{1}{4}$ in. per ft, the height of the section is

$$6\frac{1}{2} - 5 \times \frac{1}{4} = 5\frac{1}{4} \text{ in. and the width is } \frac{5\frac{1}{4}}{2} = 2\frac{5}{8} \text{ in.}$$

The dimensions as determined by the two methods are in close agreement in this case.

4. Construction of Flywheels. The simplest form of a flywheel consists of a heavy rim connected to a hub at the center by means

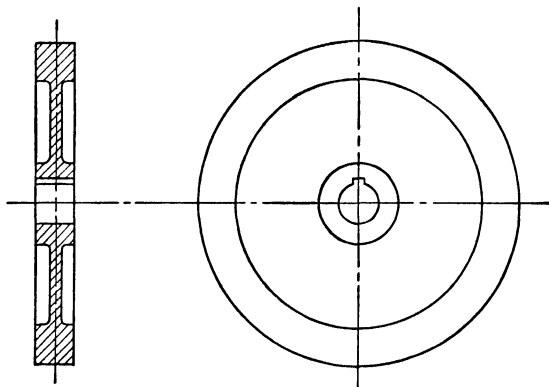


FIG. 15-4

of a web, as shown in Fig. 15-4. This style is widely used for the flywheels of motor boat and other small engines. The difficulty of securing satisfactory castings limits the application of the webbed type to wheels under 2 ft in diameter. For larger sizes the web is replaced by arms, the number of which depends upon the size of the wheel and the speed of rotation. In the smaller flywheels the

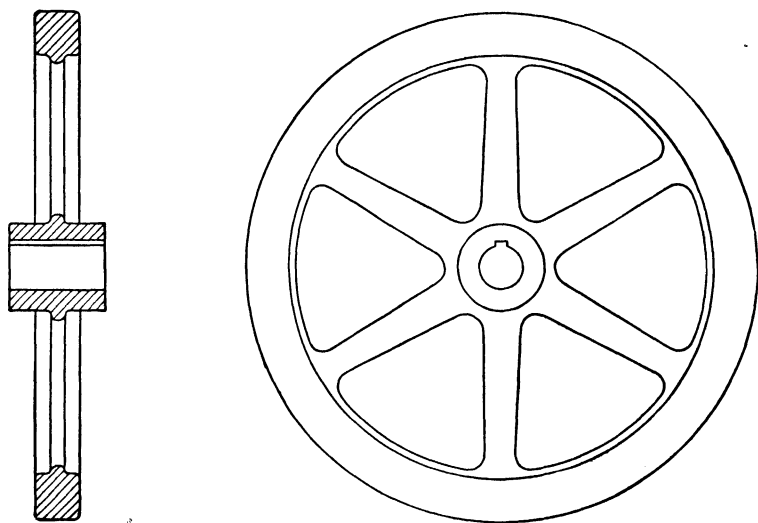


FIG. 15-5

rim, arms, and hub are cast in one piece, as shown in Fig. 15-5. As the size increases, the stresses set up by the unequal rates of cooling of the rim, and hub become so serious that, in some instances, they may cause the arms to crack. To relieve these stresses the hubs of large single-piece wheels are often split as shown in Fig. 15-6. Such hubs are then reinforced with bolts as shown. Annealing may also be resorted to with the smaller flywheels, since they can be accommodated in the furnace. By this means severe shrinkage strains may be lessened and slight ones eliminated.

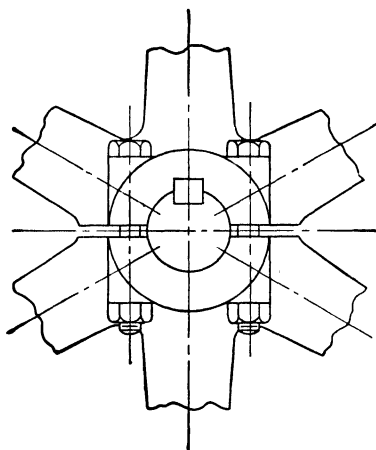


FIG. 15-6

The space available on freight cars sets a limit to the size of

wheels that can be transported, and therefore we find all wheels above 10 ft in diameter cast in more than one piece. See Fig. 15-7. The pieces composing the wheel are joined at the hub either by through bolts, as shown in Fig. 15-6, or by rings shrunk on the ends

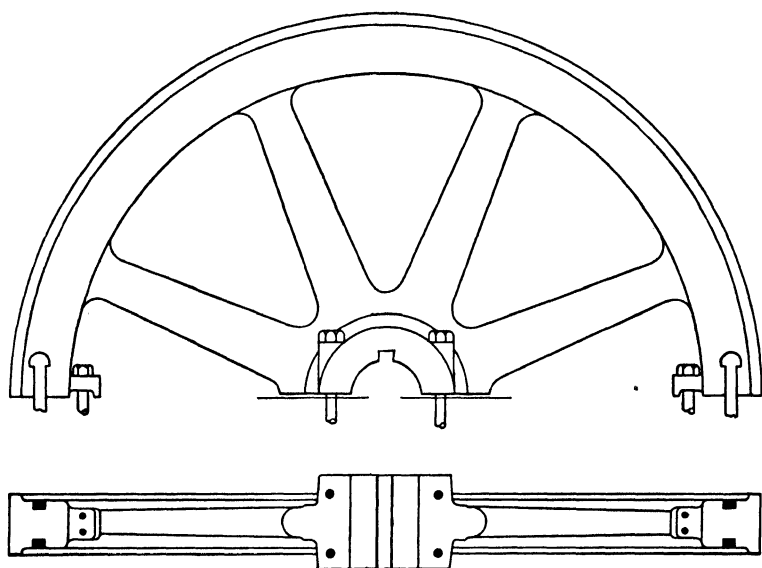


FIG. 15-7. Two-piece flywheel with through bolts and links.

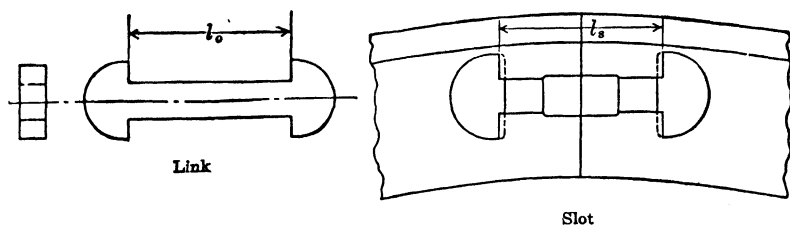


FIG. 15-8. Link connection for flywheel rim segments.

of the hub. The former practice is more common. The joints in the rim are secured by using links shrunk into place. See Fig. 15-8. The dimensions of the links are made such that the stress induced in them by the centrifugal force of the halves of the rim

will give a factor of safety about three-quarters of that present in the rim proper. The links are made enough shorter than the slots into which they fit to induce a shrinkage stress only a little below the elastic limit of the link material. This makes it certain that the segments of the rim will not separate, even though the wheel is considerably overspeeded.

Wheels for speeds up to 6000 ft per min are made of cast iron. For velocities moderately greater they are made of steel castings; and for extreme velocities of more than 10,000 ft per min they are built up from steel plates. In some cases the rims are made up of a number of layers of piano wire wrapped in a steel-plate housing to which the arms are attached. For a more detailed description of high speed flywheels the student is referred to Halsey's "Handbook."

PROBLEMS — CHAPTER XV

1. A 14- by 36-in. steam engine rotates at 115 rpm. The rectified tangential effort diagram is plotted to a force scale of 1 in. = 40 lb per sq in. and a space scale of 2 in. = 12 in. The area representing the maximum excess or deficiency of energy is found to be 1.76 sq in. Assuming a mean velocity of the rim of 61 ft per sec, determine the weight of rim necessary to keep the speed variation to 0.5 per cent. *Ans.* 8310 lb.

2. An 18- by 20-in. high speed steam engine rotates at 250 rpm. The tangential effort diagram is plotted to a force scale of 1 in. = 80 lb per sq in. and a space scale of 3 in. = 12 in. The area representing the maximum variation of energy is found to be 1.25 sq in. Assuming the flywheel to have a mean diameter of 6 ft and a width of rim face of 21 in., determine the thickness of the rim to keep the speed variation within 0.65 per cent. (Cast iron weighs 0.26 lb per cu in.) *Ans.* 4.95 in., say 5 in.

3. The flywheel of a punch must be capable of supplying 1923 ft-lb of energy in order that the machine may punch a hole. The flywheel is 36 in. in mean diameter and rotates at 210 rpm when running at normal speed. Assuming a permissible drop of 15 per cent in speed while the hole is being punched and taking the weight of cast iron as 0.26 lb per cu in., determine the cross-sectional area required for the rim. *Ans.* 12.4 sq in.

4. The tangential effort diagram of a steam engine is drawn to a force scale of 1 in. = 100 lb per sq in. of piston area and a space scale of 4 in. = 1 ft. The area on this diagram representing the maximum amount of energy to be handled by the flywheel is found to be 1.75 sq in. The engine has a bore of 20 in. and a stroke of 24 in., and turns at 240 rpm. The maximum speed variation is not to exceed 1 per cent of the mean speed. Consider the flywheel

effect of the arms and hub, and determine the weight and cross-sectional area needed for a rim having a mean diameter of 8 ft. Cast iron weighs 0.26 lb per cu in.

5. Determine the stress set up in the rim of the flywheel found in Prob. 1, assuming the arms permitted full radial expansion of the rim.

Ans. 360 lb per sq in.

6. Determine the stress induced in the rim of the flywheel found in Prob. 2; assume that the wheel has six arms and that these permit three-fourths full radial expansion.

Ans. $s = 1035$ lb per sq in.

7. A high speed steam engine rotating at 300 rpm has a cast iron flywheel 5 ft in diameter. The rim is 15 in. wide and 2 in. thick and is joined to the hub by six arms. Assume that the arms permit three-fourths full radial expansion of the rim and determine the stress induced in the rim.

Ans. $s = 1685$ lb per sq in.

8. The maximum torque of the engine described in Prob. 1 is 161,000 lb-in. The flywheel has six arms of elliptical cross section, the minor axis being one-half the major. Assume that each arm takes its share of the load and determine the dimensions for the arm at the center line of the hub and of the rim for a maximum total unit stress of 1500 lb per sq in.

Ans. $h = 7.63$ in. and $h' = 6.05$ in.

9. A certain flywheel exerts a maximum twisting moment of 8370 lb-in. on a shaft. It has six arms, elliptical in cross-section, the major axis being twice the minor. The velocity of the rim is 33 ft per sec. Assume that the arms act as simple cantilevers and permit three-fourths full radial expansion of the rim. Determine the dimensions of the cross-section of the arm at the center of the hub for a maximum stress of 2000 lb per sq in.

Ans. $h = 2.46$ in.

10. The cast iron flywheel of an engine has a mean diameter of 8 ft and rotates at 240 rpm. The rim is 18 in. wide and 3 in. thick. The wheel has six arms of elliptical cross section, the minor axis being one-half the major. The maximum torque to be transmitted is 180,000 lb-in. Determine the dimensions for the arms at the center of the hub, assuming that each arm takes its share of the load, that the rim has three-fourths full radial expansion, and that the maximum allowable stress is 2000 lb per sq in. Cast iron weighs 0.26 lb per cu in.

CHAPTER XVI

FLAT PLATES, CYLINDERS, WELDED CONSTRUCTION

1. Flat Plates. Cylinder heads, steam chest covers, piston faces, and similar machine parts subjected to pressure may be considered as flat plates. The most general shapes for such members are circles or rectangles, and for each of these the loading may be either concentrated or distributed. Also the plate may be rigidly fixed at the edges or merely supported there.

For circular plates carrying a uniformly distributed load and merely supported at the edges the maximum stress occurs at the center and is

$$s = \frac{5wr^2}{6t^2} \quad (16-1)$$

where w = load in pounds per square inch

r = radius of the plate in inches

t = thickness of the plate in inches.

If the plate is fixed at the edge the maximum stress occurs at the edge and is

$$s = \frac{2wr^2}{3t^2} \quad (16-2)$$

For circular plates carrying a concentrated load at the center applied over a small area of radius r_0 , and merely supported at the edges, the maximum stress occurs at the center and is

$$s = \frac{P}{\pi t^2} \left[\frac{4}{3} \log_e \frac{r}{r_0} + 1 \right] \quad (16-3)$$

where P = total load.

If the plate is fixed at the edge the maximum stress occurs there

and is

$$s = \frac{4P}{3\pi t^2} \log_e \frac{r}{r_0} \quad (16-4)$$

For rectangular plates having a length of L and a width of B in. Bach gives the induced stress when the plate is merely supported at the edges as

$$s = \frac{w}{2t^2} \times \frac{L^2 B^2}{L^2 + B^2} \quad (16-5)$$

When the edges are fixed the induced stress is

$$s = \frac{w}{3t^2} \times \frac{L^2 B^2}{L^2 + B^2} \quad (16-6)$$

It should be noted that the equations given above apply to flat plates only and are not to be used when the plates are ribbed.

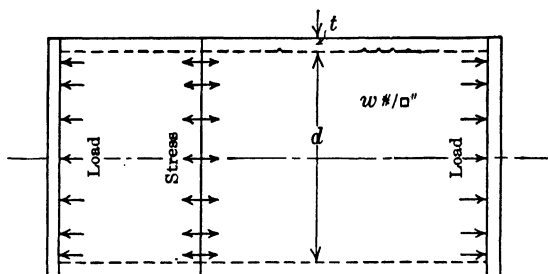


FIG. 16-1

2. Thin Tubes. When a closed thin tube is subjected to internal pressure, the stresses induced are of two kinds, longitudinal and tangential, depending upon the direction of loading considered.

Longitudinal stresses are set up when the pressure acts against the ends of the tube as in Fig. 16-1. Let

s = unit intensity of longitudinal stress, pounds per square inch

w = unit pressure in pounds per square inch

d = internal diameter of the tube in inches

t = thickness of the tube in inches

Then the total force tending to rupture the tube is $\frac{w\pi d^2}{4}$ and the total resisting strength equals $s\pi dt$. Since, for equilibrium,

$$s\pi dt = \frac{w\pi d^2}{4} \quad \text{or} \quad s = \frac{wd}{4t} \quad (16-7)$$

Tangential stresses are induced when the pressure acts on the inside periphery of the tube.

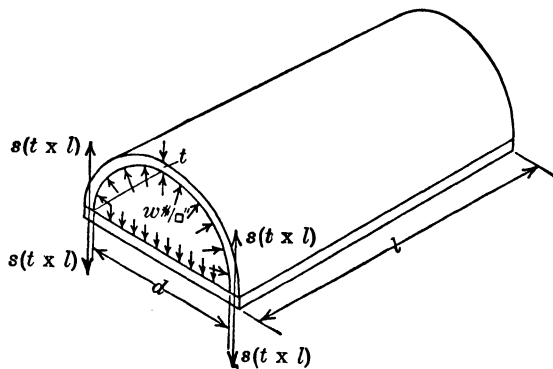


FIG. 16-2

In Fig. 16-2 let s , w , d , and t represent the same equivalents as above and l the length of the tube. Figure 16-2 represents one-half of a thin tube closed across its diameter by a plate. For equilibrium, the summation of the vertical pressure forces must equal zero. Therefore the vertical summation of the radial forces must equal $w dl$. The total resisting strength equals $2(tl)s$. Since the external force is balanced by the total resisting strength

$$2(tl)s = w dl \quad \text{and} \quad s = \frac{wd}{2t} \quad (16-8)$$

Since the longitudinal stress is only half the tangential stress, it is customary to design such tubes or cylinders on the basis of the tangential stress alone.

When the tube is subjected to *external* pressure, the relationship between the induced stress, the pressure, and the dimensions

of the tube is not so simple. This is because the wall of the tube subjected to external pressure is in a state of unstable equilibrium, the external pressure tending to increase any departure from roundness which may be present in the tube. This introduces bending in the tube walls and eventually causes collapse. It has been found that the closed ends of the tube exert a marked supporting effect on the portion of the tube adjacent to them. If the tube has a length of more than six times its diameter the support furnished by the heads does not add to the strength of the tube as a whole, for collapse will occur in the region beyond their influence. With tubes shorter than six times their diameter the supporting effect of the ends is a prominent factor in the resistance of the walls of the tube to collapse.

Basing his conclusions on the results of experiments conducted at the Illinois Engineering Experiment Station, Professor A. P. Carman has proposed two equations for tubes whose lengths exceed six times their diameters. These equations, as restated by Kimball and Barr, are for t/d less than 0.025 and pressures less than 600 lb per sq in.

$$t = d \sqrt[3]{\frac{w}{k}} \quad (16-9)$$

where t = thickness of wall in inches

w = unit collapsing pressure in pounds per square inch

= working pressure $\times$ factor of safety

d = outside diameter of the tube in inches

k = 25,150,000 for brass

= 50,200,000 for seamless or lap-welded steel.

For t/d greater than 0.025 and pressure greater than 600 lbs per sq in.

$$t = \frac{d(w + c)}{k} \quad (16-10)$$

where k = 93,365 and c = 2474 for brass

k = 95,520 and c = 2090 for seamless cold-drawn steel

k = 83,270 and c = 1025 for lap-welded steel.

When the length of the tube is less than six times its diameter Fairbairn's equation applies, and

$$t = \sqrt{\frac{wld}{9,675,600}} \quad (16-11)$$

where l = length of tube in inches.

3. Thick Cylinders. When a *thick* cylinder wall is subjected to either internal or external pressure the stress induced in the wall is not uniformly distributed. The tangential stress is a maximum at the inner and a minimum at the outer surface of the cylinder. This condition obtains whether the pressure is internal or external. When thick cylinders are used to confine gas or fluid pressure the ends are of necessity closed and the pressure acting against them produces direct longitudinal stresses. For this condition the tangential, or hoop stress at any point is

$$s = \frac{r_1^2 w_1 - r_2^2 w_2 + \frac{4r_1^2 r_2^2}{x^2} (w_1 - w_2)}{3(r_2^2 - r_1^2)} \quad (16-12)$$

where r_1 = internal radius of the cylinder in inches

r_2 = external radius of the cylinder in inches

w_1 = unit internal pressure in pounds per square inch

w_2 = unit external pressure in pounds per square inch

x = radius in inches at which stress s occurs.

x is made equal to r_1 if the maximum stress is sought and equal to r_2 if the minimum is required.

4. Welded Construction. The art of building up members out of rolled shapes joined by means of autogenous welding has developed steadily until parts made in this manner are being used widely in many fields. The most common welding systems are those of the electric arc, atomic hydrogen, and oxyacetylene. Each system has advantages peculiar to itself, but when welding is skillfully done the above methods produce results about equally good, and joints as strong as the joined pieces can be secured.

Members built up out of rolled shapes welded together possess several marked advantages over those made by casting.

(a) No patterns are required. This results in more rapid construction and lower costs when only a few parts are to be produced. However, this advantage may disappear in quantity production.

(b) Less machine work is required since rolled shapes are accurately sized.

(c) Increased strength is secured for a given weight or reduced weight for a given strength, since mild steel is about three times stronger in tension than cast iron.

(d) Increased stiffness also accrues because the modulus of elasticity of steel is about double that of cast iron.

It is evident that items (c) and (d) disappear when steel castings are considered. The principal disadvantage in welding lies in the fact that the soundness of the joints depends, to a very large extent, on the skill and carefulness of the welder.

Up to the present, welding has been employed mostly in the repair and manufacture of machine frames and bed plates, pressure vessels, pipe lines where welded joints have replaced joints of other types, automobile parts such as mufflers, and many airplane fittings. It is taking an increasingly important place as a means of construction of machine members of all kinds, from the largest to the smallest. It is probable that this trend will continue.

5. Types of Joints. Four methods are in common use for joining plates by means of autogenous welds. They are

(a) The butt weld.

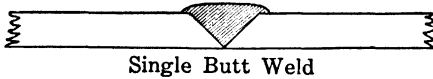
(b) The lap weld.

(c) The fillet weld.

(d) The rivet weld.

These are shown in Fig. 16-3. The butt weld is the most economical of material and permits a joint which is practically flush with the surfaces of the parts joined. When the thickness of the plates to be joined is 10 gage or less no scarfing of the edges is necessary; when greater than this they should be scarfed as shown. Either the single or double butt weld may be used; the latter is preferred for joining heavy plates.

The lap weld is used where it is desirable to lap one piece over the other. For ordinary joints the single lap weld is satisfactory and, of course, cheaper than the double lap weld which should be used where a maximum of strength is desired.



Single Butt Weld



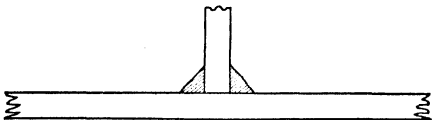
Double Butt Weld



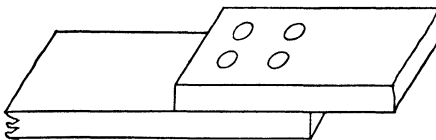
Single Lap Weld



Double Lap Weld



Fillet Weld



Rivet Weld

The fillet weld is used where two plates are to be joined with their planes at an angle to each other.

The rivet weld is employed where conditions make it undesirable to use either a lap or a butt weld. In making it, holes are drilled or punched in one of the pieces to be joined. The pieces are then lapped over each other and the welding metal deposited in the holes. It fuses to each of the plates and gives a secure joint which will not loosen under a weaving strain.

6. Strength of Welded Seams. Figure 16-4, made from data given in the manual issued by the Lincoln Electric Company, shows graphically the ultimate strength of arc-welded seams in

FIG. 16-3. Types of welded joints.

pounds per linear inch of joint, for flush top butt welds, for lap welds in cross shear and in longitudinal shear. Double welded

lap joints are about as strong as the plates joined. Butt welds can be made with a strength equal to that of the plates joined by rounding up the added metal on both sides of the plate, thus in-

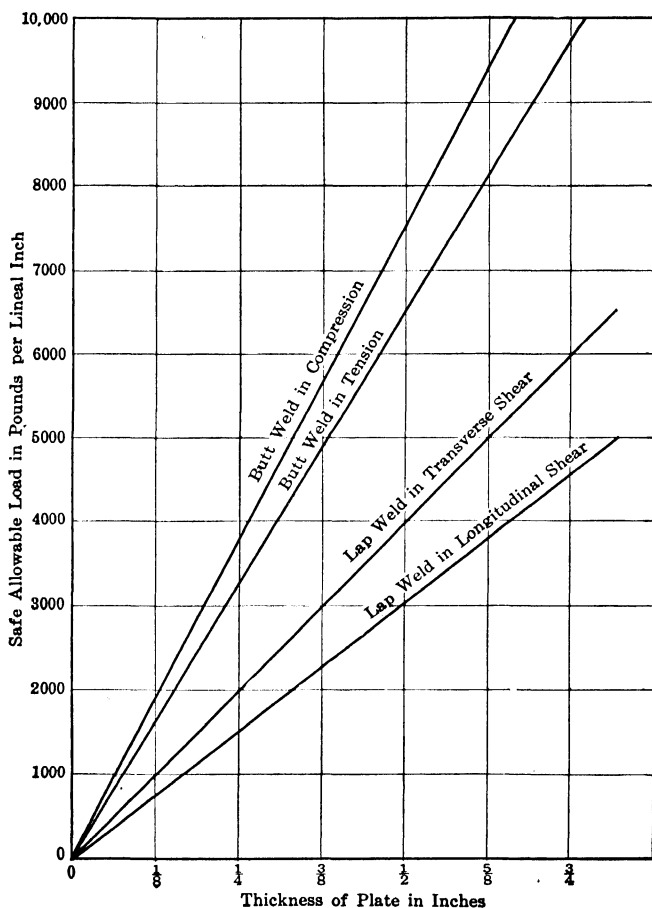


FIG. 16-4. Allowable load per lineal inch for welded joints.

surrounding sound weld metal, then machining the joint down to the thickness of the plate to avoid stress concentrations at the edge of the bead. Figure 16-5, taken from the same manual, shows a

welded joint subjected to a load acting in the direction of the arrow. Welds *d* and *e* are said to be in longitudinal shear whereas weld *c* is in transverse shear. The latter is about 30 per cent stronger than the former.

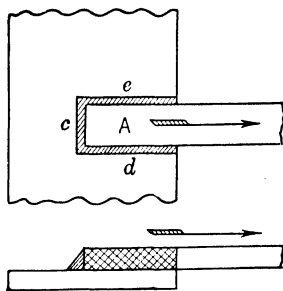


FIG. 16-5

The strength of fillet welds depends upon the amount of metal deposited, and can be made stronger than the plates joined.

Owing to difficulties in securing a good weld at the beginning and end of the seam it is recommended that $\frac{1}{4}$ in. be added to the length of all seams when Fig. 16-4 indicates a length of less than 6 in. required to carry the load.

Example. The bar *A* shown in Fig. 16-5 is to be secured by the welds at the sides *d* and *e* only and is to carry a load of 20,000 lb. The material is $\frac{3}{8}$ in. thick. Determine the length of bar *A* that must be lapped over plate *B*.

Solution. From Fig. 16-4 the safe allowable load of a weld in $\frac{3}{8}$ in. plate subjected to longitudinal shear is 2230 lb per in. of length.

$$\text{Length of joint required} = \frac{20,000}{2230} = 8.9 \text{ in., say 9 in.}$$

This is divided into two seams each $9 \div 2 = 4\frac{1}{2}$ in. long.

Since the seams are to be less than 6 in. long, the length of each should be increased by $\frac{1}{4}$ in. for starting and stopping.

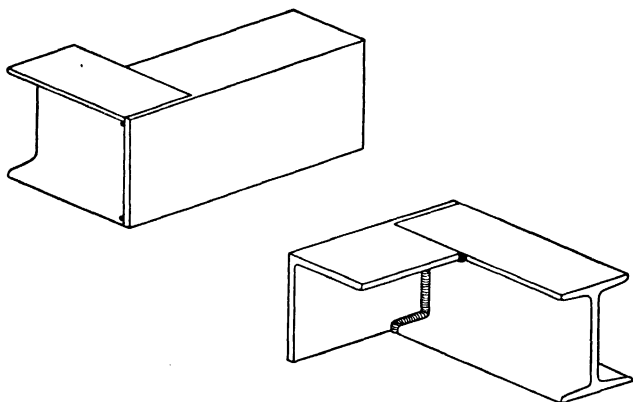
Lap of *A* over *B* should be $4\frac{1}{2} + \frac{1}{4} = 4\frac{3}{4}$ in.

7. Details of Welded Construction. In deciding on how to arrange the elements that enter into any welded structure, consideration must be given to three principal factors:

- (a) Strength.
- (b) Cost.
- (c) Appearance.

The first of these has been discussed briefly under Art. 6.

The second, cost, is exceedingly important and should not be neglected. To keep expense down to a minimum, the joints should involve as short welds as are consistent with strength; and the



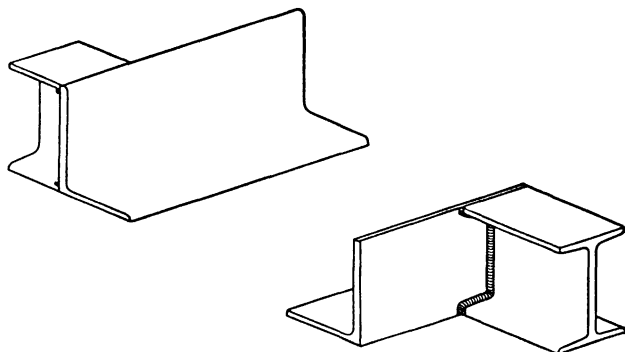
Lincoln Electric Co.

FIG. 16-6

cutting of the pieces joined should be as simple as possible in order that time may be minimized in doing the work.

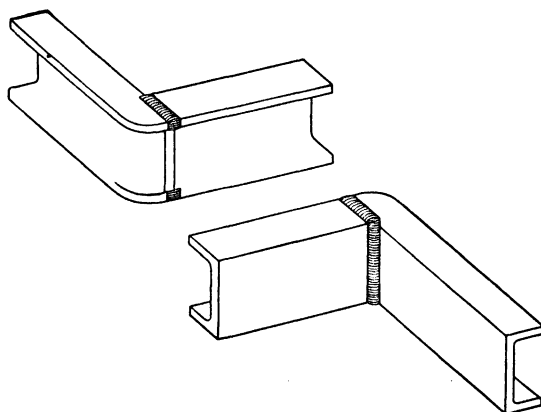
Good appearance is always an asset. Sometimes the joint that is simplest and cheapest is also the one having the best appearance. More often, good appearance can be secured only at an increased cost. The ultimate decision must rest on the judgment of the designer.

Figures 16-6 to 16-8 show several typical joints made between an angle and an I-beam. The one shown in Fig. 16-6 is the simplest and cheapest. It is made by notching the horizontal leg of the angle to admit the full width of the I-beam. Welds are located on both sides of the joint. It is a strong, serviceable joint having a fair appearance. Figure 16-7 shows another arrangement of the joint between an angle and an I-beam. This involves a minimum of cutting, the cost is low, and the appearance is good. Frequently the appearance can be improved by the simple expedient of rounding the corners as shown in Fig. 16-8. This procedure adds slightly to the cost.



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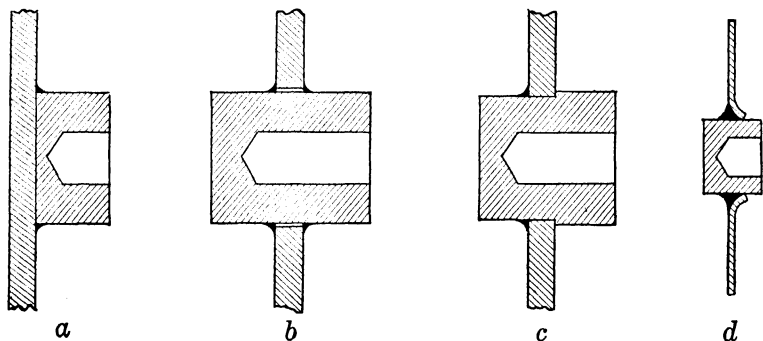
FIG. 16-7



Lincoln Electric Co.

FIG. 16-8

Frequently bosses are required on members. These are easily provided for by welding on pieces of cylindrical stock, either solid or hollow as may be required. Figure 16-9 shows four different ways in which this may be done. That shown in Fig. 16-9(a) is very cheap, is easily applied, and is quite strong if not made too long. That shown in Fig. 16-9(b) is similar to (a) but it



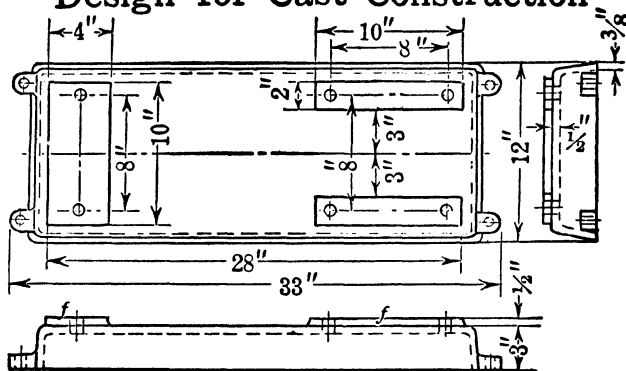
Lincoln Electric Co.

FIG. 16-9. Methods of attaching bosses to plates.

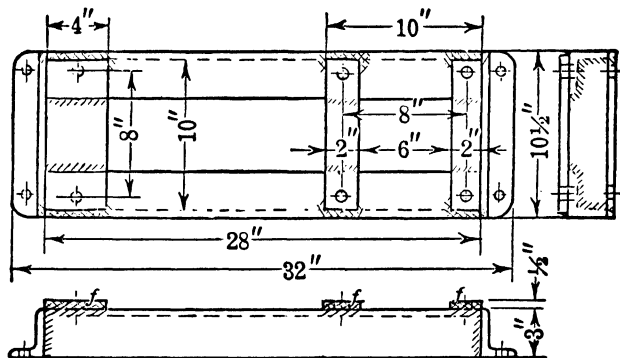
permits a longer boss, and since it has fillet welds on each side it is considerably stronger. If accurate placing of the boss is important a hole may be drilled in the plate and a plug fitted snugly into it and welded in place as shown in Fig. 16-9(c). Bosses may be attached to very thin plates by cutting a hole in the plate slightly smaller than the boss, flanging the edge, and fillet welding as shown in Fig. 16-9(d).

Figure 16-10 shows a base made of cast iron and a similar one of welded steel angles and plates. A comparison of the two shows that the cast iron base weighed 85 lb against 37.8 for the welded base. Also the factor of safety for the welded plate was found to be 14 against 10 for the cast iron.

Design for Cast Construction



Design for Arc-Welded Steel Construction



SYMBOLS

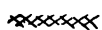
-  Filler Weld on Near Side of Joint
-  Filler Weld on Far Side of Joint
- All Welds $\frac{3}{16}$ " Bead

FIG. 16-10

PROBLEMS — CHAPTER XVI

1. The piston of a gas engine is $4\frac{1}{2}$ in. in diameter and is made of cast iron. The maximum pressure acting on it is 350 lb per sq in. Since the skirt of the piston is quite thin, and therefore flexible, the face will be considered as a flat plate merely supported at the edges. Determine the thickness for the face for a maximum tensile stress of 4000 lb per sq in.

Ans. 0.61 in., say $\frac{5}{8}$ in.

2. What thickness would be required for the piston face described in Prob. 1 if it is considered fixed at the edges?

Ans. $t = 0.543$ in.

3. A steam chest cover 10 in. long by 8 in. wide sustains a steam pressure of 125 lb per sq in. Assume the cover to be merely supported at the edges and determine the thickness for a maximum stress of 5000 lb per sq in. in tension.

Ans. $t = 0.698$ in.

4. What should be the thickness of the steam chest cover described in Prob. 3 if it is considered to be fixed at the edges?

Ans. $t = 0.57$ in.

5. A steel boiler tube $\frac{1}{8}$ in. thick has an outside diameter of 4 in. and is 16 ft long. Determine the longitudinal and transverse stresses induced if the tube is used in a water-tube boiler carrying 350 lb per sq in. steam pressure.

Ans. 2625 and 5250 lb per sq in.

6. Suppose the tube described in Prob. 5 to be 0.1 in. thick and used in a fire-tube boiler carrying 200 lb per sq in. steam pressure. Determine the collapsing pressure and factor of safety.

Ans. $w = 783$ lb per sq in.; 3.92.

7. A brass tube for an intercooler of an air compressor is to be 6 ft long and 1 in. in outside diameter. The difference in pressure between the air outside the tube and the water inside it is 1000 lb per sq in. The ultimate strength of the brass may be taken as 40,000 lb per sq in. Use a factor of safety of 8 and determine the proper thickness for the tube.

Ans. $t = 0.112$, say $\frac{1}{8}$ in.

8. The furnace flue of a Scotch marine boiler is 36 in. in diameter and is built up of sections 4 ft long. Steam at 150 lb per sq in. pressure is to be carried in the boiler. Assume a factor of safety of 6 and determine the thickness for the flue.

Ans. $t = 0.402$ in.

9. The cylinder of a hydraulic accumulator has an internal diameter of 16 in. and is 2 in. thick. The internal pressure at its maximum is to be 700 lb per sq in. Determine the maximum and minimum tangential stresses induced.

Ans. 3015 and 2070 lb per sq in.

DRAFTING ROOM PROBLEMS

The drawings are to be made on a 12×18 in. sheet with a border $1\frac{1}{4}$ in. wide on the left-hand side and $\frac{1}{2}$ in. wide on the other three sides. A space 2×6 in. is provided for the title in the lower right-hand corner.

In the illustrations, the dimensions shown in circles, thus $\textcircled{2\frac{1}{2}''}$, are distances measured from the border line. **These dimensions are always to be taken full size**, regardless of what scale is specified for the drawing.

PROBLEM 1

TABOR INDICATOR MECHANISM

1. Plot the linkage, *four times full size*, with BP horizontal as shown. Move the pencil point P upward, $\frac{1}{4}$ in. at a time, in a vertical line for a total travel of 2 in., starting 1 in. below the initial position. Locate the corresponding positions of points C , D , and E .

2. Determine and note on your drawing the **ratio of magnification** of the motion of point P to D .

3. Determine and indicate on the drawing the **radius and center of the circular arc** which most closely approximates the path of point E . Draw the **outline of the slot** which is required to guide the roller.

DATA:

$$A-B = 1\frac{1}{4} \text{ in.}$$

$$B-C = \frac{5}{8} \text{ in.}$$

$$B-P = 3\frac{1}{8} \text{ in.}$$

$$C-E = \frac{5}{8} \text{ in.}$$

$$C-D = 1 \text{ in.}$$

$$\text{Diameter of roller} = \frac{3}{16} \text{ in.}$$

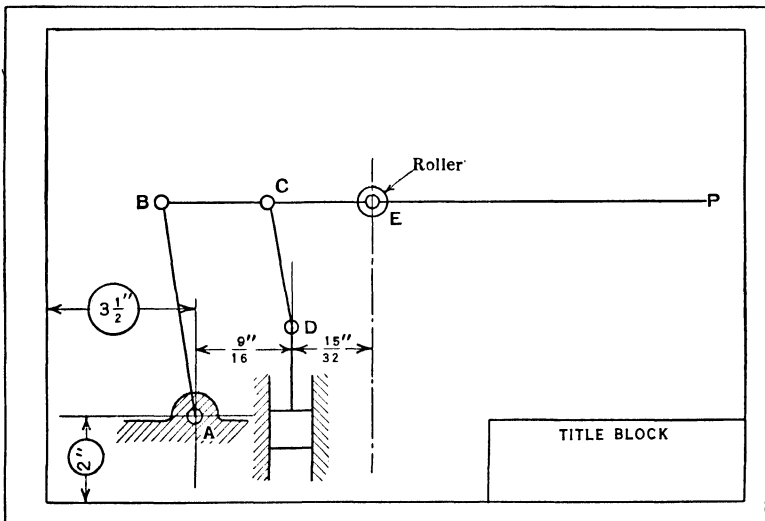


PLATE 1

PROBLEM 3

DISPLACEMENT, VELOCITY, AND ACCELERATION-TIME CURVES

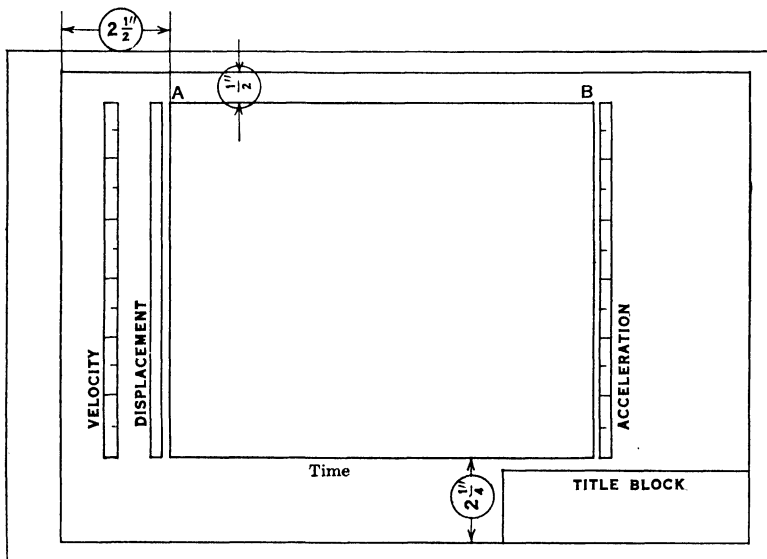
The following data relate to a moving body.

Time	Distance	Time	Distance
0 sec	0 ft	6 sec	40.00 ft
1	13.75	7	42.20
2	22.20	8	43.80
3	28.50	9	44.80
4	33.50	10	45.15
5	37.10		

PLOT: (a) Using the above data, a **distance-time curve** on a base line located as shown on the sketch below. Scales: Distance, 1 in. = 6 ft. Time, 1 in. = 1 sec.

(b) A **velocity-time curve**. Obtain points for each 1-sec. interval. Double the ordinates.

(c) An **acceleration-time curve**. Obtain points at intervals of 1 sec. Double the ordinates. Use A—B as base line for this curve. Calculate velocity and acceleration scales, expressing them in foot-second units. Construct graphical scales.



PROBLEM 4

INSTANT CENTERS

1. Draw the four mechanisms illustrated in Plate 4 to full scale.
2. Locate all the instant centers.
3. In Figs. A and B, assume that point *P* has a velocity of 12 ft per sec. Represent this velocity by a vector $1\frac{1}{2}$ in. long and find graphically the instantaneous velocities of points *Q* and *R*.
4. In Figs. C and D, assume that the link *a* is rotating at a rate of 75 rpm and find graphically the instantaneous angular velocity of links *b* and *c* in both mechanisms, in rpm. Represent the angular velocity of link *a* by a vector $1\frac{1}{2}$ in. long. Indicate the sense of rotation of *b* and *c*.

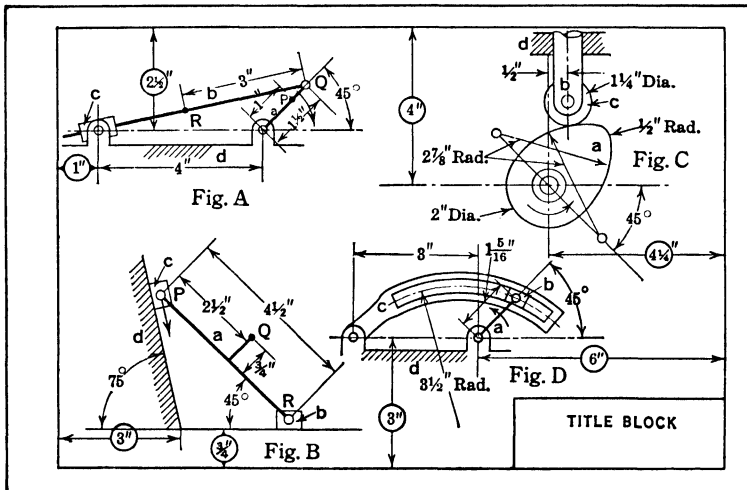


PLATE 4

PROBLEM 5**INSTANT CENTERS, LINEAR AND ANGULAR VELOCITIES**

(Refer to Plate 5.)

The skeleton drawing represents a six-link quick-return mechanism. Lay out the mechanism according to the dimensions shown on the drawing.

1. Locate all the **instant centers**.

2. Determine graphically the **linear velocities** of points *P*, *Q*, *R*, and *S*, for the position of the mechanism shown in the figure when the driving link *a* makes 15 rpm in a counterclockwise direction. Tabulate the numerical values on the drawing. Velocity scale: 1 in. = 20 ft per min.

3. Representing the angular velocity of *a* by a line 1 in. long, determine graphically the corresponding **angular velocities** of links *b*, *c*, and *d*. Tabulate the numerical values on the drawing.

Scale: 4 in. = 1 ft.

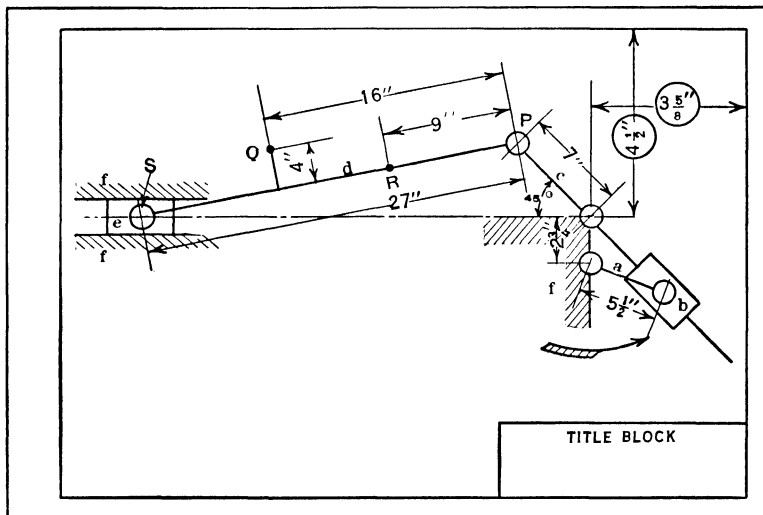


PLATE 5

PROBLEM 6

DIAGRAMS FOR SLIDER-CRANK MECHANISM

Illustrated in Plate 6 is a skeleton diagram of the **slider-crank mechanism** used to transform rectilinear motion into rotary motion. The crosshead *c* has a stroke of 24 in. The connecting rod *b* is five times as long as the crank *a*. The crank turns at a constant speed of 160 rpm in a clockwise direction.

Construct the following diagrams for a complete revolution of the crank *a*; obtain points at 15° intervals.

1. **Polar velocity diagram** showing crosshead velocities on corresponding crank positions.

2. **Velocity-displacement diagram** showing crosshead velocities on positions of the crosshead pin.

3. **Velocity-time curve**, on a base line 9 in. long, located as shown, representing the time of one revolution of the crank *a*.

4. **Acceleration-displacement diagram**, using same base line as 2.

5. **Acceleration-time diagram**, base line same as 3.

Calculate and construct graphical scales for velocity and acceleration. Show the computations on the drawing.

Scale: 2 in. = 1 ft.

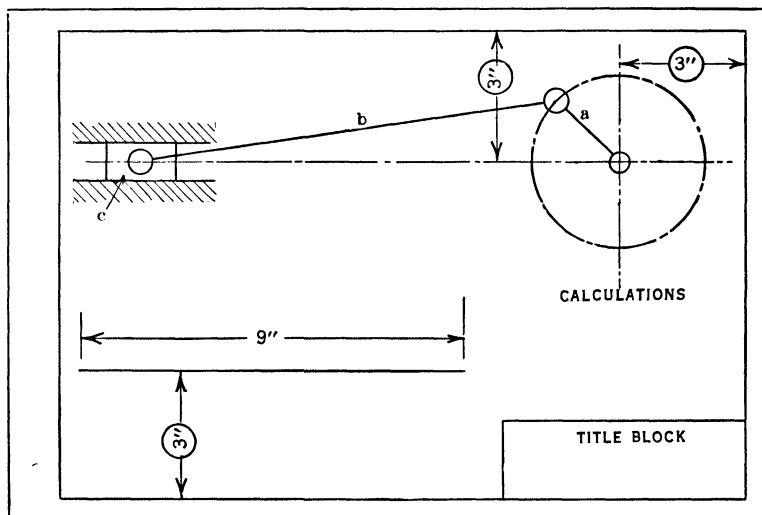


PLATE 6

PROBLEM 7

DISK CAMS WITH ROLLER AND FLAT-FACED FOLLOWERS

(See Plate 7.)

Design two disk cams, each imparting the same motion to its follower. Locate follower axes as shown, or offset them $\frac{1}{2}$ in. to the left as instructed. One follower is flat-faced; the other is provided with a roller. The outward motion takes place during a cam displacement of 135° . The follower then rests for 45° and returns during the next half-revolution. For the outward motion the velocity is constant except for periods at the beginning and end, each occupying one-twelfth of a revolution, during which acceleration and deceleration are uniform. The return motion is to be simple harmonic. The cams rotate clockwise.

Determine points on the profiles at intervals of 15° .

DATA:

Diameter of cam shafts $1\frac{3}{16}$ in. Diameter of base circles $2\frac{3}{4}$ in.

Diameter of cam hubs $2\frac{3}{8}$ in. Diameter of roller $1\frac{1}{2}$ in.

Keyways $\frac{1}{4} \times \frac{1}{8}$ in. Follower Displacement $1\frac{3}{4}$ in.

Scale: Full size.

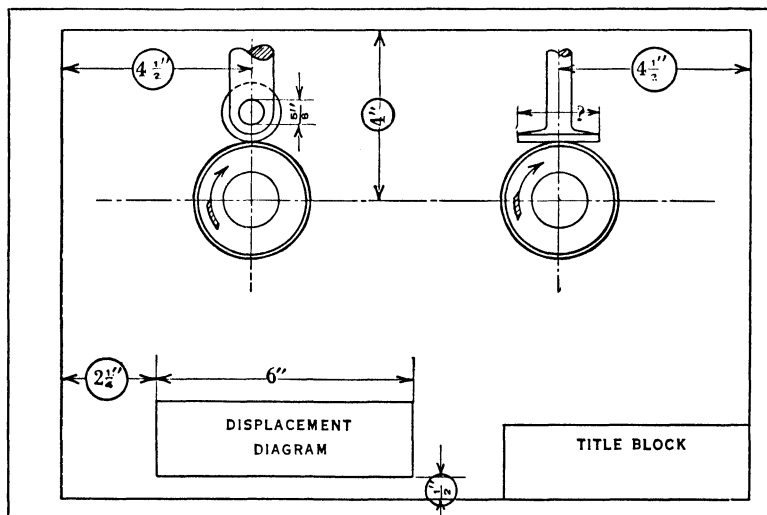


PLATE 7

PROBLEM 8**DISK CAM WITH PIVOTED FLAT-FACED FOLLOWER**

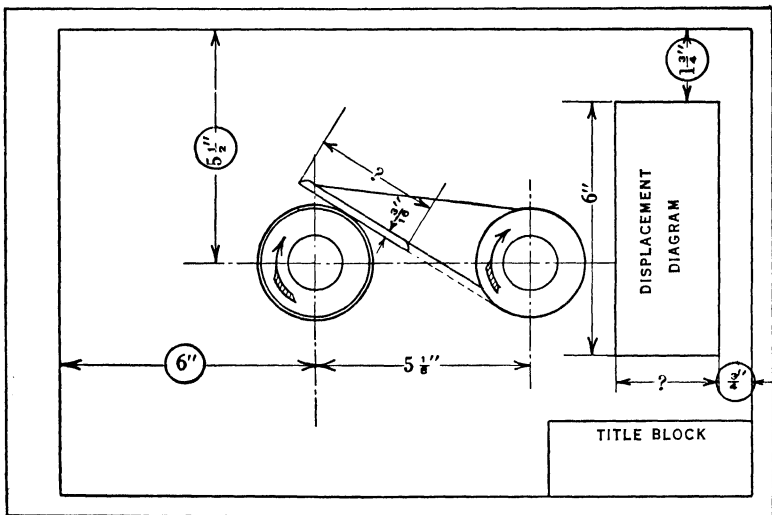
(See Plate 8.)

The follower has an angular displacement of 30° . It moves outward during 12 of the 24 equal time periods required for one revolution of the cam and returns to the initial position during the following 8 time periods. Both motions are simple harmonic. The cam rotates clockwise. The follower face when produced is tangent to the hub.

Obtain points on the cam profile at 15° intervals and determine the necessary length of the contact surface of the follower.

DATA:

Cam shaft diameter	$1\frac{3}{16}$ in.	Base circle diameter	$2\frac{3}{4}$ in.
Cam hub diameter	$2\frac{3}{8}$ in.	Follower shaft diameter	$1\frac{3}{16}$ in.
Keyways	$\frac{1}{4} \times \frac{1}{8}$ in.	Follower hub diameter	$2\frac{3}{8}$ in.

Scale: Full size.

PROBLEM 9

POSITIVE-RETURN CAM MECHANISM

(See Plate 9.)

Design a cam mechanism with a disk cam arranged for positive return to the follower. Starting from its lowest position, the follower rises with constant acceleration and deceleration during 180° of cam displacement, the acceleration-deceleration ratio being 2:1.

DATA:

Diameter of cam shaft	$1\frac{1}{8}$ in.	Diameter of rollers	$1\frac{1}{4}$ in.
Diameter of cam hub	$2\frac{1}{4}$ in.	Diameter of roller pins	$\frac{1}{2}$ in.
Diameter of base circle	3 in.	Keyway	$\frac{1}{4} \times \frac{1}{8}$ in.
		Lift	2 in.

The cam rotates counterclockwise.

Obtain points on the profiles at intervals of 15° .

Scale: Full size.

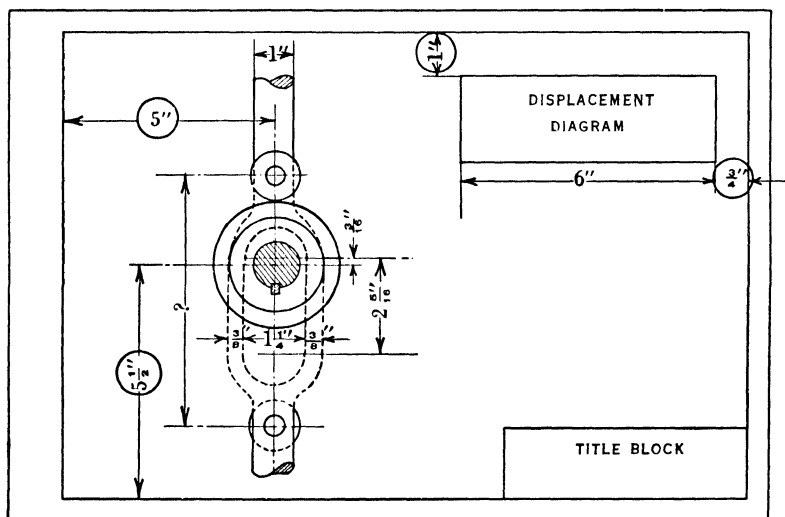


PLATE 9

PROBLEM 10**CYLINDER CAM WITH RECIPROCATING FOLLOWER**

(See Plate 10.)

Design a cylinder cam imparting motion to its roller follower which moves in a straight line parallel to the cam axis.

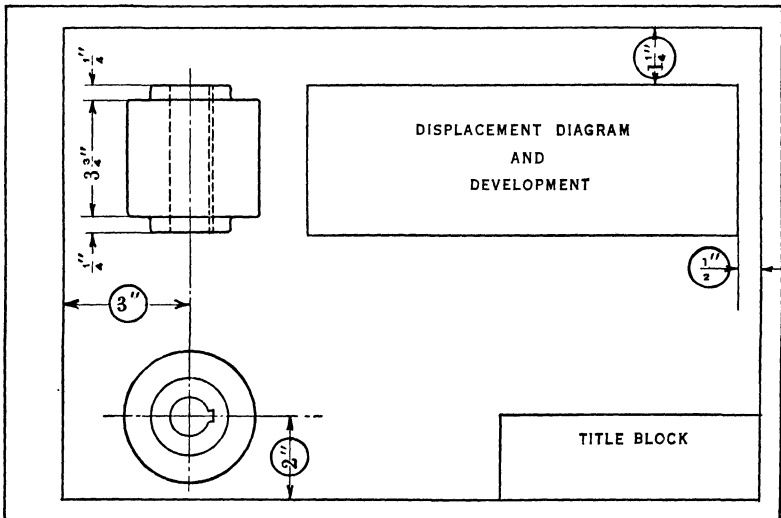
The upward motion is accomplished while the cam rotates through 180° with constant velocity except during the first and last 30° where it has uniformly accelerated and decelerated motion. The follower rests at the top position while the cam rotates through 45° . The return motion, taking place in 135° , is uniformly accelerated and decelerated during equal time periods.

DATA:

Follower displacement	$2\frac{1}{4}$ in.	Length of cam hub	$4\frac{1}{4}$ in.
Diameter of cam hub	$1\frac{7}{8}$ in.	Keyway	$\frac{1}{4} \times \frac{1}{8}$ in.
Bore of hub	$\frac{1}{16}$ in.	Depth of groove	$\frac{3}{8}$ in.
Outside diameter of cam	$3\frac{3}{16}$ in.	Diameter of roller	$\frac{1}{2}$ in.

Obtain points at intervals of 15° and draw (a) a displacement diagram and development of cam surface, (b) a plan view and elevation of the cam.

Scale: Full size.

**PLATE 10**

PROBLEM 11

INVOLUTE GEARS

(See Plate 11.)

Draw a pair of involute gears and a rack having teeth of 2 diametral pitch, zero backlash, and standard $14\frac{1}{2}^\circ$ involute proportions to meet the following requirements:

Gear: Pitch diameter 12 in., keyway $\frac{1}{2} \times \frac{1}{4}$ in., bore of hub $2\frac{3}{16}$ in.

Pinion: Pitch diameter 6 in., keyway $\frac{3}{8} \times \frac{3}{16}$ in., bore of hub $1\frac{1}{8}$ in.

1. Use the Grant odontograph method for drawing the teeth.
2. Construct the exact involute for one tooth of each gear as a check on the accuracy of the odontograph method.
3. Examine the tooth profiles for interference and, by cross-hatching, indicate the interfering portions.
4. Indicate the centers and radii used in laying out the teeth.
5. Determine the angle of action, angle of approach, and angle of recess for each gear, giving numerical values on drawing.
6. Indicate the path of the point of contact.

Scale: Full size.

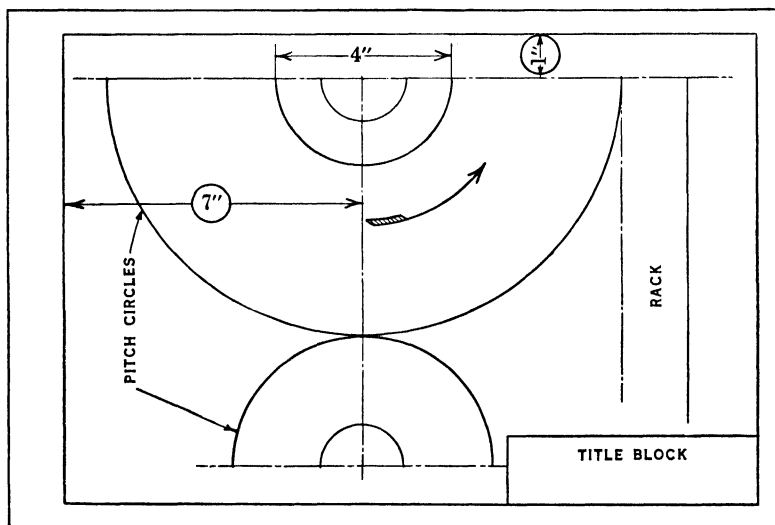


PLATE 11

PROBLEM 12**INVOLUTE BEVEL GEARS**

Make a drawing in section of a pair of involute bevel gears as illustrated below. The diametral pitch is 2. The teeth have standard $14\frac{1}{2}^\circ$ involute proportions.

Dimension the drawing and tabulate the following information: pitch diameter, edge angle, number of teeth, diameters of blanks, angle between shafts, diametral pitch, and face angle.

DATA:

Angular velocity ratio 10:7

Bore of pinion hub $1\frac{3}{8}$ in.

Pitch diameter of gear 10 in.

Keyways $\frac{3}{8} \times \frac{3}{16}$ in.

Bore of gear hub $1\frac{1}{2}$ in.

Scale: Full size.

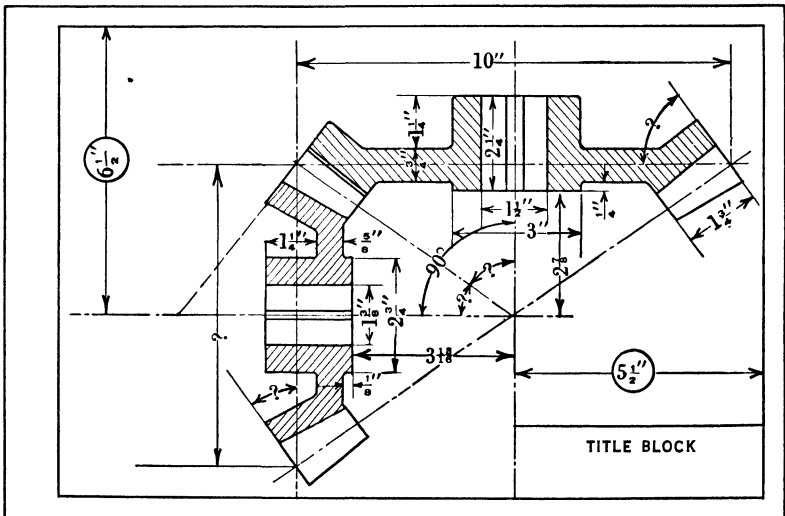


PLATE 12

PROBLEM 13

CONE PULLEYS

By the Burmester method, design a pair of cone pulleys, each with four steps, to connect, by means of an open belt, parallel shafts 30 in. center to center. The driver turns at 255 rpm, and the speed ratios of driver to driven are 3:1, 2:1, 1:1, and 1:2.5. The maximum belt speed is 1200 ft per min. Each step is $2\frac{1}{4}$ in. wide with $\frac{1}{16}$ -in. crown. The rim thickness is $\frac{1}{2}$ in. and the shafts are $1\frac{5}{8}$ -in. diameter with keyways $\frac{3}{8}$ by $\frac{3}{16}$ in. The cones are attached to the hubs by four ribs, each $\frac{1}{2}$ in. thick. The hubs are 5 in. long and $3\frac{3}{4}$ in. outside diameter, located centrally along the length of the cone pulley.

The pulley diameters found by the Burmester construction are to be rounded off to the nearest $\frac{1}{8}$ in.

Calculate the length of belt for **each pair** of steps by using formula (13-3) in the text. Tabulate these lengths on the drawing.

Scale: 3 in. = 1 ft.

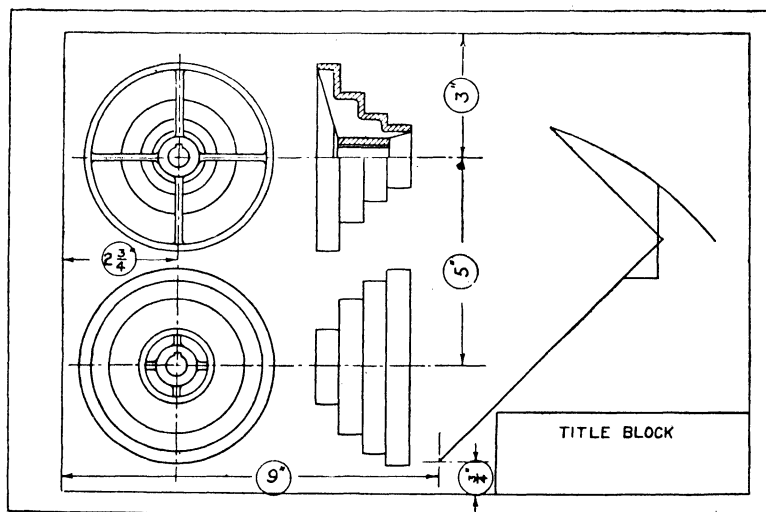


PLATE 13

PROBLEM 14

GENEVA STOP MOTION

Design a Geneva motion to give the driven member one-sixth of a revolution per revolution of the driving member, as shown in Plate 14.

The driving pin at *C* is 1 in. in diameter and the distance *BC* is 4 in. The driving shaft at *B* is $1\frac{1}{2}$ in. in diameter, and the driven shaft at *A* is $1\frac{3}{4}$ in. in diameter. The disk carrying the driving pin is $\frac{3}{4}$ in. thick and has a diameter of $9\frac{1}{2}$ in. The slotted member is $\frac{3}{4}$ in. thick.

Assume that the driving shaft rotates at a constant speed of 200 rpm and plot an angular velocity-time curve of the driven member in radians per second. Obtain the required angular velocities graphically for each 15° of motion of the driver, using a vector 4 in. long to represent the angular velocity of this member.

Construct a graphical velocity scale on the velocity diagram, recording all calculations.

Scale: 6 in. = 1 ft.

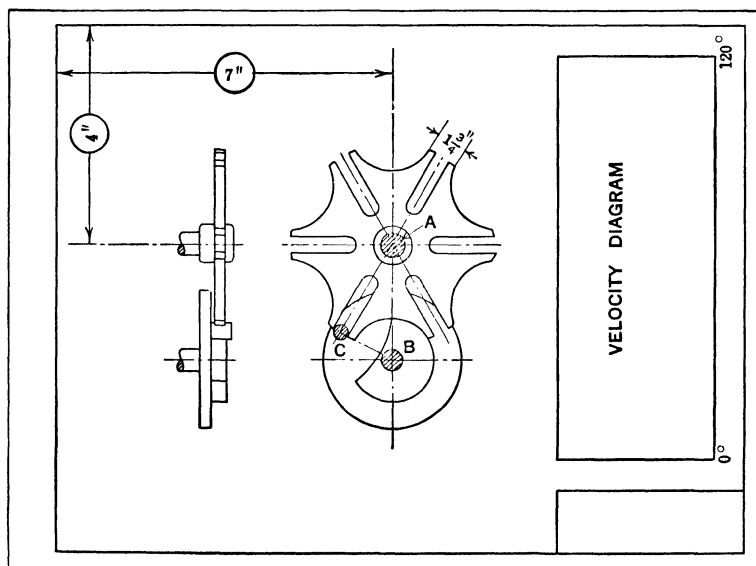


PLATE 14

PROBLEM 15

THE PANTAGRAPH

1. Draw the two forms of the pantograph shown in Plate 15.
 2. In the upper figure, move the tracing point P to other positions, Q, R, S, T, U , located at uniform intervals around the circumference of a circle $1\frac{1}{2}$ in. in diameter. Find the corresponding positions of the copying point V .
 3. In the lower figure, move the tracing point A to points B, C, D, E, F , located on a triangle of the form shown in the plate. Find the corresponding positions of the copying point G .
 4. For both pantographs, find the ratio of magnification.
- Scale:** Full size.

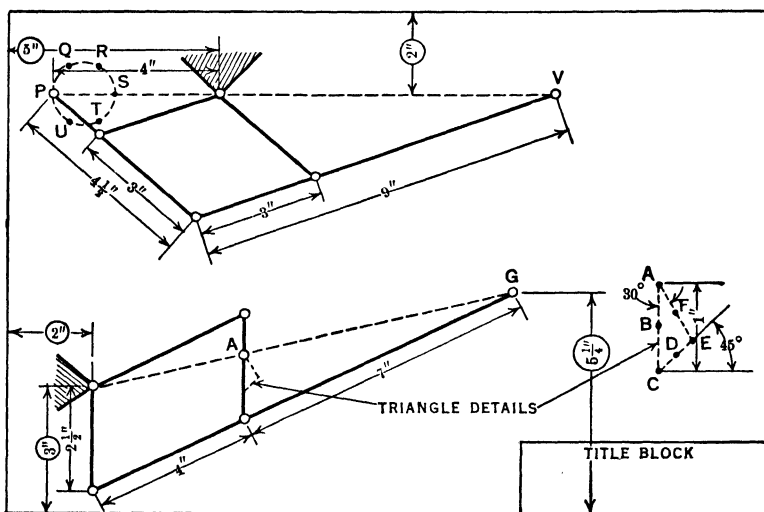


PLATE 15

TABLES

National Coarse Thread Bolts

Diam of Bolt Inches	Threads per Inch Number	Area of Bolt Body Square Inches	Diam of Root of Thread Inches	Area at Root of Thread Square Inches
$\frac{1}{4}$	20	0.0491	0.185	0.027
$\frac{5}{16}$	18	0.0767	0.240	0.045
$\frac{3}{8}$	16	0.1104	0.294	0.068
$\frac{7}{16}$	14	0.1503	0.344	0.093
$\frac{1}{2}$	13	0.196	0.400	0.126
$\frac{9}{16}$	12	0.248	0.454	0.162
$\frac{5}{8}$	11	0.307	0.507	0.202
$\frac{3}{4}$	10	0.442	0.620	0.302
$\frac{7}{8}$	9	0.601	0.731	0.420
1	8	0.785	0.837	0.550
$1\frac{1}{8}$	7	0.994	0.940	0.692
$1\frac{1}{4}$	7	1.227	1.065	0.890
$1\frac{3}{8}$	6	1.483	1.160	1.057
$1\frac{1}{2}$	6	1.766	1.284	1.293
$1\frac{5}{8}$	$5\frac{1}{2}$	2.074	1.389	1.510
$1\frac{3}{4}$	5	2.405	1.491	1.741
$1\frac{7}{8}$	5	2.761	1.610	2.050
2	$4\frac{1}{2}$	3.140	1.712	2.300
$2\frac{1}{4}$	$4\frac{1}{2}$	3.980	1.962	3.030
$2\frac{1}{2}$	4	4.910	2.176	3.719
$2\frac{3}{4}$	4	5.940	2.426	4.620
3	$3\frac{1}{2}$	7.070	2.629	5.428

Natural Trigonometric Functions

De- grees	SINES							Co- sines
	0'	10'	20'	30'	40'	50'	60'	
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01454	0.01745	89
1	0.01745	0.02036	0.02327	0.02618	0.02908	0.03199	0.03490	88
2	0.03490	0.03781	0.04071	0.04362	0.04653	0.04943	0.05234	87
3	0.05234	0.05524	0.05814	0.06105	0.06395	0.06685	0.06976	86
4	0.06976	0.07266	0.07556	0.07846	0.08136	0.08426	0.08716	85
5	0.08716	0.09005	0.09295	0.09585	0.09874	0.10164	0.10453	84
6	0.10453	0.10742	0.11031	0.11320	0.11609	0.11898	0.12187	83
7	0.12187	0.12476	0.12764	0.13053	0.13341	0.13629	0.13917	82
8	0.13917	0.14205	0.14493	0.14781	0.15069	0.15356	0.15643	81
9	0.15643	0.15931	0.16218	0.16505	0.16792	0.17078	0.17365	80
10	0.17365	0.17651	0.17937	0.18224	0.18509	0.18795	0.19081	79
11	0.19081	0.19366	0.19652	0.19937	0.20222	0.20507	0.20791	78
12	0.20791	0.21076	0.21360	0.21644	0.21928	0.22212	0.22495	77
13	0.22495	0.22778	0.23062	0.23345	0.23627	0.23910	0.24192	76
14	0.24192	0.24474	0.24756	0.25038	0.25320	0.25601	0.25882	75
15	0.25882	0.26163	0.26443	0.26724	0.27004	0.27284	0.27564	74
16	0.27564	0.27843	0.28123	0.28402	0.28680	0.28959	0.29237	73
17	0.29237	0.29515	0.29793	0.30071	0.30348	0.30625	0.30902	72
18	0.30902	0.31178	0.31454	0.31730	0.32006	0.32282	0.32557	71
19	0.32557	0.32832	0.33106	0.33381	0.33655	0.33929	0.34202	70
20	0.34202	0.34475	0.34748	0.35021	0.35293	0.35565	0.35837	69
21	0.35837	0.36108	0.36379	0.36650	0.36921	0.37191	0.37461	68
22	0.37461	0.37730	0.37999	0.38268	0.38537	0.38805	0.39073	67
23	0.39073	0.39341	0.39608	0.39875	0.40142	0.40408	0.40674	66
24	0.40674	0.40939	0.41204	0.41469	0.41734	0.41998	0.42262	65
25	0.42262	0.42525	0.42788	0.43051	0.43313	0.43575	0.43837	64
26	0.43837	0.44098	0.44359	0.44620	0.44880	0.45140	0.45399	63
27	0.45399	0.45658	0.45917	0.46175	0.46433	0.46690	0.46947	62
28	0.46947	0.47204	0.47460	0.47716	0.47971	0.48226	0.48481	61
29	0.48481	0.48735	0.48989	0.49242	0.49495	0.49748	0.50000	60
30	0.50000	0.50252	0.50503	0.50754	0.51004	0.51254	0.51504	59
31	0.51504	0.51753	0.52002	0.52250	0.52498	0.52745	0.52992	58
32	0.52992	0.53238	0.53484	0.53730	0.53975	0.54220	0.54464	57
33	0.54464	0.54708	0.54951	0.55194	0.55436	0.55678	0.55919	56
34	0.55919	0.56160	0.56401	0.56641	0.56880	0.57119	0.57358	55
35	0.57358	0.57596	0.57833	0.58070	0.58307	0.58543	0.58779	54
36	0.58779	0.59014	0.59248	0.59482	0.59716	0.59949	0.60182	53
37	0.60182	0.60414	0.60645	0.60876	0.61107	0.61337	0.61566	52
38	0.61566	0.61795	0.62024	0.62251	0.62479	0.62706	0.62932	51
39	0.62932	0.63158	0.63383	0.63608	0.63832	0.64056	0.64279	50
40	0.64279	0.64501	0.64723	0.64945	0.65166	0.65386	0.65606	49
41	0.65606	0.65825	0.66044	0.66262	0.66480	0.66697	0.66913	48
42	0.66913	0.67129	0.67344	0.67559	0.67773	0.67987	0.68200	47
43	0.68200	0.68412	0.68624	0.68835	0.69046	0.69256	0.69466	46
44	0.69466	0.69675	0.69883	0.70091	0.70298	0.70505	0.70711	45
Sines	60'	50'	40'	30'	20'	10'	0'	De- grees
	COSINES							

Natural Trigonometric Functions—(Continued)

De- grees	COSINES							Sines
	0'	10'	20'	30'	40'	50'	60'	
0	1.00000	1.00000	0.99998	0.99996	0.99993	0.99989	0.99985	89
1	0.99985	0.99979	0.99973	0.99966	0.99958	0.99949	0.99939	88
2	0.99939	0.99929	0.99917	0.99905	0.99892	0.99878	0.99863	87
3	0.99863	0.99847	0.99831	0.99813	0.99795	0.99776	0.99756	86
4	0.99756	0.99736	0.99714	0.99692	0.99668	0.99644	0.99619	85
5	0.99619	0.99594	0.99567	0.99540	0.99511	0.99482	0.99452	84
6	0.99452	0.99421	0.99390	0.99357	0.99324	0.99290	0.99255	83
7	0.99255	0.99219	0.99182	0.99144	0.99106	0.99067	0.99027	82
8	0.99027	0.98986	0.98944	0.98902	0.98858	0.98814	0.98769	81
9	0.98769	0.98723	0.98676	0.98629	0.98580	0.98531	0.98481	80
10	0.98481	0.98430	0.98378	0.98325	0.98272	0.98218	0.98163	79
11	0.98163	0.98107	0.98050	0.97992	0.97934	0.97875	0.97815	78
12	0.97815	0.97754	0.97692	0.97630	0.97566	0.97502	0.97437	77
13	0.97437	0.97371	0.97304	0.97237	0.97169	0.97100	0.97030	76
14	0.97030	0.96959	0.96887	0.96815	0.96742	0.96667	0.96593	75
15	0.96593	0.96517	0.96440	0.96363	0.96285	0.96206	0.96126	74
16	0.96126	0.96046	0.95964	0.95882	0.95799	0.95715	0.95630	73
17	0.95630	0.95545	0.95459	0.95372	0.95284	0.95195	0.95106	72
18	0.95106	0.95015	0.94924	0.94832	0.94740	0.94646	0.94552	71
19	0.94552	0.94457	0.94361	0.94264	0.94167	0.94068	0.93969	70
20	0.93969	0.93869	0.93769	0.93667	0.93565	0.93462	0.93358	69
21	0.93358	0.93253	0.93148	0.93042	0.92935	0.92827	0.92718	68
22	0.92718	0.92609	0.92499	0.92388	0.92276	0.92164	0.92050	67
23	0.92050	0.91936	0.91822	0.91706	0.91590	0.91472	0.91355	66
24	0.91355	0.91236	0.91116	0.90996	0.90875	0.90753	0.90631	65
25	0.90631	0.90507	0.90383	0.90259	0.90133	0.90007	0.89879	64
26	0.89879	0.89752	0.89623	0.89493	0.89363	0.89232	0.89101	63
27	0.89101	0.88968	0.88835	0.88701	0.88566	0.88431	0.88295	62
28	0.88295	0.88158	0.88020	0.87882	0.87743	0.87603	0.87462	61
29	0.87462	0.87321	0.87178	0.87036	0.86892	0.86748	0.86603	60
30	0.86603	0.86457	0.86310	0.86163	0.86015	0.85866	0.85717	59
31	0.85717	0.85567	0.85416	0.85264	0.85112	0.84959	0.84805	58
32	0.84805	0.84650	0.84495	0.84339	0.84182	0.84025	0.83867	57
33	0.83867	0.83708	0.83549	0.83389	0.83228	0.83066	0.82904	56
34	0.82904	0.82741	0.82577	0.82413	0.82248	0.82082	0.81915	55
35	0.81915	0.81748	0.81580	0.81412	0.81242	0.81072	0.80902	54
36	0.80902	0.80730	0.80558	0.80386	0.80212	0.80038	0.79864	53
37	0.79864	0.79688	0.79512	0.79335	0.79158	0.78980	0.78801	52
38	0.78801	0.78622	0.78442	0.78261	0.78079	0.77897	0.77715	51
39	0.77715	0.77531	0.77347	0.77162	0.76977	0.76791	0.76604	50
40	0.76604	0.76417	0.76229	0.76041	0.75851	0.75661	0.75471	49
41	0.75471	0.75280	0.75088	0.74896	0.74703	0.74509	0.74314	48
42	0.74314	0.74120	0.73924	0.73728	0.73531	0.73333	0.73135	47
43	0.73135	0.72937	0.72737	0.72537	0.72337	0.72136	0.71934	46
44	0.71934	0.71732	0.71529	0.71325	0.71121	0.70916	0.70711	45
Co- sines	60'	50'	40'	30'	20'	10'	0'	De- grees
SINES								

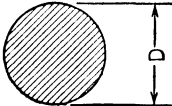
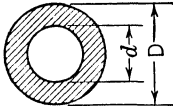
Logarithms of Numbers

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10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

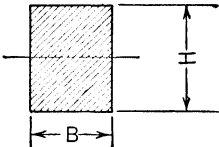
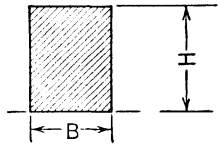
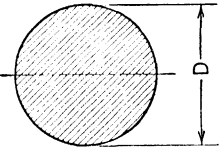
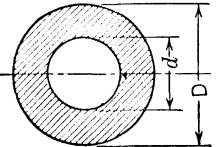
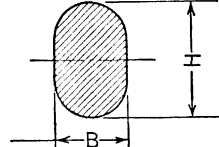
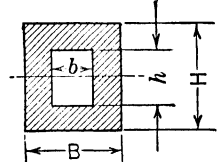
Logarithms of Numbers—(Continued)

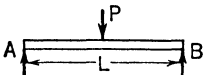
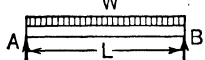
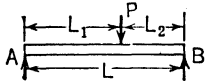
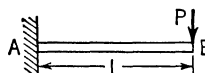
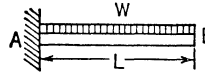
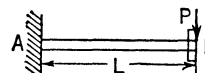
N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Properties of Sections

Type of Section	Polar—Use for Torsion Only	
	Moment of Inertia I_p or J	Section Modulus I_p/c
	$\frac{\pi D^4}{32}$	$\frac{\pi D^3}{16}$
	$\frac{\pi (D^4 - d^4)}{32}$	$\frac{\pi (D^4 - d^4)}{16D}$

Properties of Sections. Rectangular—Use for Bending

Type of Section	Moment of Inertia I	Section Modulus $Z = I/c$
	$\frac{BH^3}{12}$ About C of G	$\frac{BH^2}{6}$
	$\frac{BH^3}{3}$ About base	
	$\frac{\pi D^4}{64}$	$\frac{\pi D^3}{32}$
	$\frac{\pi (D^4 - d^4)}{64}$	$\frac{\pi (D^4 - d^4)}{32D}$
	$\frac{\pi BH^3}{64}$	$\frac{\pi BH^2}{32}$
	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$

Beam Loadings	Maximum Bending Moment M	Maximum Deflection Δ
Simple Beam—Concentrated Load at Middle 	At middle $\frac{PL}{4}$	At middle $\frac{PL^3}{48EI}$
Simple Beam—Uniform Load 	At middle $\frac{WL}{8}$	At middle $\frac{5WL^3}{384EI}$
Simple Beam—Concentrated Load 	At load $\frac{PL_1L_2}{L}$	
Cantilever Beam—Concentrated Load 	At A PL	At B $\frac{PL^3}{3EI}$
Cantilever Beam—Uniform Load 	At A $\frac{WL}{2}$	At B $\frac{WL^3}{8EI}$
Cantilever Beam—Guided One End Fixed 	At A or B $\frac{PL}{2}$	At B $\frac{Pl^3}{12EI}$

The bending moment at *any* transverse section in *any* beam is the algebraic summation of the moments of the *external* forces acting on the beam to the left or right of the section.

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